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ERRATA TO "GENERAL PRODUCT MEASURES"

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Theorem 5.9 is false and the proof we gave of Theorem 7.7 used 5.9. Fortunately a proof of 7.7, known earlier to us, makes no use of 5.9. For the record we also note that the conclusion about B in Theorem 7.2.4 is also false but this was not used anywhere in the paper.

The proof of Theorem 7.7 can be repaired by replacing Part 2 thereof (p. 282) by the following.

PART 2. If $\mathfrak{F}\subset\mathfrak{B}$ and $\sigma\mathfrak{F}=S$ then there is such a countable subfamily $\mathfrak{G}$ of $\mathfrak{F}$ that $.\psi(S\sim\sigma\mathfrak{G})=0$.

Proof. The desired conclusion is found in Step 2 below.

Let $V_u=\text{tpr}\,u$ whenever $u\in\text{fnt}\cap\text{sb}\,t$ and $F_p=\text{E}A$ (for some $B\in\mathfrak{F}$ and $u\subset t$, $\text{dmn}\,u=\text{dmn}\,p$, $A\in V_u$ and $\text{cyl}\,AS\subset B$) whenever $p\in\text{fnt}\cap\text{sb}\,m$. Now we take

STEP 1. If $p\in\text{fnt}\cap\text{sb}\,m$ then there is such a countable subfamily $\mathfrak{G}$ of $\mathfrak{F}$ that

$$.\psi(\text{cyl}\,\sigma F_pS\sim\sigma\mathfrak{G})=0.$$

Proof. Suppose $\mu=\text{prm}\,p$, $u\subset t$, $\text{dmn}\,u=\text{dmn}\,p$, $\mathfrak{B}'=\text{tpr}\,u$, $\mathfrak{F}'=F_p$, $S'=\text{spc}\,p$ and $\mathfrak{H}'=\bigcup B\in\mathfrak{H}\ \text{sng prj}\,BS'$. Clearly $\mathfrak{F}'\subset\mathfrak{B}'$ and $\sigma\mathfrak{F}'\in\mathfrak{B}'$. Noting that $\mu\in\text{Clin}\,\mathfrak{B}'$ and $\mathfrak{H}'\subset\text{dmn}'\,\mu$ we can and do select such a function w on $\mathfrak{H}'$ that, for each $A\in\mathfrak{H}'$, $.wA\in\text{cbl}\cap\text{sb}\,\mathfrak{F}'$ and

$$.\mu((\sigma\mathfrak{F}'\sim .wA)\cap A)=0.$$

We let $\mathfrak{G}'=\bigcup A\in\mathfrak{H}'\ .wA$ and check that $\mathfrak{G}'\in\text{cbl}\cap\text{sb}\,\mathfrak{F}'$ and

$$\begin{aligned}0&\leq .\mu((\sigma\mathfrak{F}'\sim\sigma\mathfrak{G}')\cap\sigma\mathfrak{H}')\\&= .\mu(\textstyle\bigcup A\in\mathfrak{H}'(\sigma\mathfrak{F}'\sim\sigma\mathfrak{G}')\cap A)\\&\leq .\mu(\textstyle\bigcup A\in\mathfrak{H}'(\sigma\mathfrak{F}'\sim .wA)\cap A)\\&\leq \textstyle\sum A\in\mathfrak{H}'\ .\mu((\sigma\mathfrak{F}'\sim .wA)\cap A)=0.\end{aligned}$$

Hence taking $\mathfrak{G}$ to be such a countable subfamily of $\mathfrak{F}$ that each member A of $\mathfrak{G}'$ is related to some member B of $\mathfrak{G}$ by having $\text{cyl}\,AS\subset B$ we have $\text{cyl}\,\sigma\mathfrak{G}'S\subset\sigma\mathfrak{G}$ and infer

$$.\phi((\text{cyl}\,\sigma\mathfrak{F}'S\sim\sigma\mathfrak{G})\cap\sigma\mathfrak{H})=0$$

and

$$\begin{aligned} 0 &\leqq .\psi(\text{cyl}\,\sigma\mathfrak{F}'S \sim \sigma\mathfrak{G}) \\ &= .\phi((\text{cyl}\,\sigma\mathfrak{F}'S \sim \sigma\mathfrak{G}) \cap T) \\ &\leqq .\phi((\text{cyl}\,\sigma\mathfrak{F}'S \sim \sigma\mathfrak{G}) \cap \sigma\mathfrak{H}) + .\phi(T \sim \sigma\mathfrak{H}) \\ &= 0+0 = 0 \end{aligned}$$

to complete our proof.

STEP 2. There is such a countable subfamily $\mathfrak{G}$ of $\mathfrak{F}$ that $.\psi(S \sim \sigma\mathfrak{G}) = 0$.

Proof. Let $P = \text{fnt} \cap \text{sb}\, m$, note that $P \in \text{cbl}$, and using Step 1 secure a function f on P for which $.fp \in \text{cbl} \cap \text{sb}\, \mathfrak{F}$ and $.\psi(\text{cyl}\, \sigma F_p S \sim \sigma\ .fp) = 0$ whenever $p \in P$. Taking $\mathfrak{G} = \bigcup p \in P\ .fp$ we have

$$\begin{aligned} S &= \bigcup p \in P\, \text{cyl}\, \sigma F_p S, \\ S &\sim \sigma\mathfrak{G} \subset \bigcup p \in P (\text{cyl}\, \sigma F_p S \sim\ .fp) \end{aligned}$$

and consequently

$$0 \leqq .\psi(S \sim \sigma\mathfrak{G}) \leqq \sum p \in P\ .\psi(\text{cyl}\, \sigma F_p S \sim\ .fp) = 0$$

which completes the proof.

REFERENCES

1. E. O. Elliott and A. P. Morse, *General product measures*, Trans. Amer. Math. Soc. **110** (1964), 245–282.

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