

# VLBI observation biases induced by antenna gravitational flexures

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# The early “warnings”

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## Comparison of Geodetic and Radio Interferometric Measurements of the Haystack-Westford Base Line Vector

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A three-dimensional geodetic measurement of the 1.24-km Haystack-Westford base line vector was performed to verify the accuracy of previously published radio interferometric measurements. The differences between the geodetic and the very long base line interferometry (VLBI) measurements of the base line length, the horizontal  $X$  and  $Y$  components, and the vertical  $Z$  component were  $-4$ ,  $2$ ,  $4$ , and  $-19$   $\text{mm}$  respectively. After a correction was applied to the VLBI determination of the vertical component to

TABLE 1. Results From Geodetic Measurements of Haystack-Westford Base Line Vector

Base Line Characteristic*	Value, † m
X component	$-1149.592 \pm 0.002$
Y component	$-462.196 \pm 0.003$
Z component	$-30.024 \pm 0.003$
Length	$1239.390 \pm 0.001$

TABLE 2. Results From VLBI Measurements of Haystack-Westford Base Line Vector

Base Line Characteristic*	Mean of 11 Determinations, † m [Rogers <i>et al.</i> , 1978]	Twelfth Determination, ‡ m (See Text)	Difference, §
X component	$-1149.594 \pm 0.003$	$-1149.597 \pm 0.001$	0.003
Y component	$-462.200 \pm 0.005$	$-462.196 \pm 0.001$	-0.004
Z component	$-30.005 \pm 0.007$	$-30.002 \pm 0.003$	-0.003
Length	$1239.394 \pm 0.003$	$1239.394 \pm 0.001$	0.000

TABLE 3. Comparison of the Geodetic and VLBI Measurements of the Haystack-Westford Base Line Vector

Base Line Characteristic*	Difference Between Geodetic Value and Mean VLBI Value, m	
	Uncorrected†	Partially Corrected‡
X component	0.002	0.002
Y component	0.004	0.004
Z component	-0.019	-0.006
Length	-0.004	-0.004

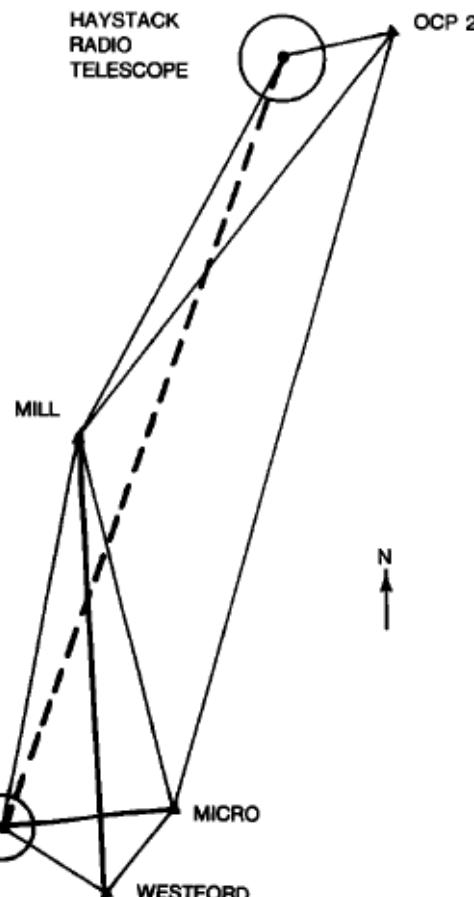


Fig. 3. Sketch of the geodetic network. The dashed line represents the 1.24-km Haystack-Westford base line vector.

## Deformations in VLBI Antennas

T. A. Clark

*Goddard Space Flight Center  
Greenbelt, Maryland*

and

P. Thomsen

*Interferometrics, Inc.  
Vienna, Virginia*



National Aeronautics and  
Space Administration

Goddard Space Flight Center  
Greenbelt, Maryland 20771

1988

# Signal path variation (SPV)

(Clark & Thomsen, 1988)

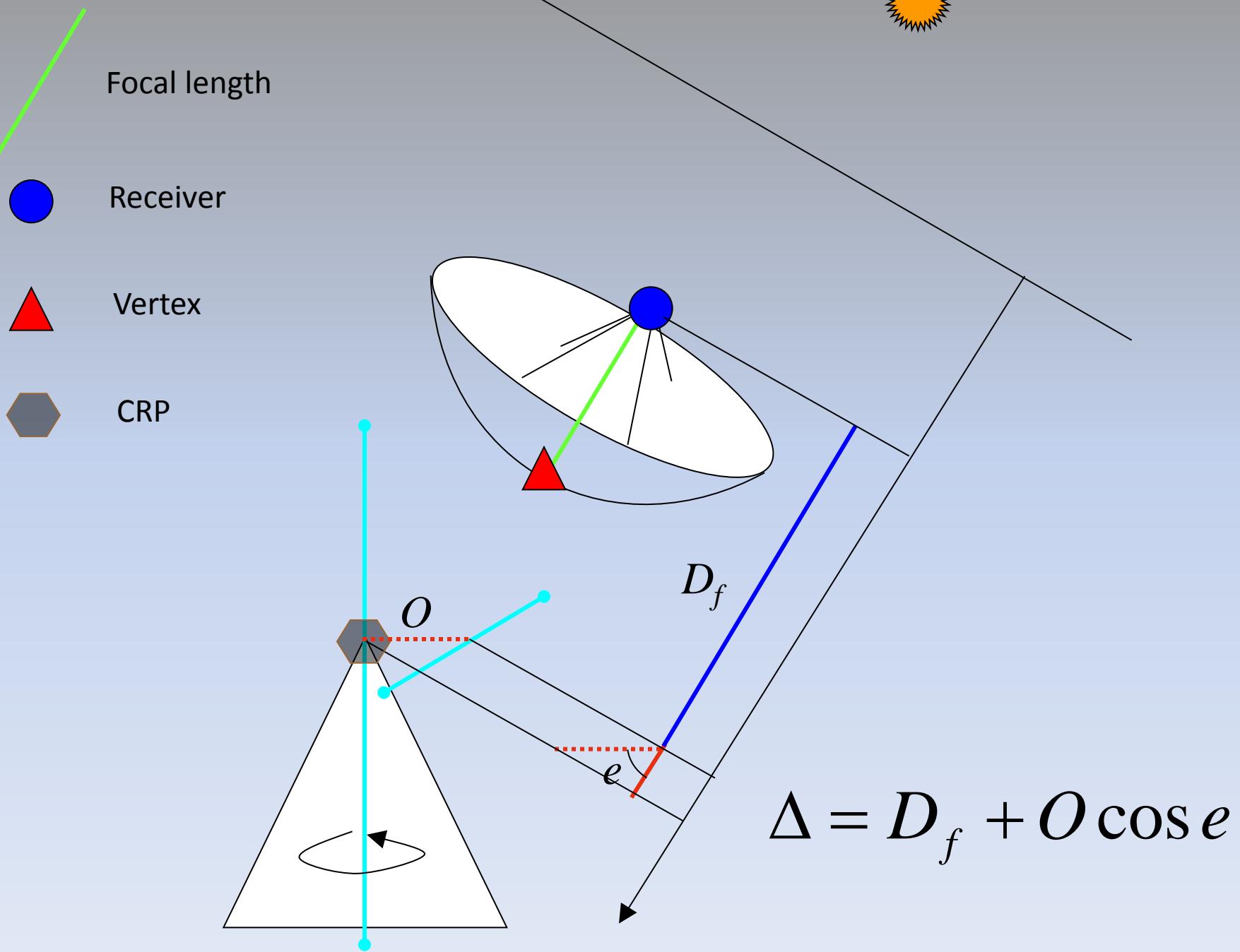
$$\Delta L(e) = \alpha_R \Delta R(e) + \alpha_F \Delta F(e) + \alpha_V \Delta V(e)$$

- Originated by gravity-induced flexure of radio telescopes
  - It is a linear combination of three contributing terms:
    - Displacement of the receiver along the line of sight
    - Deformation of the primary reflector
    - Displacement of the reflecting system along the line of sight
  - *SPV depends on the pointing elevation!*

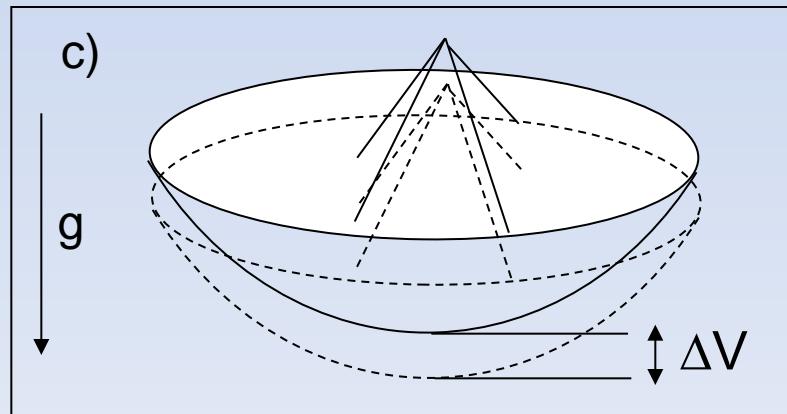
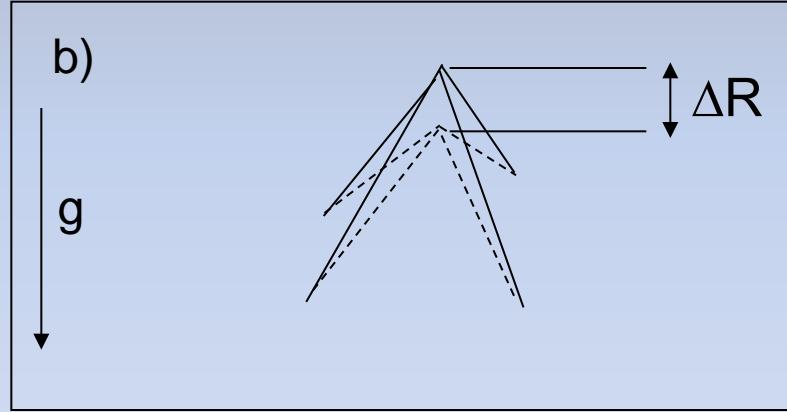
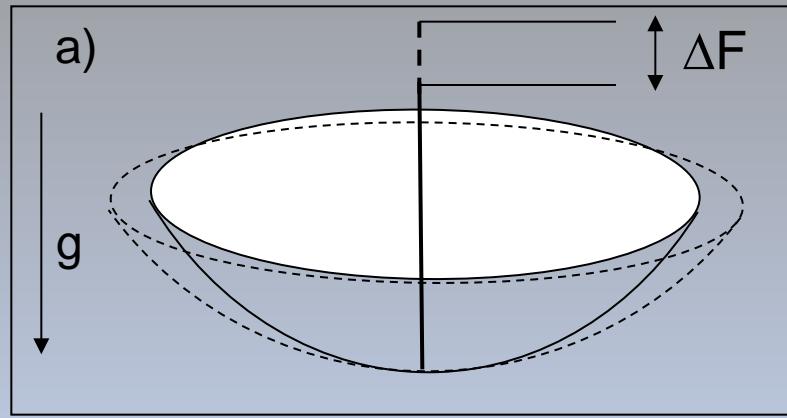
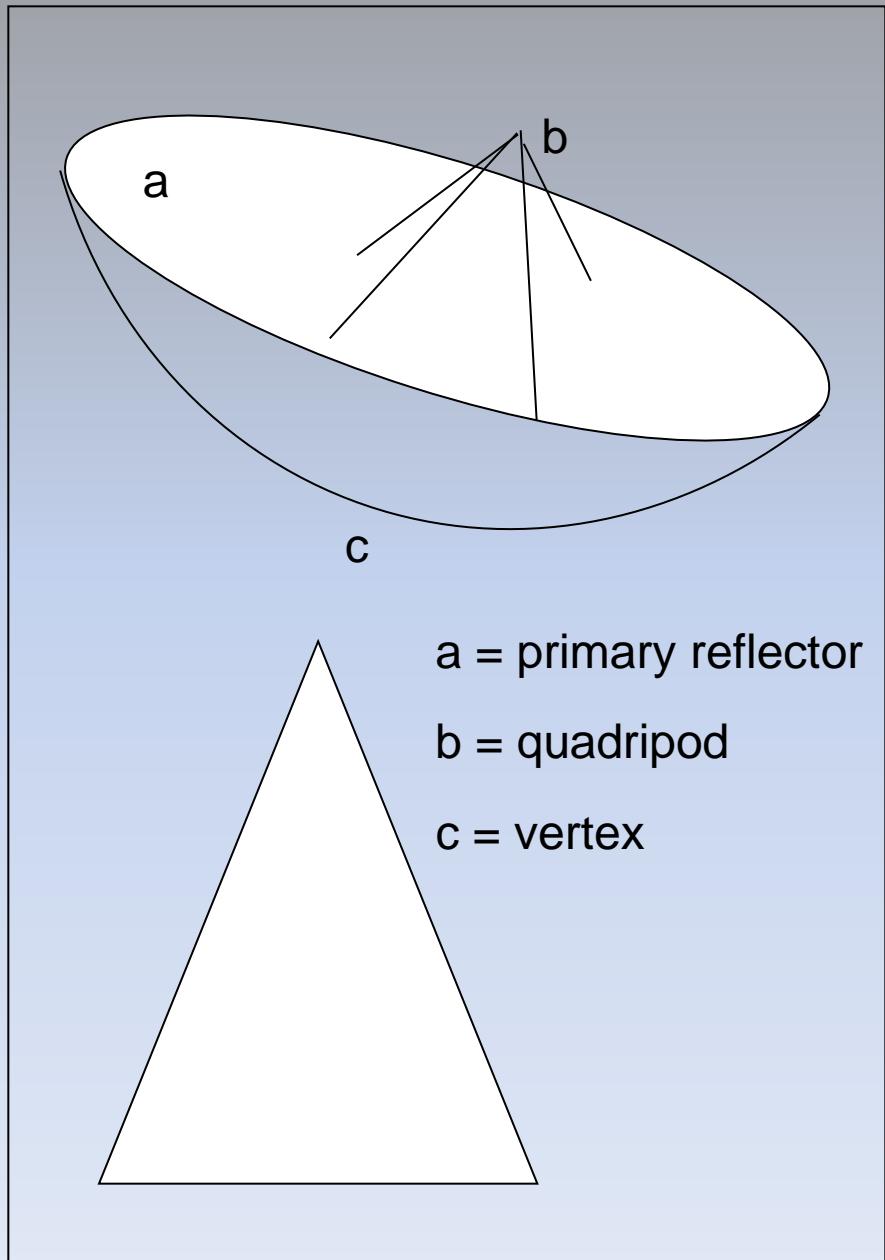
# Clark & Thomsen (1988)

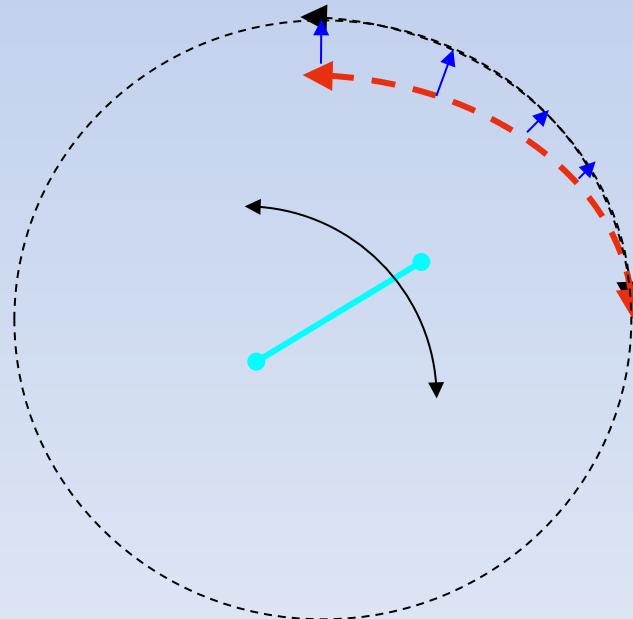
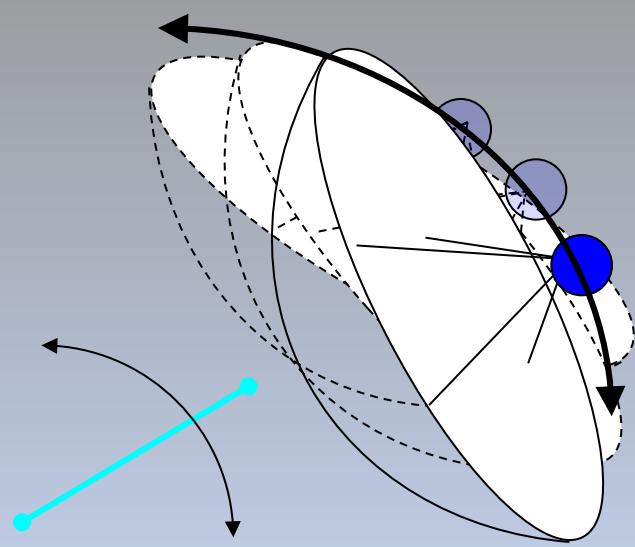
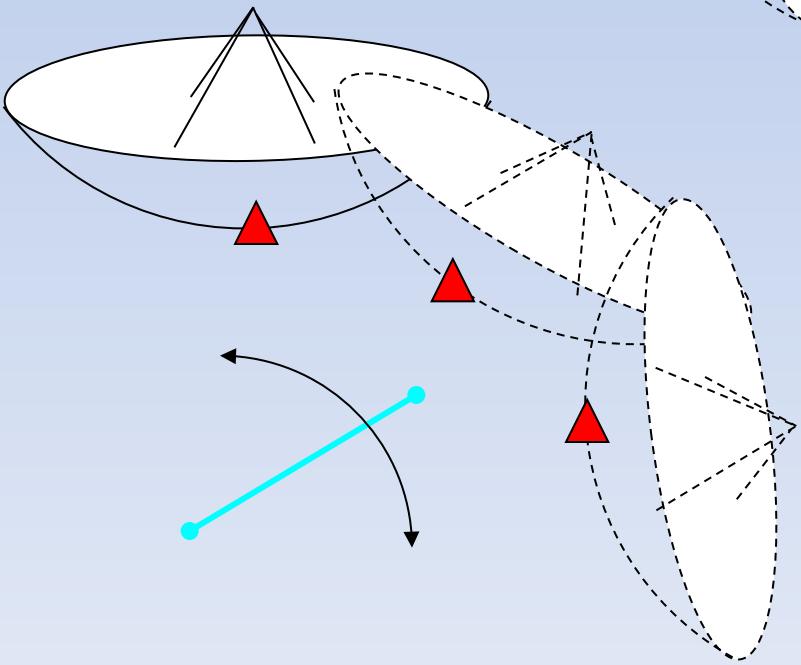
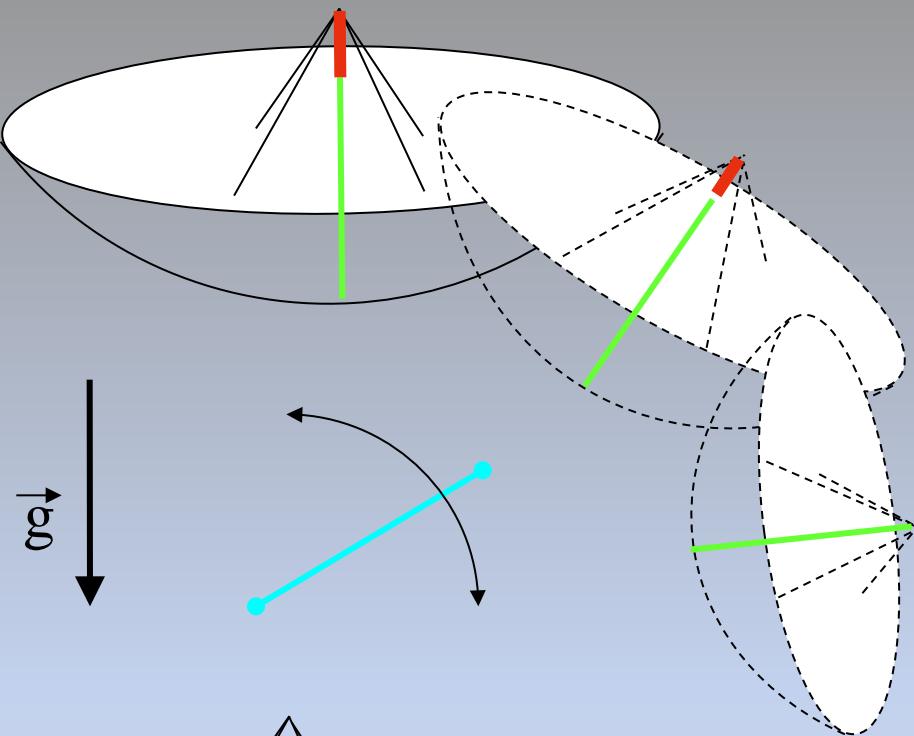
- Focused on the Fairbanks antenna (Alaska):
  - 26 m dish
  - X-Y mount
  - S/X receiver in primary focus position
- Used a Finite Element Model of the antenna
- The coefficients  $\alpha_i$  were determined assuming a uniform grading on the primary reflector
- The resulting  $\Delta L(e) \approx 0$  at all elevations!

**How do gravitational flexures  
impact VLBI observations?**



# How does gravity deform the telescope?







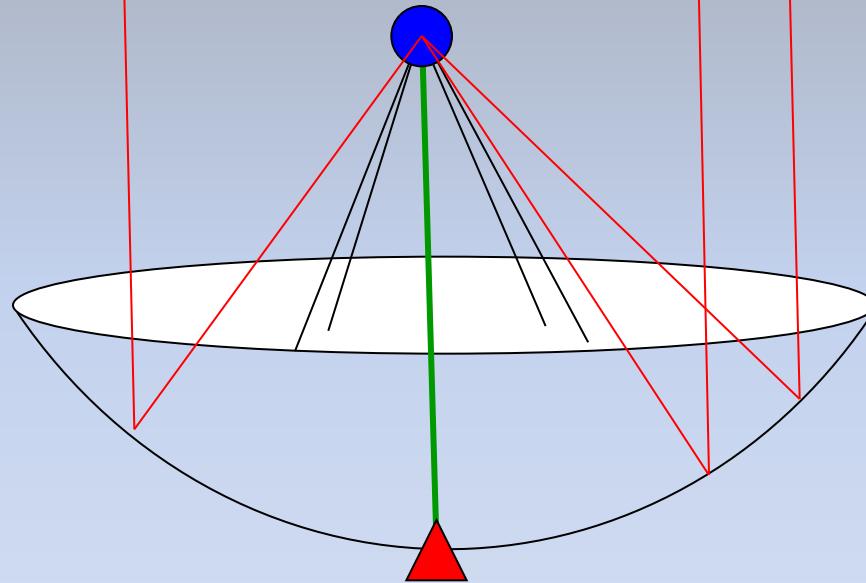
Focal length



Receiver

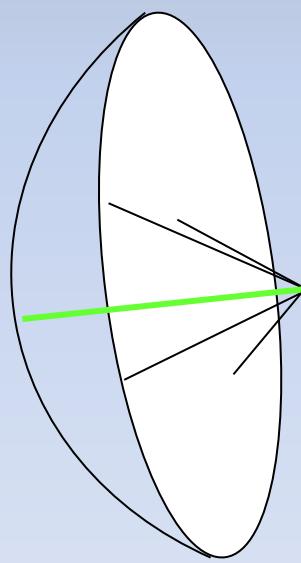


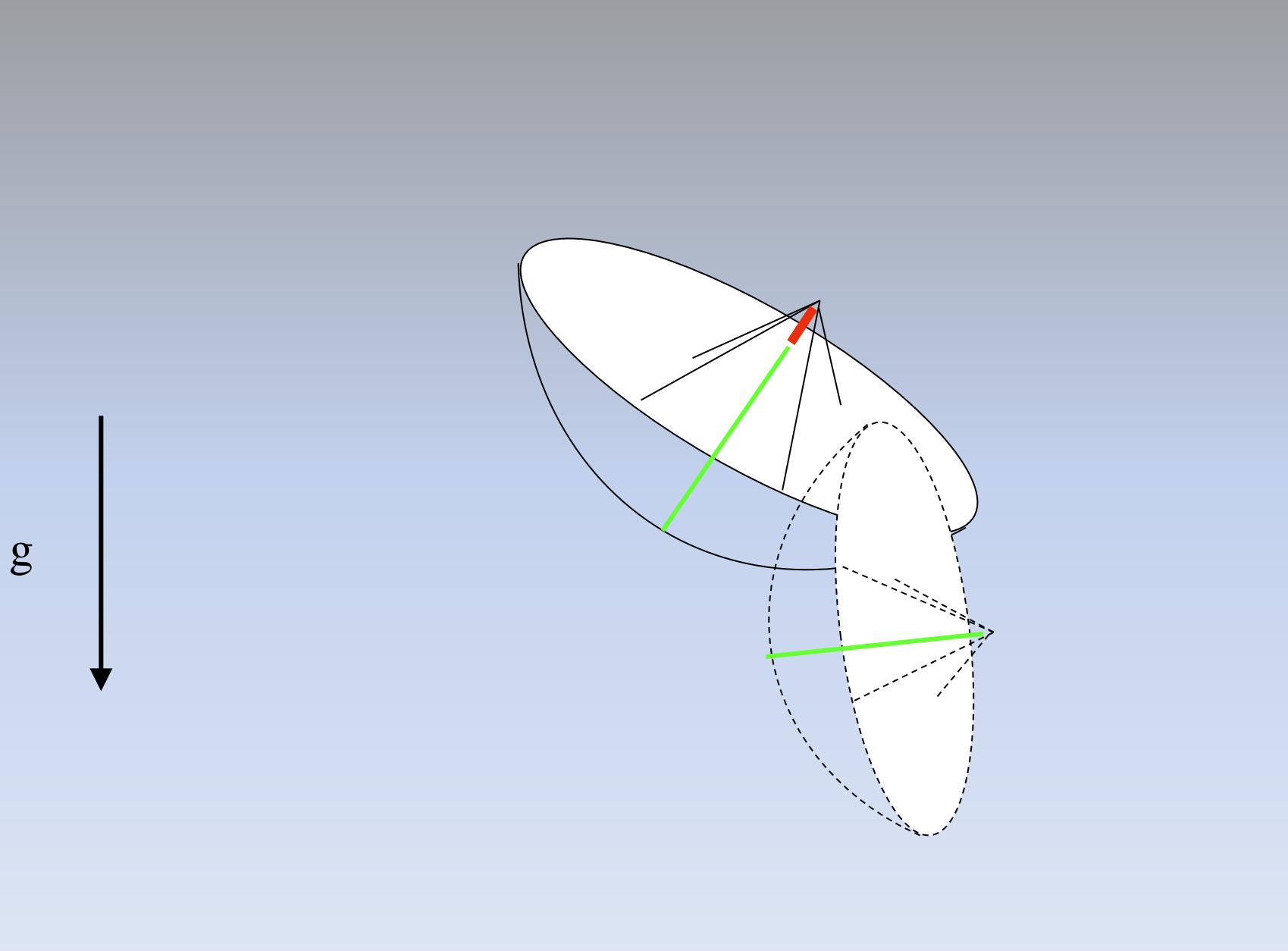
Vertex

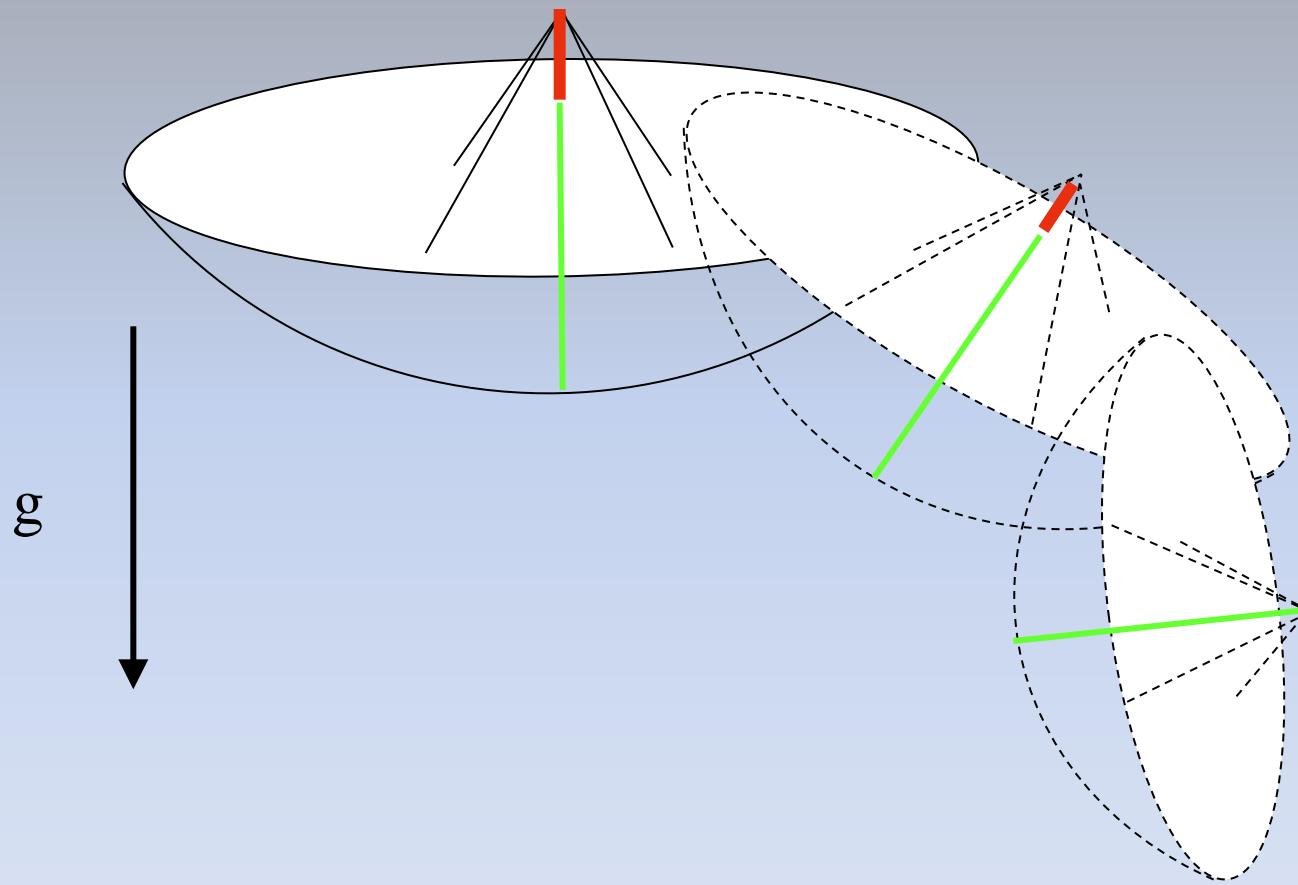


$$\Delta L(e) = \alpha_F \Delta F(e) + \alpha_R \Delta R(e) + \alpha_V \Delta V(e)$$

*g*

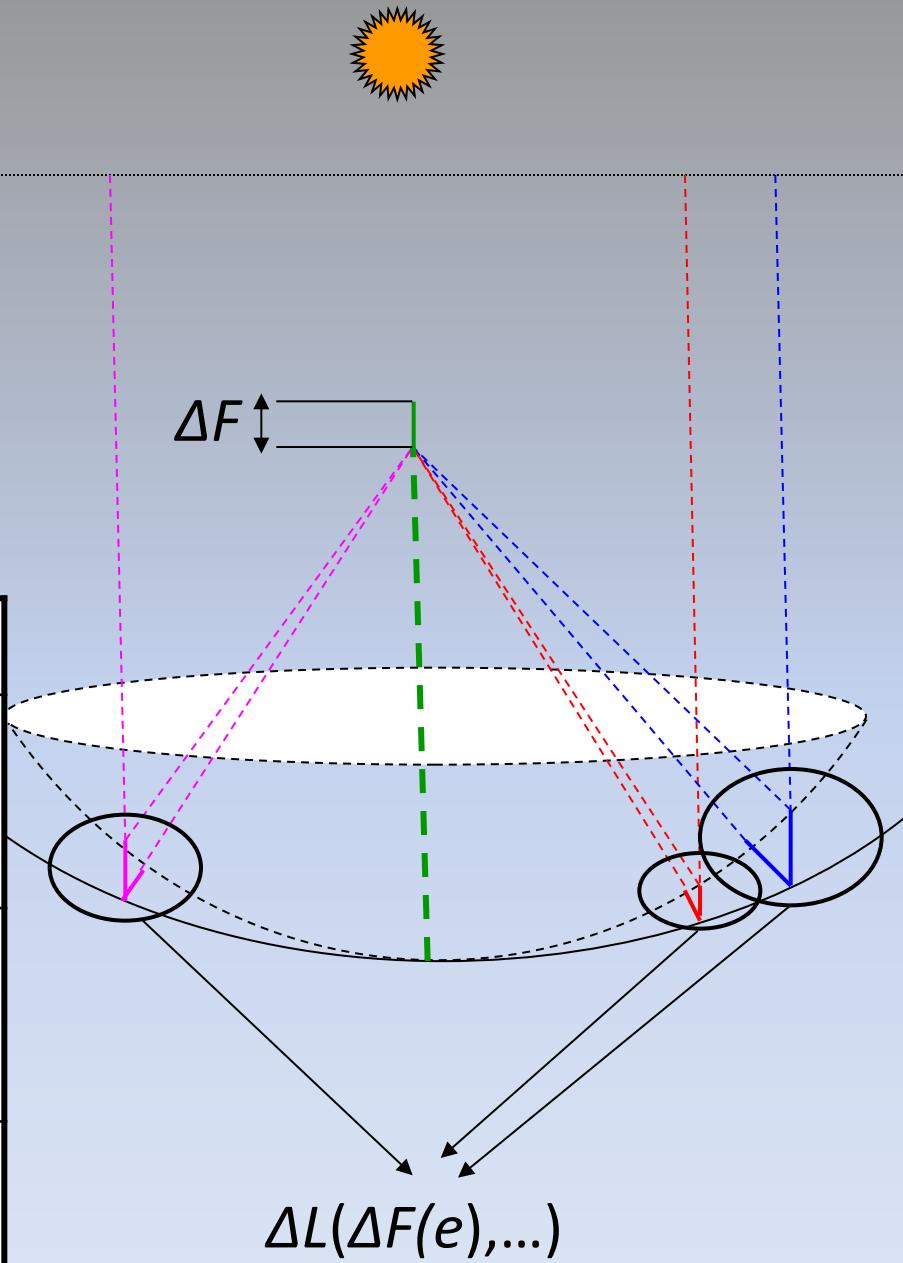


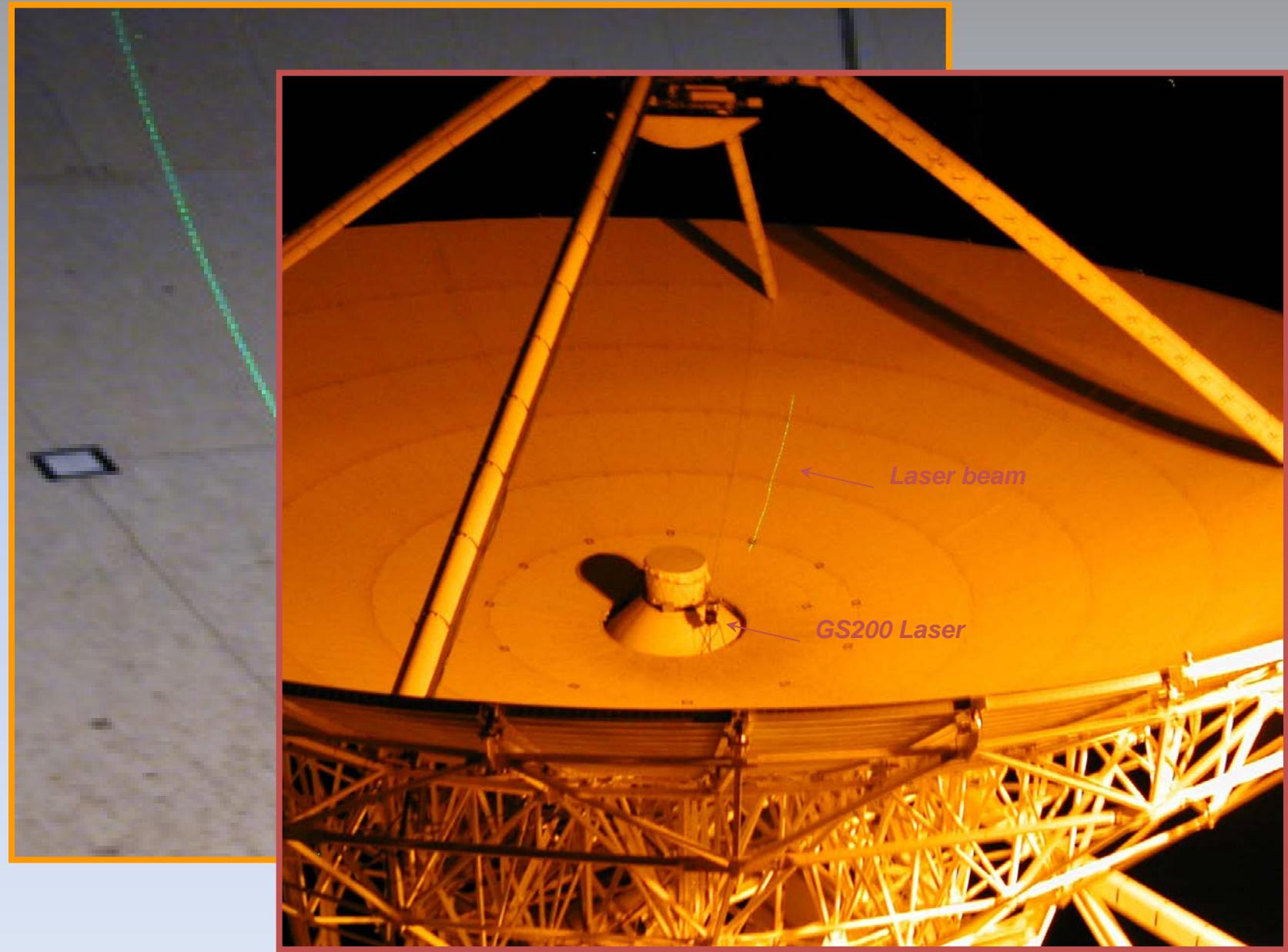




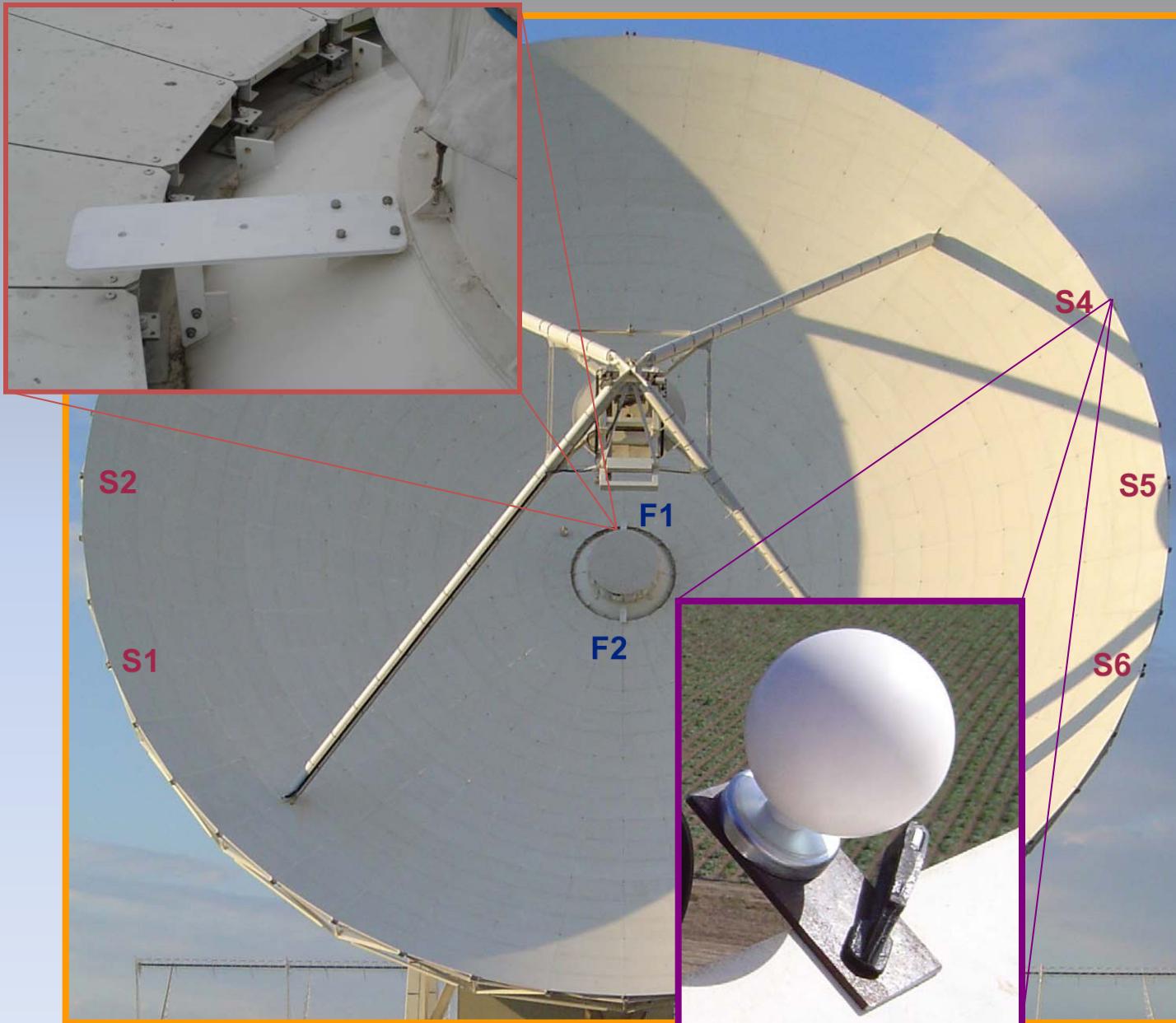
Focal length

$\Phi$ (m)	$\Delta F_{max}$ (mm)
26	4.5 (Clark & Thomsen 1988)
32	36.3/24.4 (Sarti et al. 2009)
100	12.6 (Holst et al. 2012)





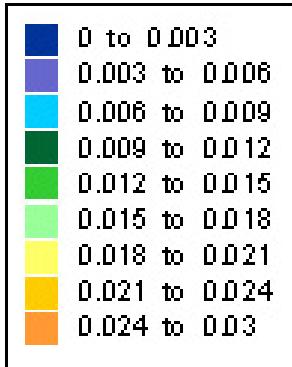
*Standpoint*



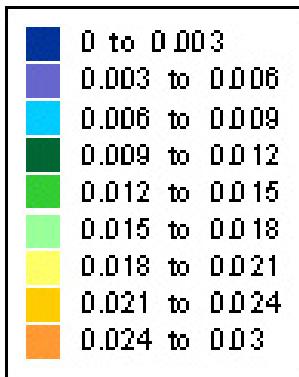
*Target*



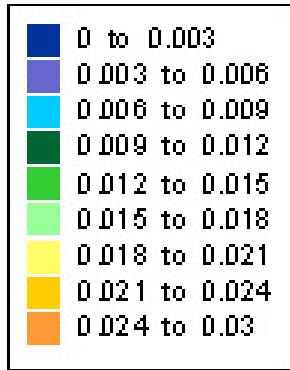
Elevation 90 deg



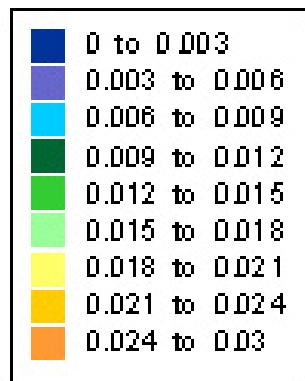
Elevation 75 deg



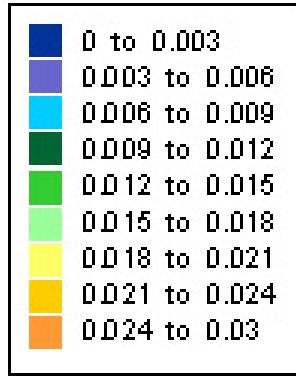
Elevation 60 deg



Elevation 45 deg



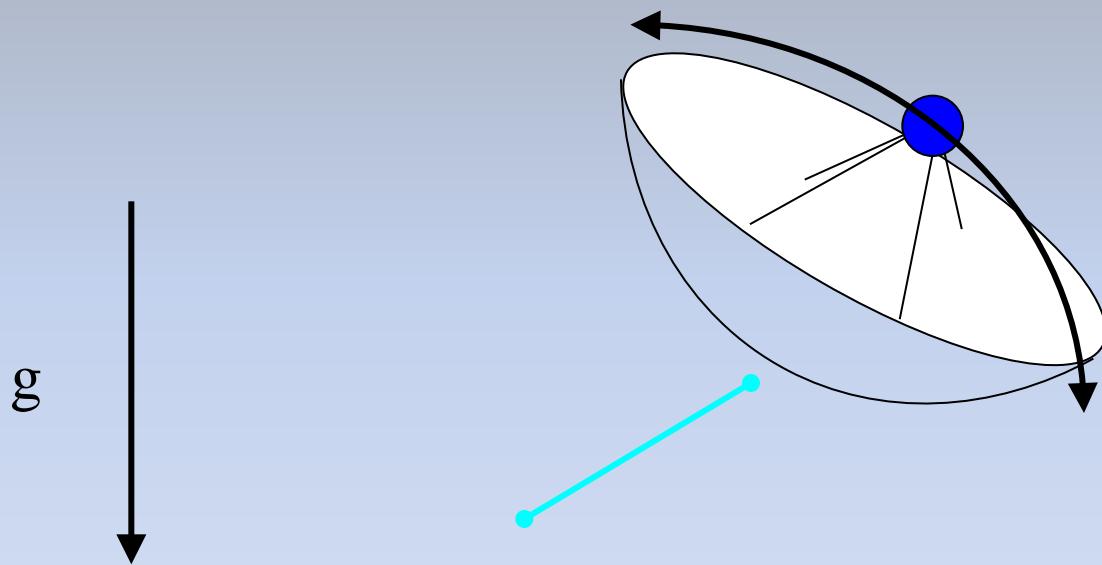
Elevation 30 deg



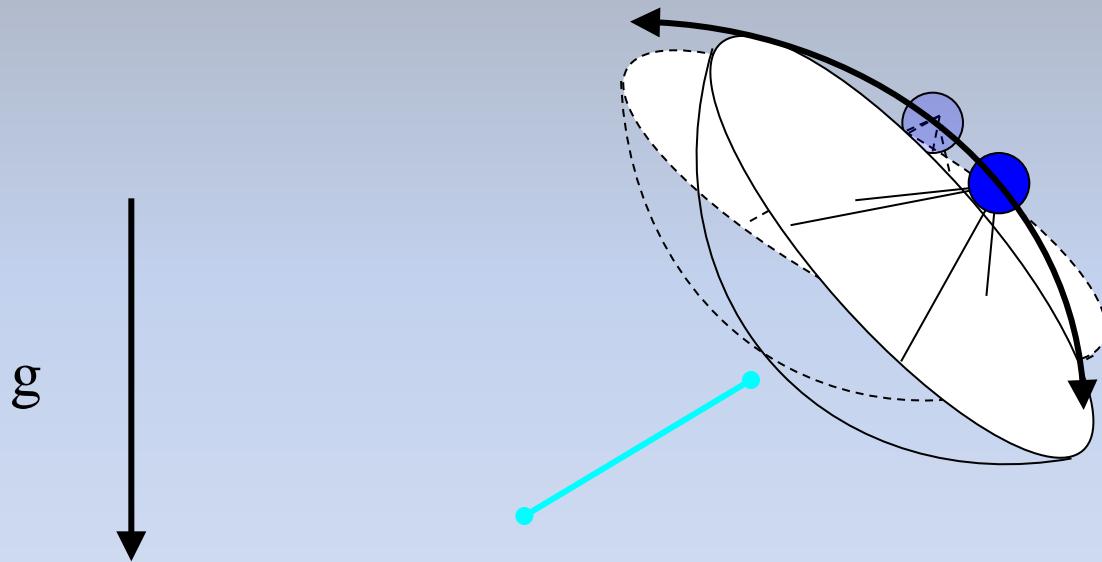
Elevation 15 deg

$$\Delta L(e) = \alpha_F \Delta F(e) + \alpha_R \Delta R(e) + \alpha_V \Delta V(e)$$

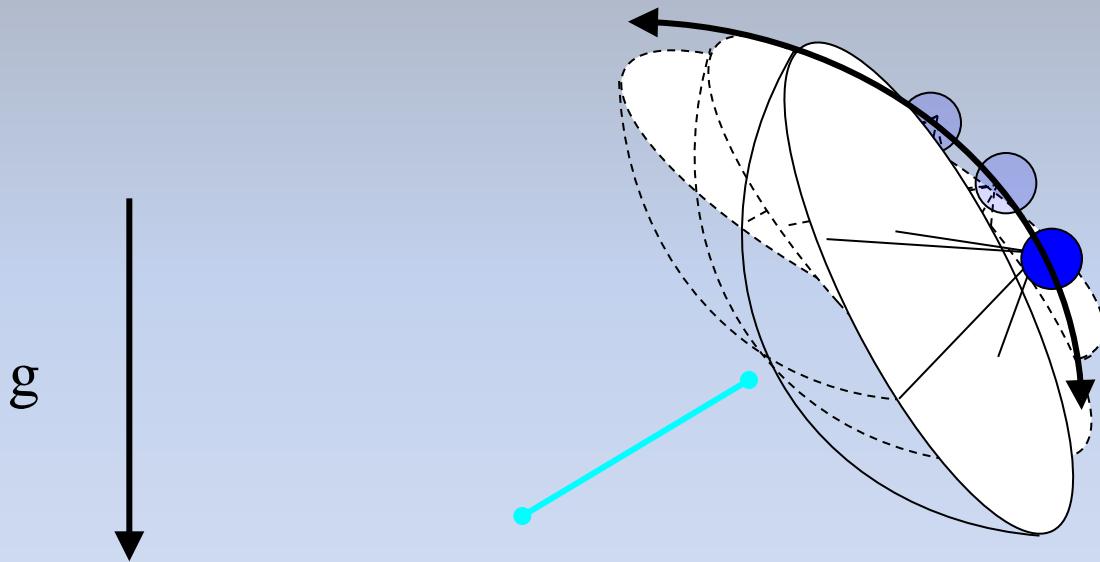
# Displacement of the receiver



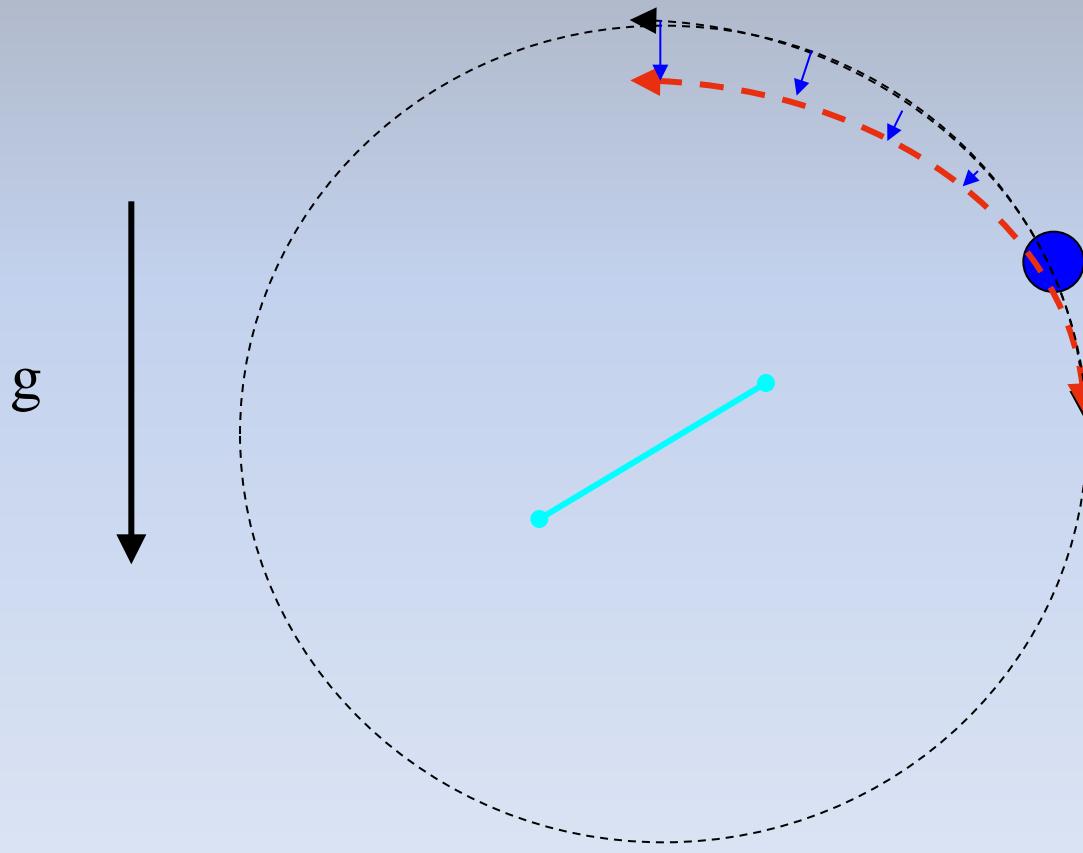
# Displacement of the receiver

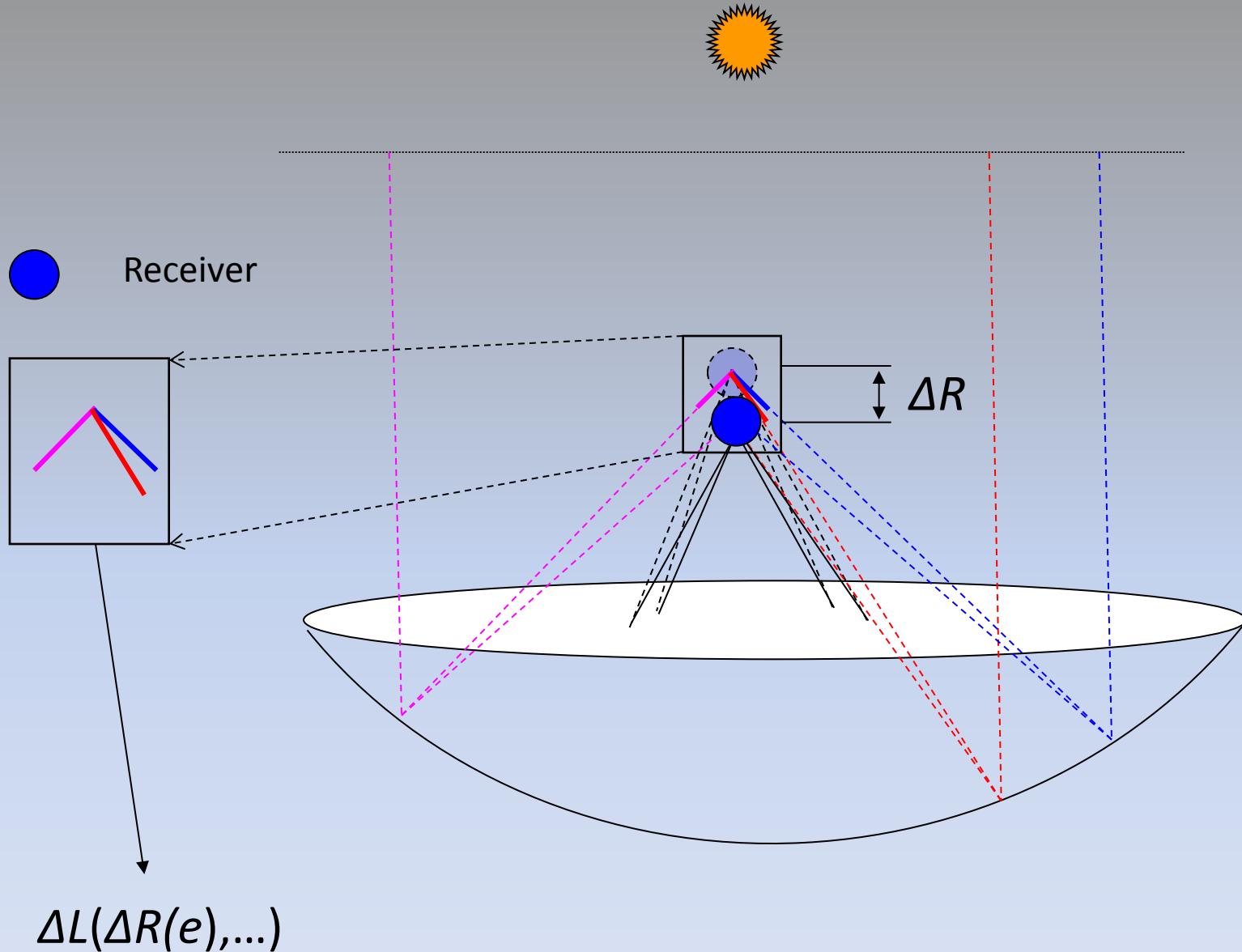


# Displacement of the receiver



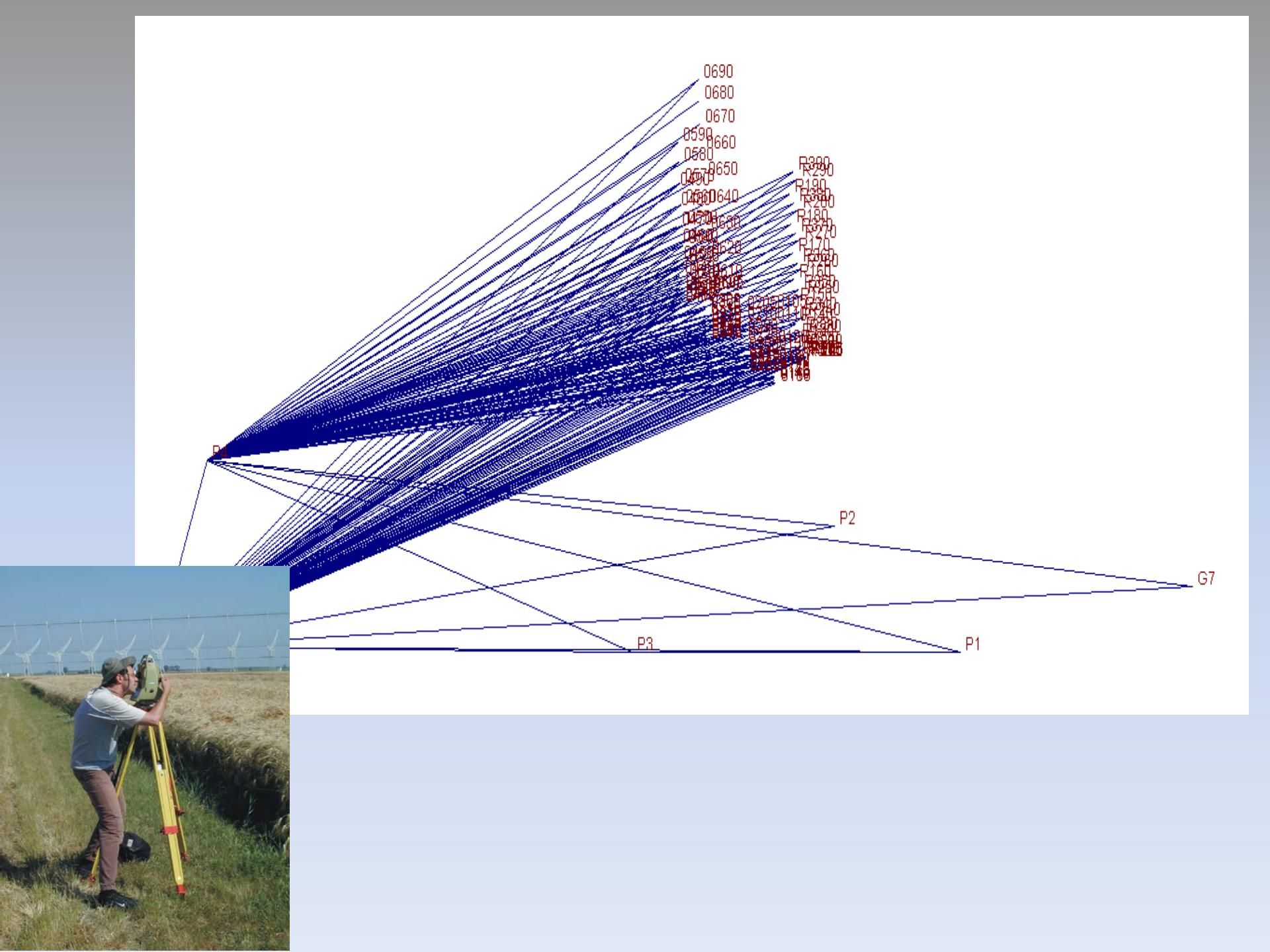
# Displacement of the receiver

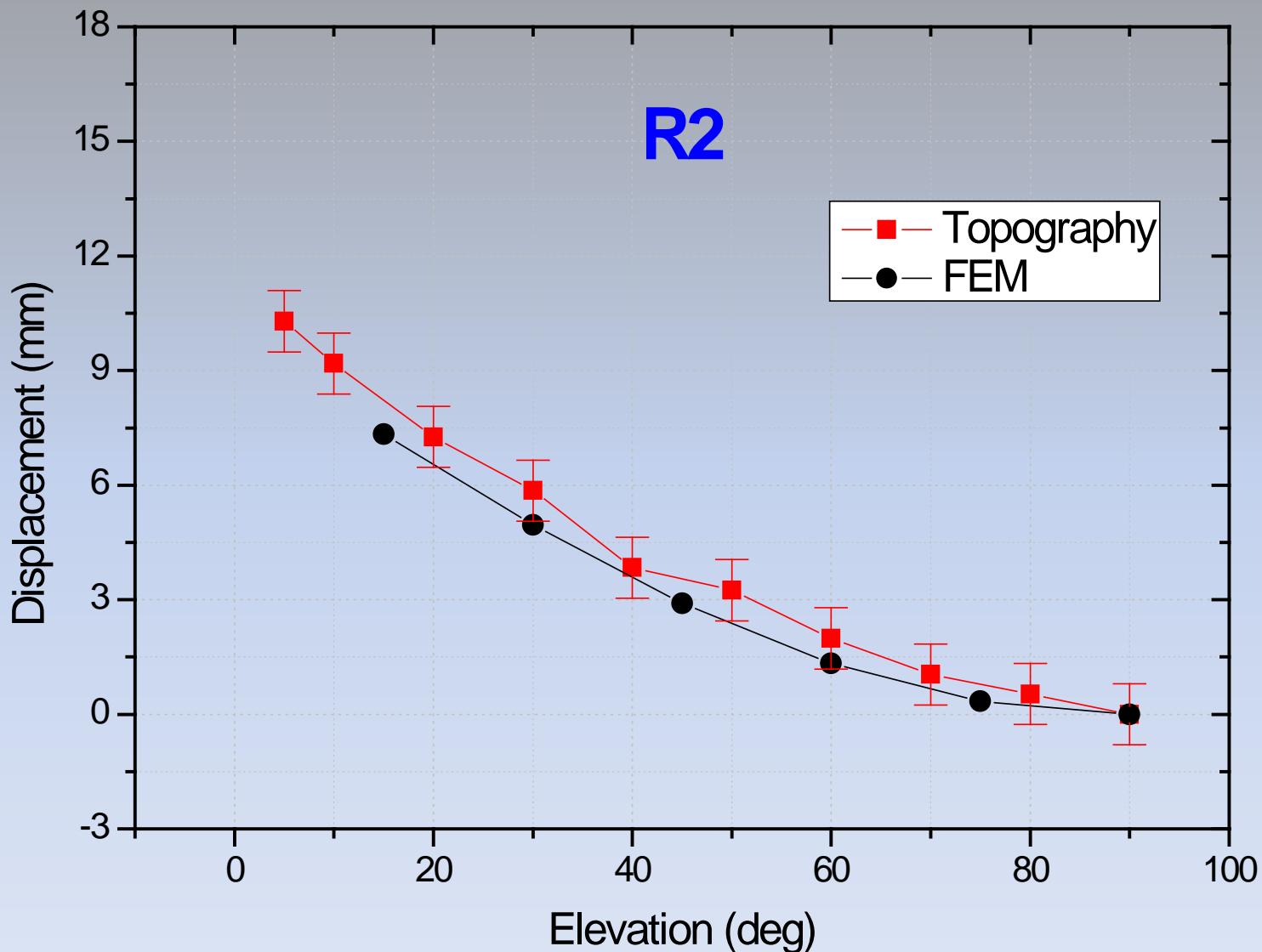


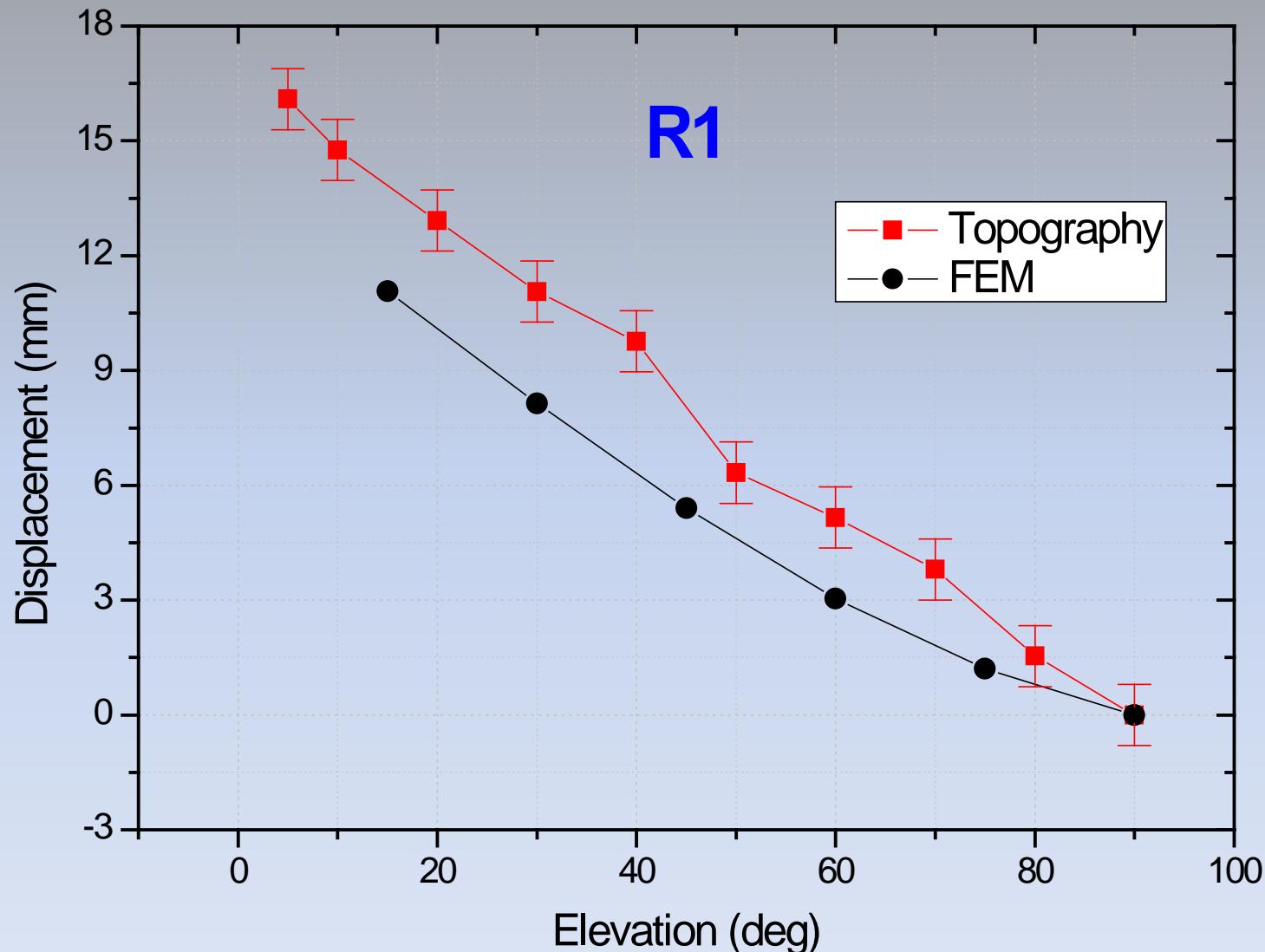


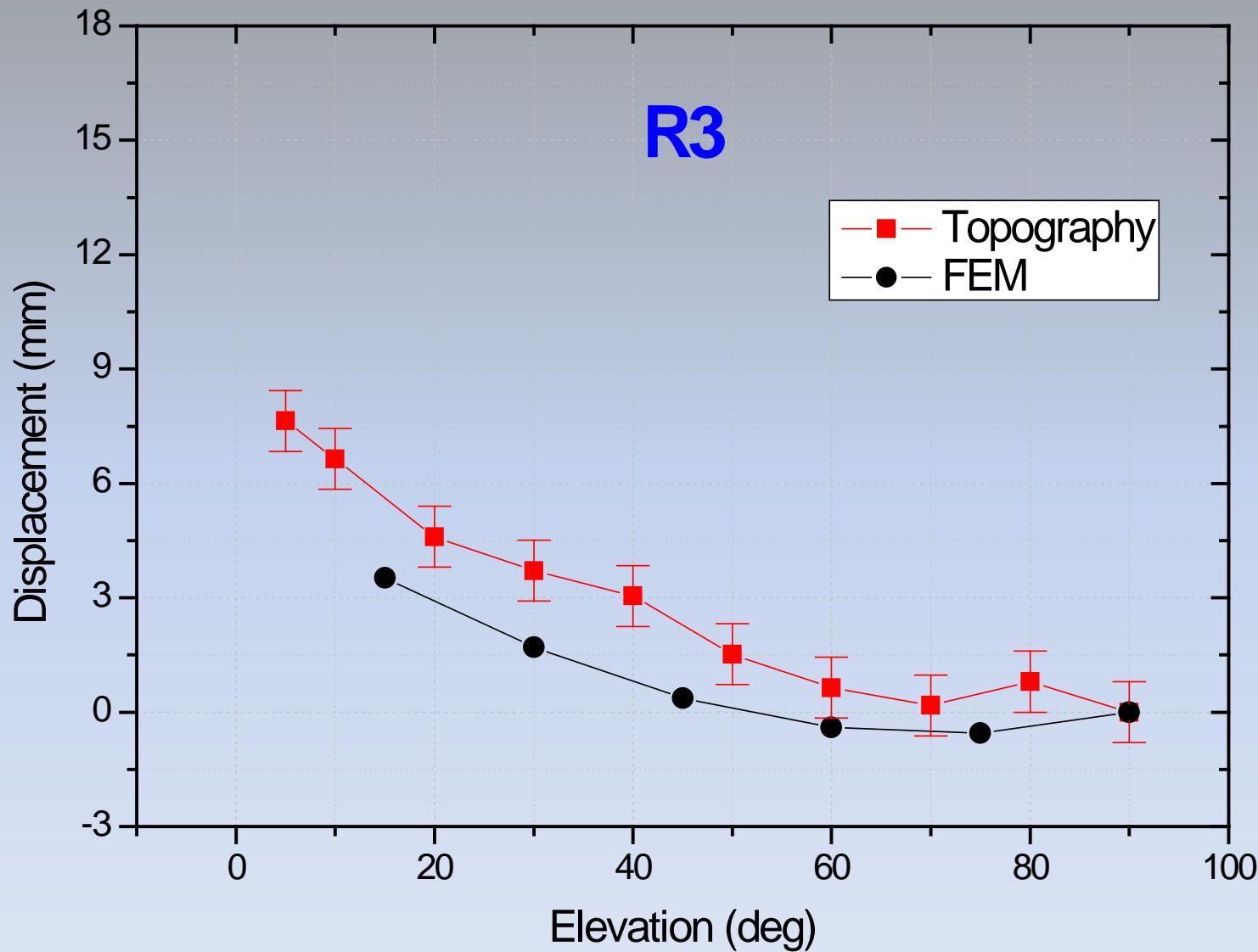
$$\Delta L(\Delta R(e), \dots)$$

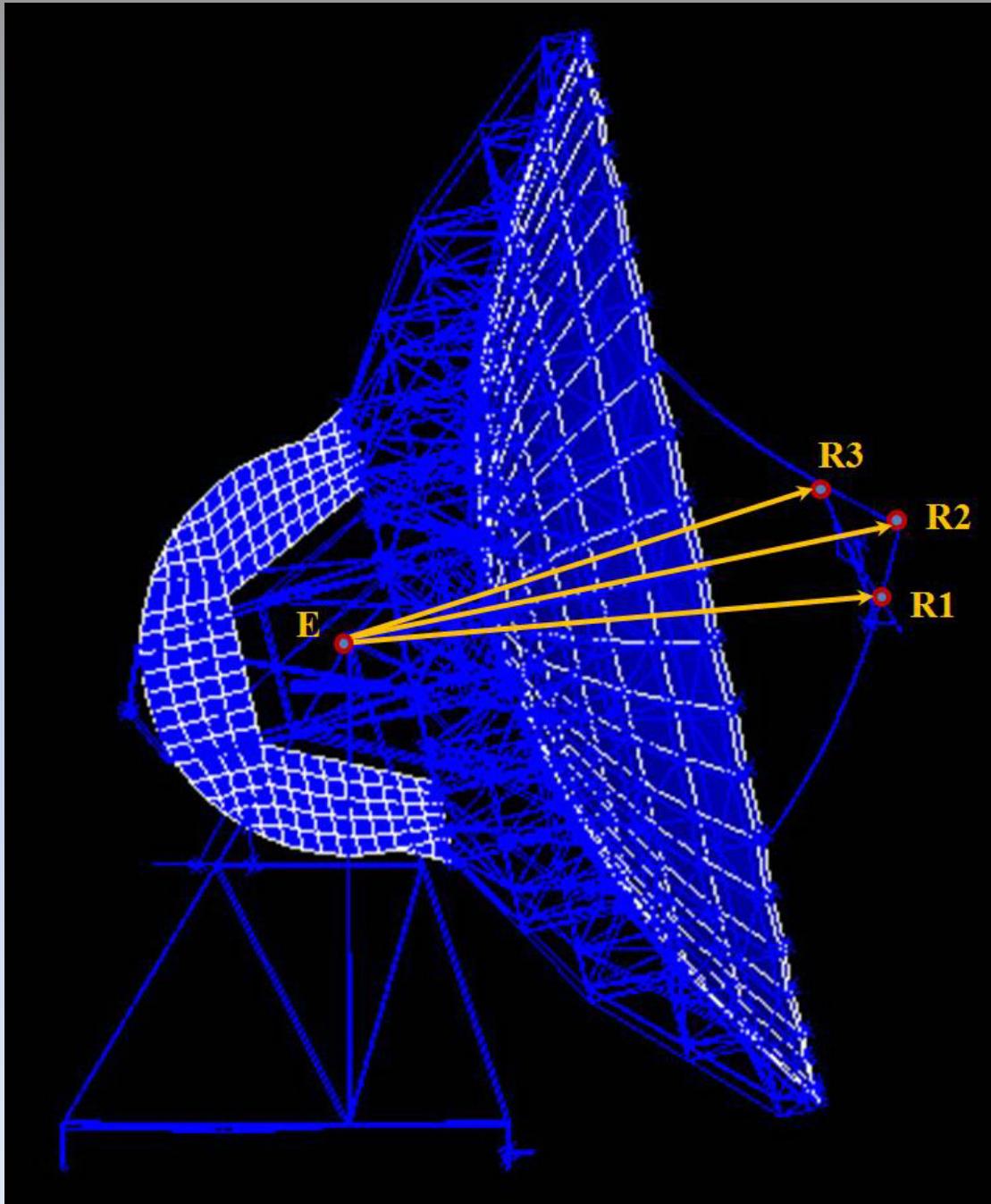
 $R_3$  $R_2$  $R_1$

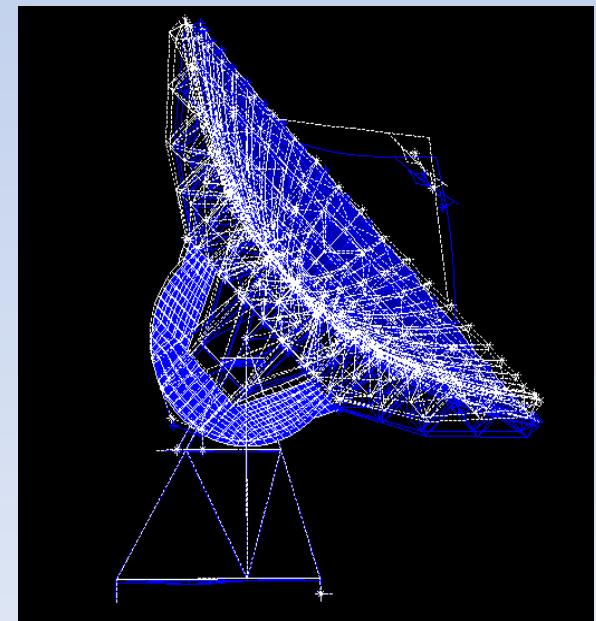
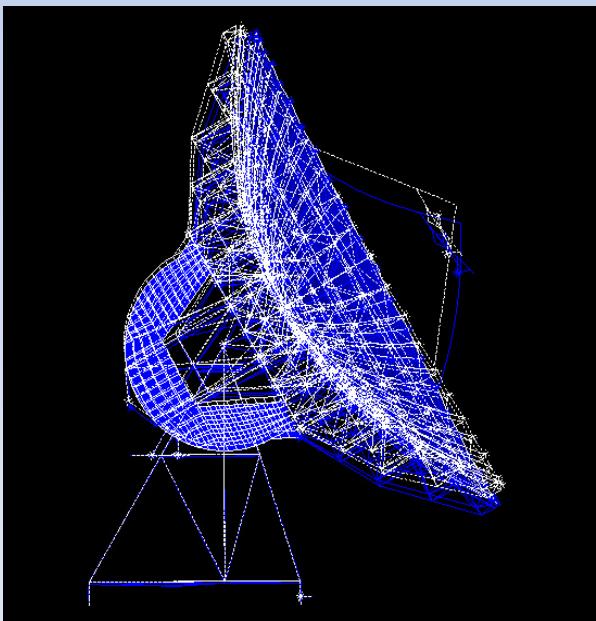
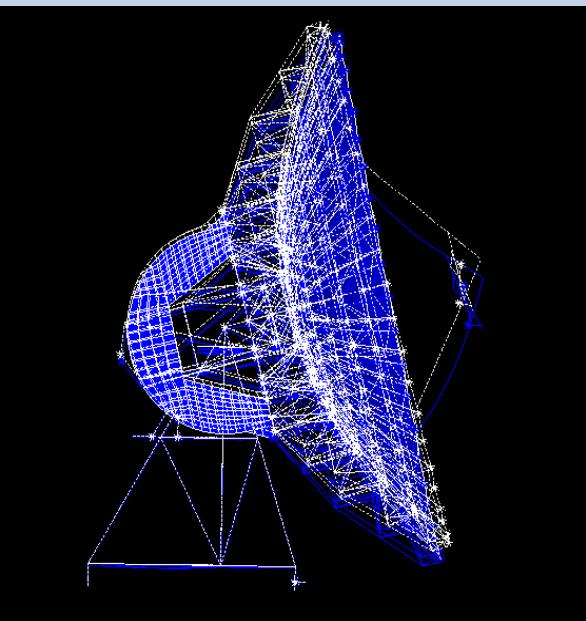
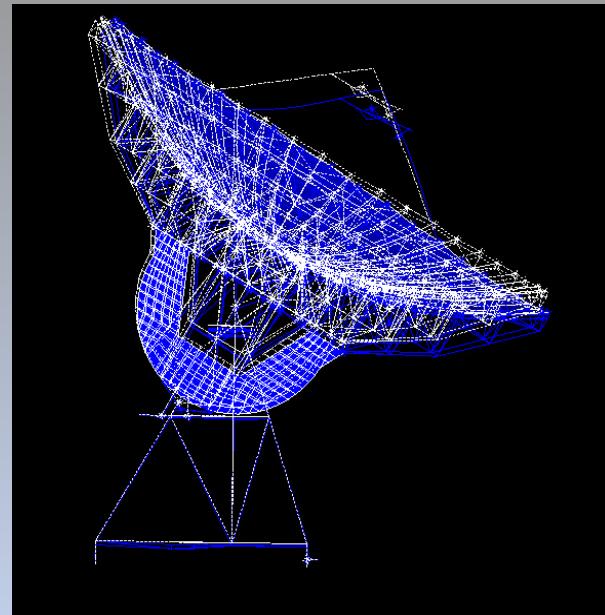
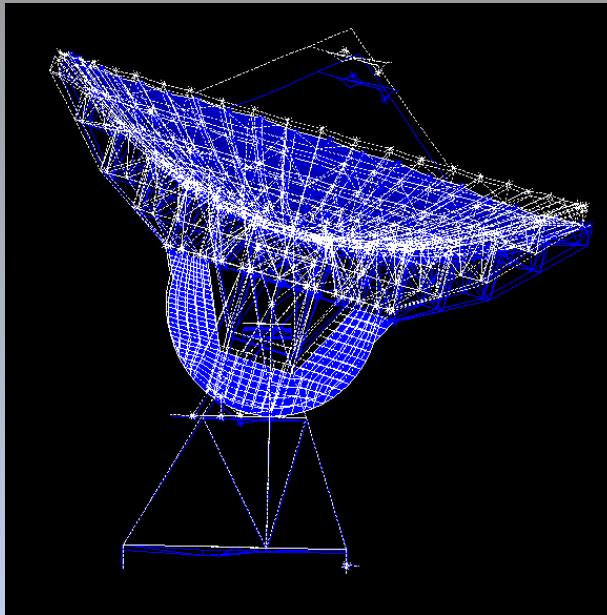
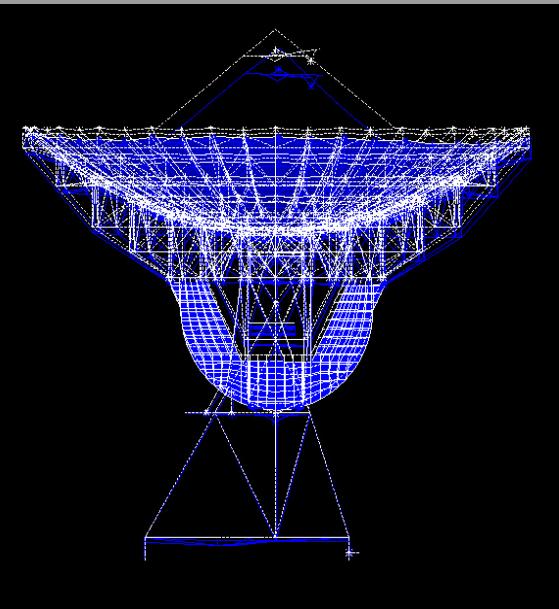








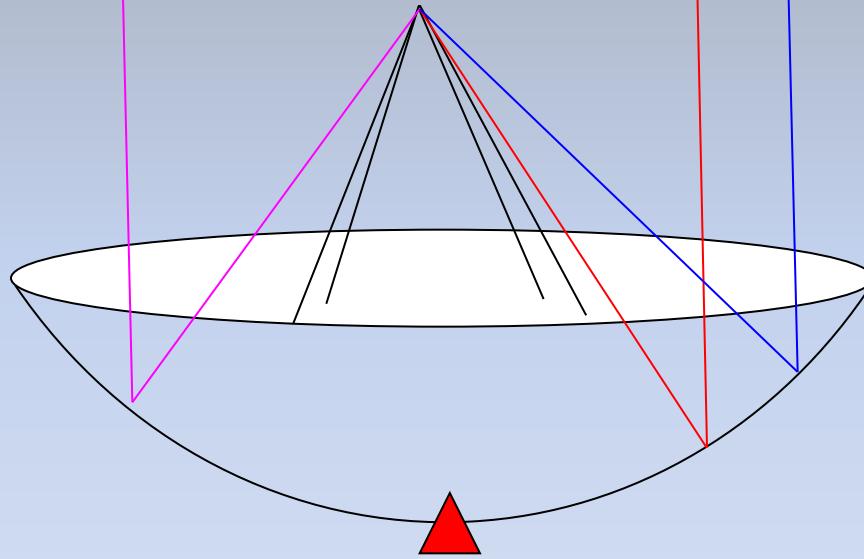




$$\Delta L(e) = \alpha_F \Delta F(e) + \alpha_R \Delta R(e) + \alpha_V \Delta V(e)$$

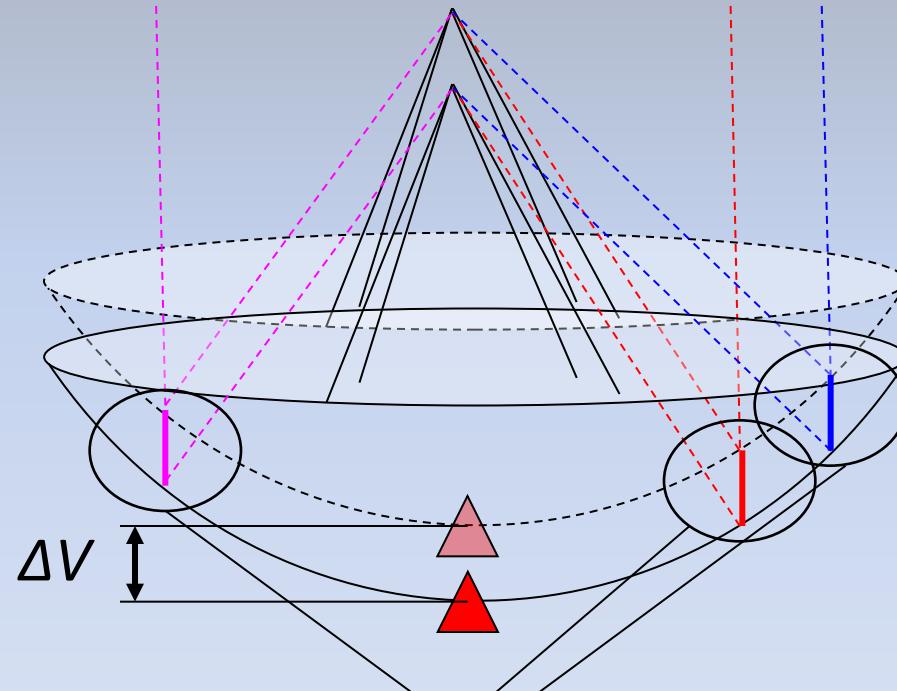


Vertex

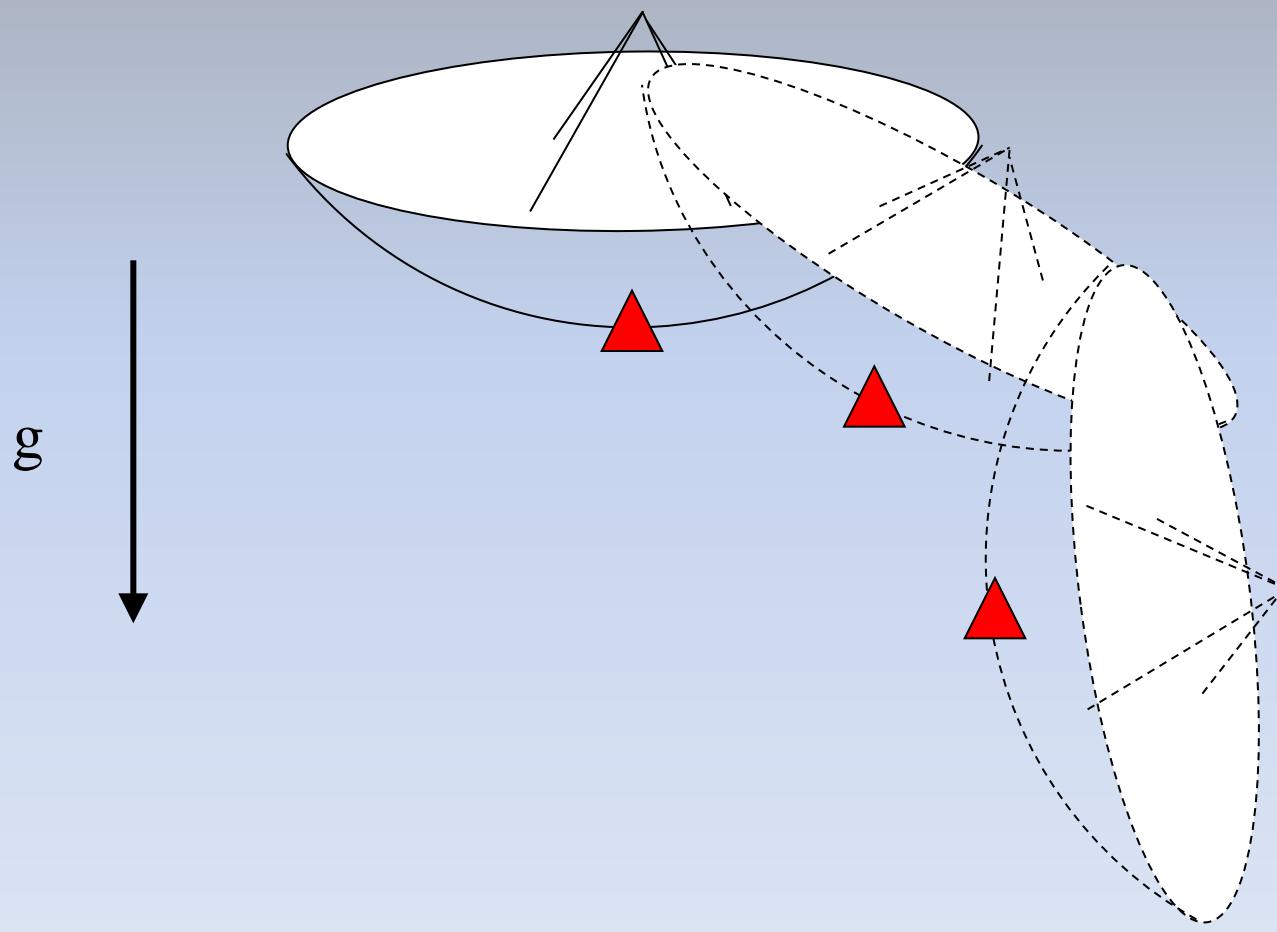




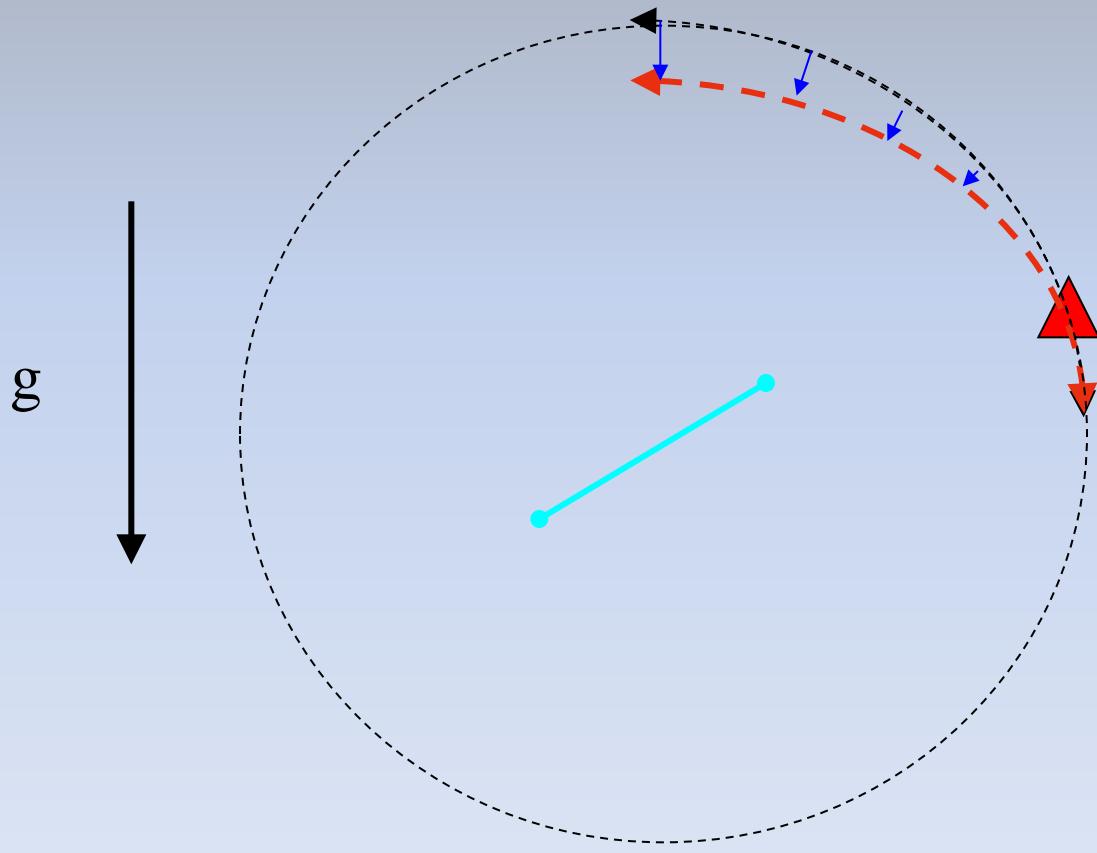
Vertex

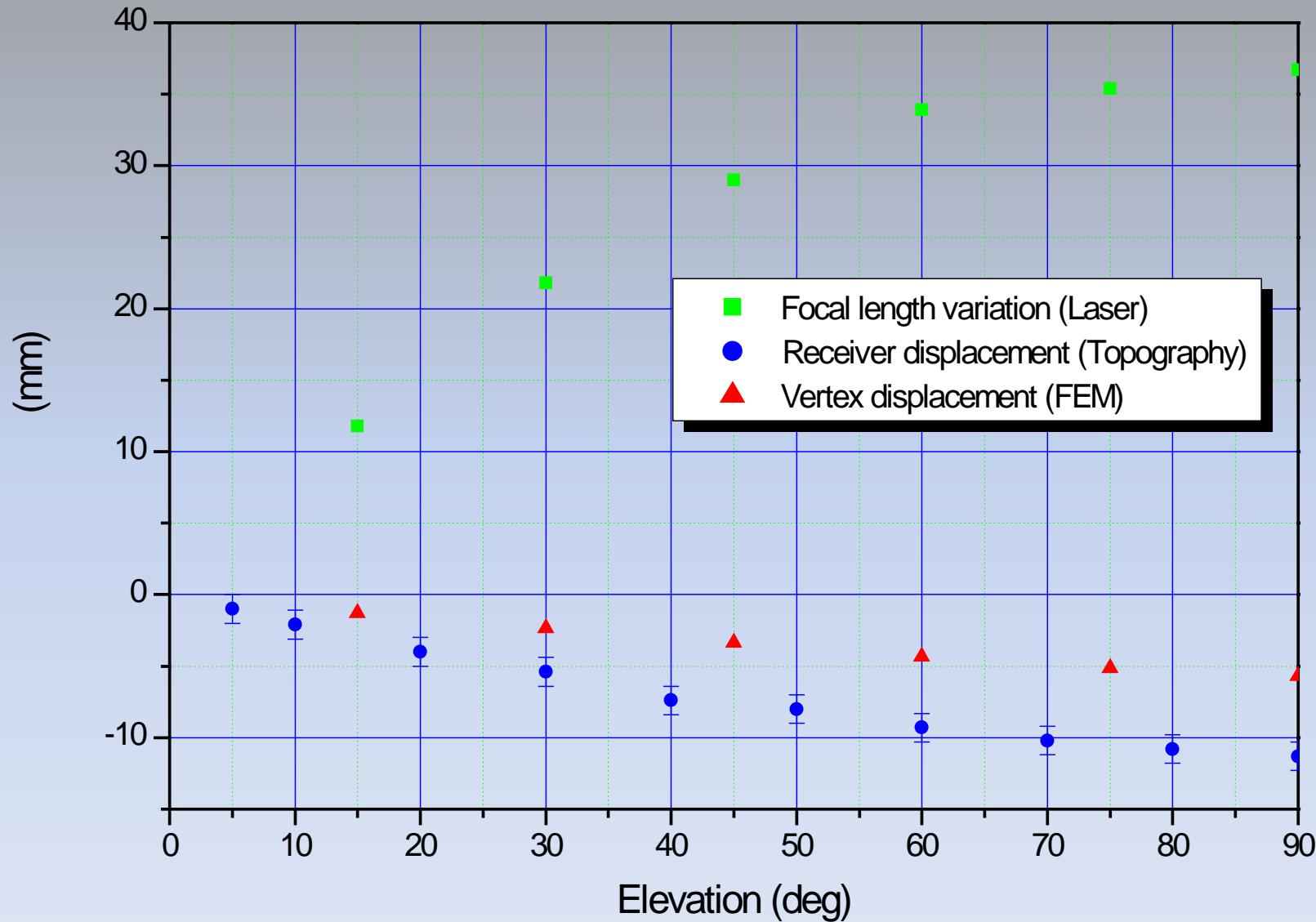


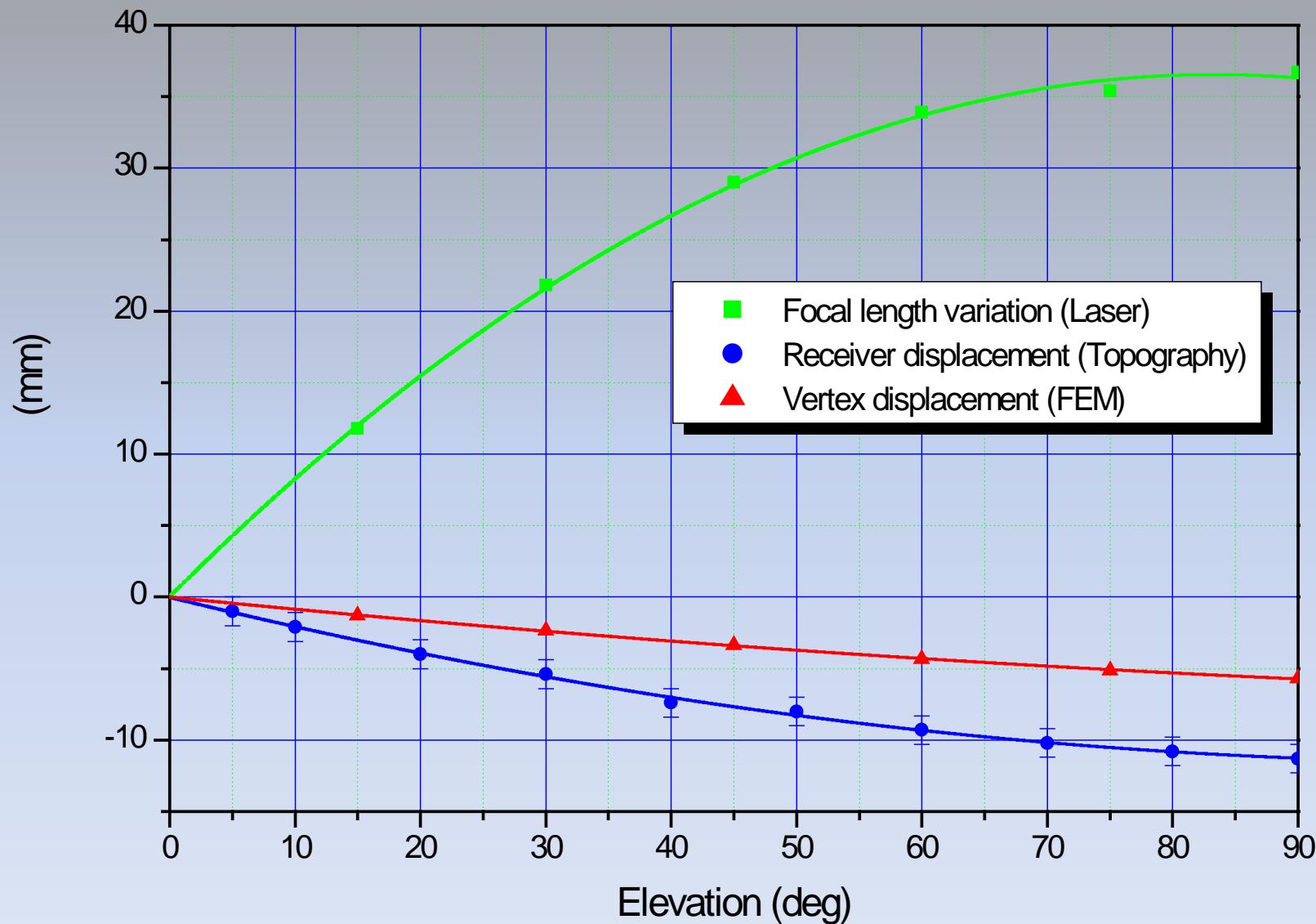
$$\Delta L(\Delta V(e), \dots)$$



# Displacement of the vertex







# How were the $\Delta$ s determined so far?

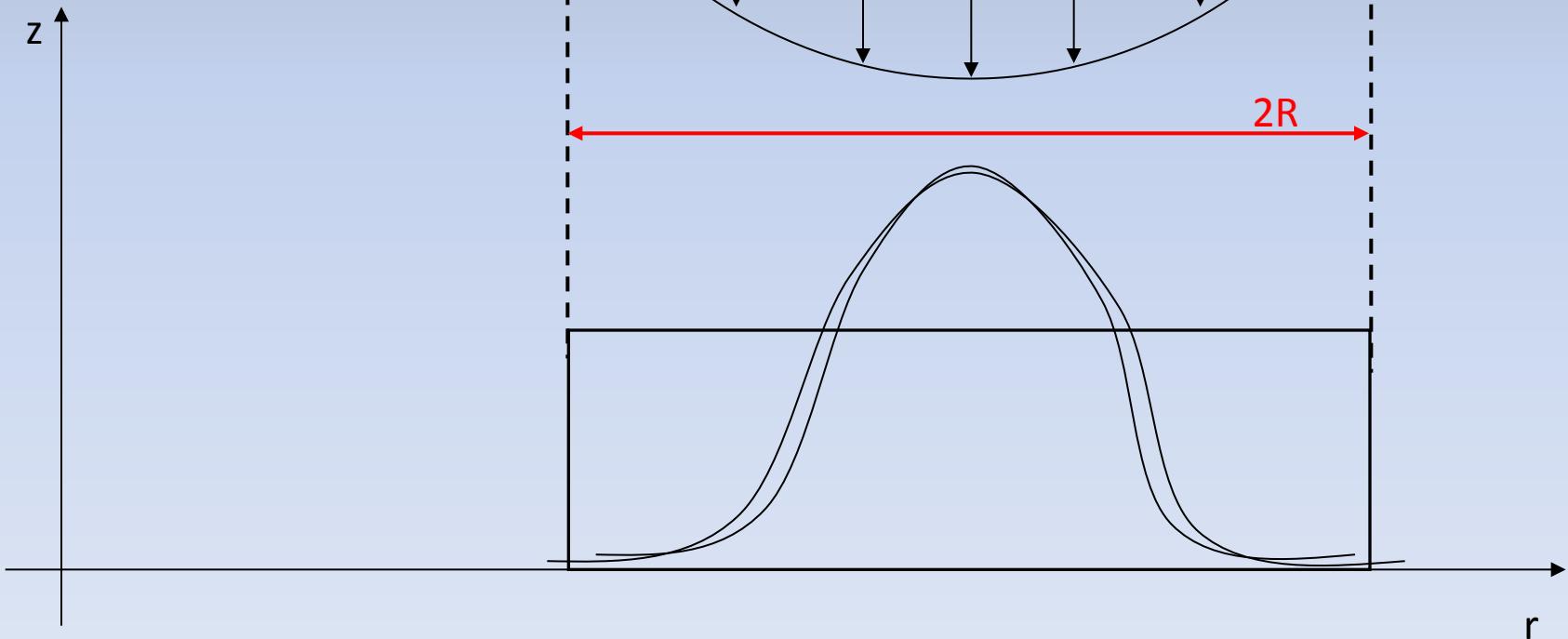
- Laser scanning for  $\Delta F$ : *Sarti et al. 2009; Holst et al. 2012; (J Surv Eng)*
- Finite Element Model: all  $\Delta$ s in Clark and Thomsen 1988 (NASA Tech Rep);  $\Delta F$  in *Sarti et al. 2009 (J Geodesy)*
- Triangulation and trilateration:  $\Delta R$  in *Sarti et al. 2009 (J Geodesy)*

$$\Delta L(e) = \alpha_F \Delta F(e) + \alpha_R \Delta R(e) + \alpha_V \Delta V(e)$$

$$I_u(r) = k_u$$

$$I_g(r) = k_g e^{-\beta_g r^2}$$

$$I_b(r) = k_b \left[ 1 - \left( \frac{r}{R} \right)^2 \right]^{\beta_b}$$



$$\alpha_R^i = \int_S h(r) I(r) dS$$

Primary focus configuration:

$$\alpha'_F = 1 - \alpha'_R$$

$$\alpha'_V = -1 - \alpha'_R$$

Cassegrainian configuration:

$$\alpha''_F = 2 - 2\alpha''_R$$

$$\alpha''_V = -1 - 2\alpha''_R$$

*Abbondanza & Sarti (2010), J Geodesy*

**Table 1** Values of the linear coefficients  $\alpha$  computed for the Medina and Noto telescopes for primary and Cassegrainian configurations adopting (a) IFun1: uniform, (b) IFun2: Gaussian and (c) IFun3: binomial illumination functions

	IFun 1	IFun 2	IFun 3	4 <sup>a</sup>
Primary focus				
$\alpha'_R$	0.56	0.75	0.68	0.62
$\alpha'_V$	-1.56	-1.75	-1.68	-1.62
$\alpha'_F$	0.44	0.25	0.32	0.38
Secondary focus				
$\alpha''_R$	0.77	0.84	0.83	0.81
$\alpha''_V$	-2.54	-2.68	-2.66	-2.62
$\alpha''_F$	0.46	0.32	0.34	0.38

Last column contains values based on a rule of thumb as derived by and adapted from [Cha \(1987\)](#)

<sup>a</sup> Adapted from [Cha \(1987\)](#)

<sup>b</sup> Derived by [Cha \(1987\)](#)

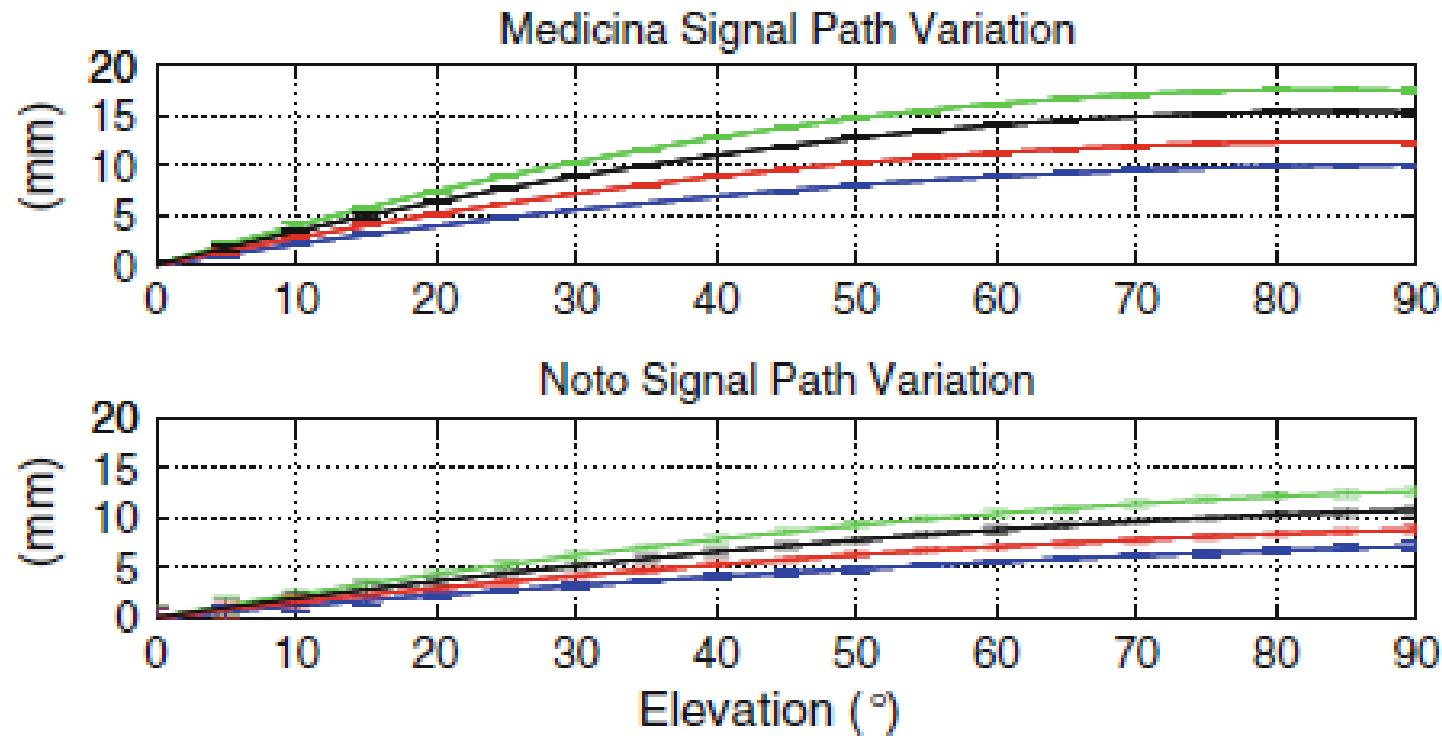


Fig. 3 Signal path variations of Medicina and Noto telescopes computed with  $\alpha$  coefficients contained in Table 1: the *green curve* corresponds to the uniform illumination function; the *black curve* to the simplified computation reported in *column 4*; the *red* and *blue curves* correspond to the binomial and Gaussian illumination function, respectively

# $\Delta L$ in VLBI data analysis

(Sarti *et al.* 2011, *J Geodesy*)

- VTD/Solve allows computation of theoretical VLBI path delay (Petrov 2008, Ref Manual)
- 7.08 millions of VLBI observations processed with VTD/Solve
- 345 experiments for Medicina; 150 for Noto
- Ran two solutions:
  - R1: reference solution without SPV model
  - A1: as R1 but SPV models for Medicina and Noto applied

Station	$\Delta U$ (mm)	$\Delta E$ (mm)	$\Delta N$ (mm)	# Sess
DSS65	0.0	0.0	0.0	86
MATERA	0.0	0.0	0.0	632
MEDICINA	8.9	0.0	0.0	345
NOTO	6.7	0.0	0.0	150
NYALES20	0.0	0.0	0.0	912
ONSALA60	0.0	0.0	0.0	632
WETTZELL	0.0	0.0	0.0	2612

# Conclusions

- Gravitational deformations
  - may deform the structure of the telescope to a non negligible extent
  - modify the Up component of the telescopes Reference Point
  - do not modify the estimate of other S-geodetic parameters
  - may impact the scale of the VLBI network
- $\Delta L$  models
  - are strictly telescope-dependent and as such should be determined specifically for each VLBI telescope
  - must be computed properly:
    - with  $\alpha_i$  coefficients computed with realistic IF
    - with  $\Delta R$ ,  $\Delta V$  and  $\Delta F$  models deriving from *ad hoc* high precision terrestrial surveys and/or FEM
  - Cannot be inferred from the telescope dimensions (NO scaling!)
- The height bias cannot be determined relying on VLBI data alone!