PREDICTION OF VAPOR-LIQUID EQUILIBRIUM FROM TERNARY LIQUID-LIQUID EQUILIBRIUM DATA BY MEANS OF LOCAL COMPOSITION EQUATIONS

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ABSTRACT

Ternary vapor-liquid equilibrium can be predicted by use of the extended UNIQUAC and LEMF equations whose energy parameters are obtained from the correlation of the corresponding liquid-liquid equilibrium data. Three objective functions are used in parameter estimation, and objective functions are usually preferred defined in terms of liquid-phase mole fractions than in terms of activities. The extended UNIQUAC equation shows better performance than the LEMF equation. Prediction of binary vapor-liquid equilibrium from ternary liquid-liquid equilibrium data confirms the same trends as observed in ternary vapor-liquid equilibrium predictions.

INTRODUCTION

The correlation of the ternary liquid-liquid equilibrium (LLE) data with an appropriate expression for the activity coefficient leads to the evaluation of the parameters for the corresponding binary systems. These parameters can be used in the calculation of the vapor-liquid equilibrium (VLE) relation of the same systems or in the prediction of ternary (or multicomponent) VLE relation, combined with parameters obtained from VLE for other binary systems.

Joy and Kyle [l] used the NRTL equation [2] in the correlation of LLE data for four ternary systems and the parameters obtained were applied to the prediction of binary and ternary VLE. Their approach worked well for three systems and failed for the fourth one. By means of the Black [3] and NRTL equations, Newsham and Vahdat [4] computed ternary VLE from LLE data for three systems including water and two alcohols with reasonable accuracy. Yee et al. [5] presented a more comprehensive study of the prediction of binary and ternary VLE from LLE data for eight systems using

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Ternary systems studied

the NRTL, LEMF [6] and UNIQUAC [7] equations and concluded that the LEMF equation appears to give somewhat better results for both binary and ternary VLE predictions.

This paper presents another study of the prediction of binary and ternary VLE from ternary LLE data for seven type I systems which include one partially miscible binary and two completely miscible binaries for each

Fig. 1. Classification of ternary solubility envelopes.

ternary. Table 1 gives LLE and VLE data sources for the seven ternary systems, which are arbitrarily classified into three types. Figure 1 shows schematically these three types: type IA, with a binary solubility gap in the range 0.05-0.95 mole fraction; type IB, with a solubility gap whose one point remains within the afore-mentioned range and the other point outside this composition range; type IC, with a large solubility gap outside the same range. Two local composition equations, LEMF and extended UNIQUAC [g], are used to obtain the proper binary parameters in evaluating binary parameters from ternary LLE data, since several sets of binary parameter could be obtained from different objective functions in the regression procedure.

ACTIVITY COEFFICIENT EQUATIONS

The LEMF equation gives the activity coefficient of any component in a multicomponent system by

$$
\ln \gamma_i = \frac{\sum_j \tau_{ji} G_{ji} x_j}{\sum_k G_{ki} x_k} + \sum_j \frac{x_j G_{ij}}{\sum_k G_{kj} x_k} \left[\tau_{ij} - \frac{\sum_k x_i \tau_{rj} G_{rj}}{\sum_k G_{kj} x_k} \right]
$$
(1)

where x is the liquid-phase mole fraction. τ_{ii} and G_{ii} are related to the binary parameter a_{ii} :

$$
\tau_{ji} = a_{ji}/T \tag{2}
$$

$$
G_{ji} = \exp(\tau_{ji})
$$
 (3)

The extended UNIQUAC equation expresses $\ln \gamma_i$ by

$$
\ln \gamma_i = \ln \left(\frac{\phi'_i}{x_i} \right) + 1 - \frac{\phi'_i}{x_i} - \left(\frac{Z}{2} \right) q_i \left[\ln \left(\frac{\phi_i}{\theta_i} \right) + 1 - \frac{\phi_i}{\theta_i} \right] - q_i^* \ln \left(\sum_j \theta_j \tau_{ji} \right) + q_i \sum_j \left(\frac{q_j^*}{q_j} \right) \theta_j - q_i \sum_j \frac{\left(q_j^* / q_j \right) \theta_j \tau_{ij}}{\sum_k \theta_k \tau_{kj}}
$$
(4)

where Z is the coordination number (taken as 10). ϕ' , ϕ , θ and τ are expressed by

$$
\phi_i' = x_i r_i^{2/3} / \sum_i x_j r_j^{2/3}
$$
 (5)

$$
\phi_i = x_i r_i / \sum_i x_j r_j \tag{6}
$$

$$
\theta_i = x_i q_i / \sum_j x_j q_j \tag{7}
$$

$$
\tau_{ji} = \exp(-a_{ji}/T) \tag{8}
$$

r, *q* and q^* are the pure-component molecular constants. The values of *r* and q were taken from Prausnitz et al. [9], and those of q^* were empirically taken as follows: 0.96 for water; 0.95 for methanol; 0.92 for ethanol; 0.89 for propanols; 0.88 for butanols; $q^* = q^{0.1}$ for other components studied in this work [8].

COMPUTATIONAL PROCEDURE

All experimental ternary tie lines were checked by use of Hand's plot [10] and Othmer-Tobias's plot [11]. Tie lines deviating greatly from these plots were rejected. Various objective functions have been proposed for the evaluation of the binary parameters from the ternary LLE data. We used the following three objective functions as described in the supplement to ref. 12 (eqn. (9) in terms of activities; eqns. (10) and (11) in terms of mole fractions):

$$
F_1 = \sum_{k} \sum_{i} \left[\frac{a_{ik}^{\text{I}} - a_{ik}^{\text{II}}}{a_{ik}^{\text{I}} + a_{ik}^{\text{II}}} \right]^2 + Q \sum_{n} P_n^2 \tag{9}
$$

where $i = 1, 2, \ldots$ N (components), $k = 1, 2, \ldots$ M (tie lines), $P_n =$ parameters value in Kelvin, a_{ik}^* = activity given by $x_{ik}^i \gamma_{ik}^i$, x_{ik}^i is the experimental mole fraction of component *i* in phase I at tie line k . The calculated activity coefficients depend on the experimental mole fractions and the parameters. Q is a penalty factor set as 10^{-6} .

$$
F_2 = \sum_{k} \min \sum_{i} \sum_{j} \left[x_{ijk} - \hat{x}_{ijk} \right]^2 + Q \sum_{n} P_n^2 \tag{10}
$$

where $i = 1, 2, ..., N$ (components), $j = I$, II (phases), $k = 1, 2, ..., M$ (tie lines), x_{ijk} is the mole fraction of component *i* in phase *j* at tie line *k*. The calculated mole fractions \hat{x}_{ijk} depend on the parameters. Q is taken as 10^{-10} in eqns. (10) and (11).

$$
F_3 = \sum_{k} \min \sum_{i} \sum_{j} \left[x_{ijk} - \hat{x}_{ijk} \right]^2 + Q \sum_{n} P_n^2 + \left[\ln \left(\frac{a_{2,\infty}^{\text{I}}}{a_{2,\infty}^{\text{II}}} K_{\text{D2},\infty} \right) \right]^2 \tag{11}
$$

 ∞ = infinite dilution

 $K_{D2,\infty}$ = calculated limiting distribution coefficient defined by

$$
\lim_{\substack{x \to 0 \\ x^{\text{II}} \to 0}} K_{\text{D2}}, \text{ where } K_{\text{D2}} = x_2^{\text{I}} / x_2^{\text{II}} = \gamma_2^{\text{II}} / \gamma_2^{\text{I}} \tag{12}
$$

 $a_{2,\infty}^{\text{I}}/a_{2,\infty}^{\text{II}}$ is calculated using the parameters evaluated from objective function (9) and $K_{D2,\infty}$ is derived from the parameters obtained by objective function (11).

Both the extended UNIQUAC and LEMF equations have two binary parameters per binary mixture. For a ternary mixture these equations have generally six parameters. This means that we may fit a total of six binary parameters of the extended UNIQUAC (or LEMF) equation to a set of ternary LLE data. For a type I mixture, we can prefix the two parameters obtained from the mutual solubility data for the partially miscible system and then we may determine the remaining four parameters. Five methods were used to evaluate which method gives the most acceptable parameters to predict ternary or binary VLE: method I, objective function (9) and four parameters; method II, objective function (9) and six parameters; method III, objective function (10) and four parameters; method IV, objective function (10) and six parameters; method V, objective function (11) and six parameters. Our experiences showed that objective function (9) gave a set of similar parameters, regardless of the various starting values and, in the other methods, the initial estimates of the binary parameters lead to many sets of the final parameters obtained in regression. The parameters derived from objective function (9) are not always the suitable starting parameters for methods III-V. In such cases the binary parameters for the two completely miscible binaries, l-2 and 2-3, are estimated from the UNIFAC method [22,23] whenever the pertinent interaction parameters are available. The estimated UNIFAC binary parameters can be converted to the corresponding extended UNIQUAC or LEMF parameters as the starting values in regression using the infinite dilution activity coefficients only [24]. The initial values of the partially miscible binary $1-3$ are the parameters obtained from the binary mutual solubilities.

RESULTS AND DISCUSSION

Table 2 shows predicted results of ternary and binary VLE obtained by use of the extended UNIQUAC equation with either four or six binary parameters derived from the correlation of ternary LLE data, indicating that one may use six binary parameters for correlating ternary LLE data in order to obtain a better prediction of ternary or binary VLE and method V gives much more reasonable parameters than method I does whenever the convergence is obtained. VLE calculations included vapor-phase non-ideality. The fugacity coefficients were calculated by use of the volume-explicit virial equation of state, truncated after the second virial coefficients. The second virial coefficients were estimated from the correlation of Hayden and O'Connell [27] with the related parameters tabulated by Prausnitz et al. [9]. The Poynting correction was taken into account using pure-liquid molar volumes calculated by the modified Rackett equation [28]. Pure-component vapor pressures were calculated from the Antoine equation whose constants were taken from the literature [29-31]. Table 3 gives the extended UN-

 $A \neq I$ = method I whose objective function is eqn. (9) and four parameters are estimated.

 b II = method I whose objective function is eqn. (9) and six parameters are estimated.</sup>

 C III = method III whose objective function is eqn. (10) and four parameters are estimated.

 d IV = method IV whose objective function is eqn. (10) and six parameters are estimated.

 e V = method V whose objective function is eqn. (11) and six parameters are estimated.

 $f \Delta y_1 =$ deviation in vapor-phase mole fraction given by $\Delta y_1 = \sum_i^N |y_1(\exp) - y_1(\text{calc})|/N$.

 $\Delta P =$ deviation in pressure given by $\Delta P = \sum_{i=1}^{N} |P_i(\exp) - P_i(\text{calc})| / N$.

TABLE 3

System Temp. Prefixed parameters Parameters for binaries ($^{\circ}$ C) for binary 1-3 (K) 1-2 and 2-3 (K) Acetonitrile(1)- 45 a_{13} a_{31} a_{12} a_{21} a_{23} a_{13} 330.96 974.93 I a 9.95 80.44 6.10 6.74 $benzene(2)$ sulfur dioxide(3) III $^{\rm b}$ - 127.37 282.57 9.82 - 25.18 Water(1)- 70 307.92 421.38 I -10.34 -30.09 -0.73 23.11 ethanol(2)- $III \qquad -61.58 \qquad -343.42 \qquad -4.43 \qquad -737.49$ ethyl acetate(3) Benzene(1)- 25 1238.8 352.43 I 175.13 285.51 23.60 - 180.53
ethanol(2)- III 828.05 63.09 62.30 66.13 ethanol(2)- III 828.05 63.09 62.30 66.13 water(3)

Binary parameters obtained in fitting the extended UNIQUAC equation to ternary liquid-liquid equilibria

 $I =$ method I.

 b III = method III.</sup>

IQUAC parameters obtained with methods I and III. Table 4 presents predicted results of ternary or binary VLE obtained from the extended UNIQUAC and LEMF equations with six parameters, which are listed in Table 5. For the extended UNIQUAC equation, method IV is better than method II for the four systems, except for the water-methanol-ethyl acetate and water-ethanol-ethyl acetate systems. However, method V gives much improved results for these two systems and the acetonitrile-benzene-nheptane system. Only method II gives a set of the acceptable parameters for

Fig. 2. Predicted activity coefficients obtained from the extended UNIQUAC equation with method V for (a) acetonitrile(1)-benzene(2) and (b) benzene(1)- n -heptane(2) at 45 °C. Experimental (O) , ref. 13; calculated $($ —— $)$.

 $\label{eq:2} \frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^$

 $=$ II = method II.

 $V =$ method IV.

^a II = method II.

^b IV = method IV.

^c V = method V.

^d A = extended UNIQUAC equation.

^e B = LEMF equation. d A = extended UNIQUAC equation. $\mathsf{v} = \mathsf{method} \mathsf{v}.$

 $B = LEMF$ equation.

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Binary parameters obtained in fitting the extended UNIQUAC and LEMF equations to ternary liquid-liquid equilibria by use of six parameters

System			Method Eqn. Parameters (K)					
			a_{12}	a_{21}	a_{13}	a_{31}	a_{23}	a_{32}
Acetonitrile(1)- $benzene(2) -$ n -heptane(3)	\mathbf{I}	1 ^a	32.78	189.60	412.32	586.36	22.72	16.21
		2 ^b	68.00	63.69	273.19	308.34	-41.60	-56.44
	IV	$\mathbf{1}$	-105.93	294.03	344.85	967.31	-39.65	25.40
		$\overline{2}$	19.59	86.47	281.15	310.21	-22.45	-104.18
	V	1	4.15	281.22	323.82	985.80	3.45	84.66
		$\overline{2}$	100.02	89.49	280.73	310.96	-45.51	23.32
n -Pentane(1)- $benzene(2) -$ sulfur dioxide(3)	П	$\mathbf{1}$	76.81	46.41	466.68	407.35	18.22	-11.29
		$\overline{\mathbf{c}}$	20.67	15.37	254.36	185.18	-9.84	-20.06
	IV	$\mathbf{1}$	73.77	106.74	636.47	420.19	3.43	-28.75
		$\overline{2}$	324.34		76.79 256.60	204.26	147.89	116.64
	$\mathbf V$	$\mathbf 1$	77.23		106.96 636.58	420.14	3.26	-24.76
		$\mathbf{2}$	293.02	86.29	256.71	203.99	141.26	92.42
$Water(1) -$	\mathbf{I}	$\mathbf{1}$	-58.55	-54.95	327.06	222.83	77.84	65.15
$method(2)-$		$\overline{2}$	-6.66	42.69	131.52	404.32	13.08	10.37
ethyl acetate(3)	IV	$\mathbf{1}$	110.10	-554.85	500.91	373.61	-446.63	-36.57
		$\overline{2}$	-316.44	359.10	211.38	426.14	328.20	221.54
	$\mathbf V$	$\mathbf{1}$	368.94	-226.60	148.86	452.35	-310.45	636.81
		$\overline{2}$	-61.36	286.01		210.54 420.05	256.50	244.76
$Water(1) -$ $ethanol(2) -$ ethyl acetate(3)	$_{II}$	1	1.23	8.91	340.33	225.41	8.34	2.34
		\overline{c}	38.54	195.42	137.43	407.68	-25.89	-33.54
	IV	$\mathbf{1}$	-13.27	-438.09	395.41	471.89	-252.45	-709.18
		\overline{c}	-470.78	336.16	281.30	406.46	358.63	-699.22
	V	1	535.76	-146.56	89.67	488.04	-59.81	423.58
		$\overline{2}$	-471.03	333.21	259.13	408.93	343.07	-701.43
$Benzene(1)$ - $ethanol(2) -$ water(3)	\mathbf{I}	1	240.29	212.28	761.94	729.54	75.89	-153.93
		\overline{c}	57.74	261.93	478.12	338.57	254.17	-54.84
	IV	$\mathbf{1}$	871.32	56.61	1600.3	371.85	4.72	162.30
		$\overline{2}$	166.71	164.78	412.21	326.98	170.03	-2.81
$Benzene(1) -$ 1 -propanol (2) - water(3)	\mathbf{I}	$\mathbf{1}$	135.44	111.38	653.36	583.55	145.96	15.26
		$\overline{\mathbf{c}}$	48.23	216.40	446.31	356.19	293.33	33.23
	IV	$\mathbf{1}$	668.02	-0.30	1534.6	573.03	-19.33	386.48
		$\overline{2}$	62.31		200.50 458.82	348.50	286.54	68.57
$Water(1) -$	\mathbf{I}	$\mathbf{1}$	307.95	122.71	136.15	72.10	-74.73	-84.27
1 -butanol (2) -		$\overline{2}$	45.45	416.50	28.76	351.47	-21.76	-20.08
1 -propanol (3)								

 a^a 1 = extended UNIQUAC equation.

 $P = LEMF$ equation.

 $I =$ extended UNIQUAC equation.

 $H = LEMF$ equation.

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Fig. 3. Predicted activity coefficients obtained from the extended UNIQUAC equation with method V for (a) ethanol(1)-water(2) and (b) ethyl acetate(1)-ethanol(2) at 70 \degree C. Experimental (0). ref. 20; calculated (-).

the water-1-butanol-l-propanol system and the other methods did not work, probably because the solubility envelope for this system is too small to evaluate six parameters. For the benzene-ethanol-water and benzene-lpropanol-water systems method II provided too erroneous values of the calculated limiting distribution coefficients $K_{D2,\infty}$, so that method V was not applied for these two systems and method IV is recommended for use. For the LEMF equation, method II sometimes gives better results than the other methods as shown in the water-methanol-ethyl acetate and benzene-ethanol-water systems. The best results obtained from the extended UNIQUAC equation are better than those from the LEMF equation if the water-1-butanol-l-propanol system is excluded. The parameters which best predicted ternary VLE were used to predict binary VLE for the two completely miscible systems constituting each ternary system. Table 6 shows that the extended UNIQUAC equation provides smaller deviations in vapor-phase mole fraction and pressure than the LEMF equation does, except for the 1-butanol-1-propanol and l-propanol-water systems. Figures 2 and 3 compare the calculated values with the experimental results of the activity coefficients for four systems.

CONCLUSIONS

Ternary VLE predictions are reasonable when made with the extended UNIQUAC and LEMF equations, whose parameters are obtained by using the LLE data. VLE predictions for the completely miscible binary systems are of a similar accuracy as those for the ternary systems. The extended UNIQUAC equation gives somewhat better results in binary and ternary VLE predictions than the LEMF equation does.

NOMENCLATURE

- a_{ij} extended UNIQUAC or LEMF parameter
- *F* objective function
- *Gij* coefficient defined by eqn. (3)
- K_{D2} distribution coefficient defined by $x_2^{\text{I}}/x_2^{\text{II}}$
- P_n parameter value
- \overline{Q} penalty factor
- q, q^* pure-component surface parameters in the extended UNIQUAC equation
- R universal gas constant
- *r* pure-component size parameter in the extended UNIQUAC equation
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- *T* absolute temperature
- x liquid-phase mole fraction
- z liquid-phase coordination number, taken as 10

Greek letters

- activity coefficient γ
- segment fraction $\dot{\phi}$
- θ surface fraction
- τ coefficient related to binary parameter defined by eqn. (2) or (8)

Superscripts

- I, II phases
- E excess

Subscripts

- 1, 2, 3 components
- i, j components
- j phase
- k tie line
- ∞ infinite dilution

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