# Spectral Oscillation of Impurity Photoconductivity and the Photo-Hall Effect in $p$-Type InSb†

V. J. MAZURCZYK* AND H. Y. FAN
*Department of Physics, Purdue University, Lafayette, Indiana 47907*
(Received 1 December 1969)

Photoconductivity and the photo-Hall effect associated with the impurity in $p$-type InSb were studied with applied magnetic fields up to 8 kG, in the temperature range 5–~20°K. Both showed a similar oscillation with wavelength, but the magnitude of oscillation was different. A wavelength dependence of Hall mobility was deduced. The effect of the strength of magnetic field was investigated. The normal conductivity of one type of samples studied was dominated by free holes generated by background radiation and thermal ionization. In the second type of samples at 5°K, the Hall coefficient was strongly influenced by free holes but the conductivity was determined chiefly by impurity-band conduction. The results show that a mobility variation was mainly responsible for the oscillation of photoconductivity in the second type of samples at 5°K. The conclusion could be true in the other cases also, according to the results, but it was not proven definitely.

## I. INTRODUCTION

SPECTRAL oscillation of photoconductivity has been observed and studied in a number of semiconductors at low temperatures.[1] The period $\Delta h\nu$ of oscillation is related to the longitudinal optical-phonon energy $\hbar\omega_0$. The basic cause of the oscillation is that the energy distribution of carriers is nonequilibrium and depends on $h\nu$ in an oscillatory manner because of LO-phonon emission. The energy with which carriers are generated depends on $h\nu$. A fluctuation of the energy distribution occurs if the lifetime $\tau_l$ of excess carriers is long in comparison with the time $\tau_{0p}$ required for the emission of LO phonons and short compared with the time $\tau_\epsilon$ needed for energy thermalization: $\tau_\epsilon \gtrsim \tau_l > \tau_{0p}$. The photoconductivity oscillates if the mobility or the lifetime $\tau_l$ has an energy dependence. It is of interest to obtain good evidence regarding the relative role of each factor in a given case.

An energy dependence of mobility follows from the normal dependence of scattering on energy. If the mobility increases with energy at energies close to the band edge, as is usually the case at low temperatures, minima of intrinsic photoconductivity occur at

$$h\nu = E_g + (1 + m_e/m_h) n\hbar\omega_0 , \tag{1}$$

assuming that the electrons are mainly responsible for the photoconductivity. The expression for extrinsic photoconductivity is

$$h\nu = E_i + n\hbar\omega_0 . \tag{2}$$

$E_g$ is the energy gap, $E_i$ is the impurity ionization energy, and $n=0, 1, 2, \ldots$. Different expressions may apply in case many-valley energy bands are involved.[2] It has been pointed out that carriers with energy close to $\hbar\omega_0$ may show as a group a strong reduction of mobility due to the scattering by LO phonons since the carriers, with random velocity in the direction of acceleration, have their energy increased by the field and will be subject to scattering by the emission of LO phonons.[3] The same expressions (1) and (2) apply for the positions of minima. For brevity, this effect is referred to as a mechanism of preferential momentum loss. Since the effect is expected only for carrier energies close to $\hbar\omega_0$, it should produce dips in photoconductivity which sharpen with decreasing electric field.

The recombination rate may be expected to depend on the carrier energy. If the rate decreases with energy, the lifetime of excess carriers and the photoconductivity would also have minima given by (1) and (2). When the lifetime is especially low at a certain energy $\epsilon_1$, minima in intrinsic photoconductivity would occur at

$$h\nu = E_g + (1 + m_e/m_h)(\epsilon_1 + n\hbar\omega_0) , \tag{3}$$

and the minima in impurity photoconductivity would occur at

$$h\nu = E_i + \epsilon_1 + n\hbar\omega_0 . \tag{4}$$

One of the mechanisms likely to give this situation is recombination involving some state as an intermediate step. Efficient recombination may occur when energetically the carrier can form this state by the emission of LO phonons. This case will be referred to as resonance recombination. For several semiconductors, it was found from the positions of minima that the observed oscillation of intrinsic photoconductivity was caused by recombination involving the exciton state.[4] As to impurity photoconductivity, two sets of minima were observed in CdTe, one set of which was attributed

† Work supported in part by the U. S. Office of Naval Research.
* Present address: U. S. Army Electronics Command, Night Vision Laboratory, Fort Belvoir, Va. 22060.

[1] For a summary and references, see H. Y. Fan, in *Proceedings of the Ninth International Conference on the Physics of Semiconductors, Moscow, 1968* (Publishing House, Nauka, Moscow, 1968), p. 135.

[2] G. G. Kovalevskaya, D. N. Nasledov, and S. V. Slobodchikov, Phys. Status Solidi **23**, 755 (1967); H. J. Stocker, Solid State Commun. **6**, 125 (1968); A. Onton, Bull. Am. Phys. Soc. **14**, 369 (1969).

[3] V. F. Elesin and E. A. Manykin, Zh. Eksperim. i Teor. Fiz. **50**, 1381 (1966) [Soviet Phys. JETP **23**, 917 (1966)]; H. J. Stocker and H. Kaplan, Phys. Rev. **150**, 619 (1966).

[4] D. W. Langer, Y. S. Park, and R. N. Euwema, Phys. Rev. **152**, 788 (1966); A. Coret, R. Levy, J. B. Grun, A. Mysyrowicz, and S. Nikitine, Phys. Letters **23**, 596 (1966); P. Denham, E. G. Lightowlers, and P. J. Dean, Phys. Rev. **161**, 762 (1967); Y. S. Park and C. McConn, Phys. Letters **26A**, 483 (1968).

TABLE I. Electrical properties of the samples.

| $T$ | $\rho$ ($\Omega$ cm) 77°K | $\rho$ ($\Omega$ cm) 4.2°K | $R$ (cm³/C) 77°K | $R$ (cm³/C) 4.2°K | $p$ (cm⁻³) 77°K | $\mu_H$ (cm²/V sec) 77°K | $\mu_H$ (cm²/V sec) 4.2°K |
|---|---|---|---|---|---|---|---|
| Sample $T$ | 0.5 | $3.2\times10^4$ | $1.66\times10^3$ | $\sim10^6$ | $3.8\times10^{15}$ | 3320 | $\sim30$ |
| Sample $C$ | 1.6 | $\sim10^9$ | $1\times10^4$ | | $6.3\times10^{14}$ | 6340 | |

to resonance recombination with the ground state of the impurity.[5]

In the work reported here, the effects of magnetic field, particularly the photo-Hall effect, were studied in connection with photoconductivity for the purpose of obtaining a clearer understanding of the oscillation. For the interpretation of Hall effect, it is important to have a nearly uniform photoexcitation in the sample. This condition can be met in the study of impurity photoconductivity where the absorption can be sufficiently low. The results show that the usual energy dependence of mobility plays a significant part in the oscillation in $p$-type InSb. In one case, it is possible to show that the mobility variation is mainly responsible for the oscillation.

## II. EXPERIMENTAL DETAILS

The sample to be measured was mounted on the cooled copper plate in a Dewar. A sample temperature of $\sim 20°$K was obtained. Lower temperatures down to $\sim 5°$K were reached with the use of an exchange gas can which was provided with a window of high-purity InSb in order to pass the radiation for excitation. The exciting radiation was chopped at 13 cps. When the sample resistance was high, the load resistance used in the measurement was kept below $10^7$ $\Omega$ in order to have a speed of response sufficiently fast for the chopping frequency. In photo-Hall effect, the quantity of interest is $\Delta R/R=\Delta V_H/V_H-\Delta i/i$, where $V_H$ is the Hall voltage and $i$ is the sample current. The effect of $\Delta i$ was made negligible by using a current-limiting resistor in series with the sample and battery.

Studies were carried out on two types of samples. Samples $T$ were cut from material supplied by Texas Instruments, and samples $C$ were cut from material supplied by Cominco. The electrical properties measured are listed in Table I. The slope of the Hall coefficient $R$ versus $1/T$ corresponded to an acceptor ionization energy of $\epsilon_i\simeq 8$ meV for samples $T$ and $\epsilon_i\simeq 29$ meV for samples $C$. Photoluminesence measurements showed impurities with $\epsilon_i\sim 8$ meV in sample $T$. The presence of such shallow impurity was also indicated by photoluminescence of sample $C$. However, the shallow impurity was not revealed by the Hall coefficient of sample $C$. Apparently, it was compensated hence it could not have contributed to the photoexcitation. At low temperatures, $T<10°$K, the electrical property of both types of samples were determined by impurity conduction.

At 77°K, the Hall coefficient of both samples showed a dependence on magnetic field. The behavior was caused by the presence of light holes with a much larger mobility. With two groups, 1 and 2, of the same type of carriers, the Hall coefficient at magnetic field $B$ is given by

$$R(B)=(F_1/ep_1)F\,, \tag{5}$$

where $F_1=R_1P_1$ is the usual parameter characterizing the carriers of group 1 alone, and

$$F=\frac{p_1+p_2}{p_1}\,\frac{1+(p_2/p_1)(\mu_{H2}/\mu_{H1})^2+(\mu_{H2}B/c)^2(1+F_1p_2/F_2p_1)}{(1+\sigma_2/\sigma_1)^2+(\mu_{H2}B/c)^2(1+F_1p_2/F_2p_1)^2}\,. \tag{6}$$

In case $p_2\ll p_1$, and

$$\mu_{H2}B/c\gg 1+(p_2/p_1)(\mu_{H2}/\mu_{H1})^2\,;$$

we get

$$R(B)\sim F_1/ep_1. \tag{7}$$

Experimentally, $R$ becomes nearly constant, $\sim 0.66$ of the value at low fields, for $B\gtrsim 3$ kG showing that the effect of light holes was suppressed. The result is consistent with the reported values of $p_2/p_1=1.17\times10^{-2}$ and $\mu_{H2}/\mu_{H1}=7.4$ for the light and heavy holes. In photo-Hall measurements at low temperature, the effect of light holes should be small at even lower fields with the presence of impurity conduction.

In the Dewar for optical measurements, the sample was exposed to background radiation which produced free holes and affected the electrical properties of the sample. Consequently, the measured resistivity and Hall coefficient were nearly constant at temperatures below $\sim 18°$K. When sample $C$ was shielded from background radiation, the measured resistivity was very high, $>10^9$ $\Omega$ cm, at low temperatures. A value of $2.2\times10^4$ $\Omega$ cm was measured with background radiation. Evidently, the conductivity and Hall coefficient were determined by free holes generated by the background radiation. For sample $T$ at $\sim 5°$K, $\rho=3.2\times10^4$ $\Omega$ cm with $\mu_H\sim 30$ cm²/V sec in the absence of background radiation. With background radiation, the measurement did not show a big drop of $\rho$ but gave a con-

[5] A. L. Mears, A. R. L. Spray, and R. A. Stradling, J. Phys. C 1, 1412 (1968).

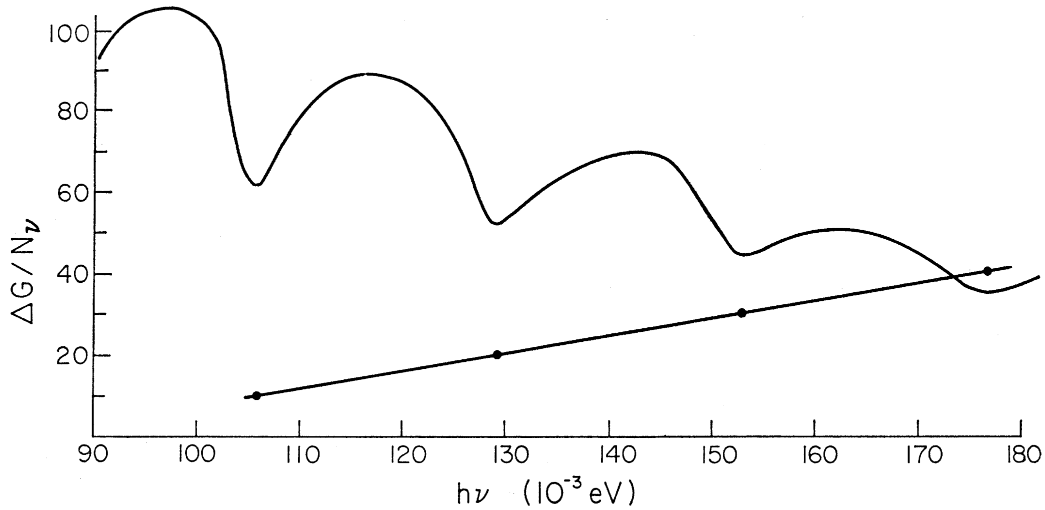


FIG. 1. Spectrum of photoconductivity per unit number of absorbed photons, $\Delta G/N_\nu$, in arbitrary units. Sample $T$ at 5°K. The points connected by the straight line are arbitrary order numbers of the minima in the spectrum.

siderably larger value of $\mu_H \sim 270$ cm²/V sec. Sample $T$ had a much higher conductivity than that of sample $C$. Estimates show that the conductivity of holes generated by background radiation had the same order of magnitude in both samples and it was not sufficient to affect the conductivity of sample $T$, the increase of $\mu_H$ being the result of the fact that $\mu_H$ was much larger for free holes than for impurity conduction. Because of the effect of background radiation, the magnitude of photoconductive signal did not vary strongly with the temperature of the sample below 18°K.

### III. PHOTOCONDUCTIVITY

Oscillation in the extrinsic photoconductivity[6] was observed in both samples $C$ and $T$. The period of the oscillation was equal to the energy $\hbar\omega_0 = 24.4$ meV of longitudinal optical phonons. This is to be expected in contrast to the case of intrinsic photoconductivity where the period should be $(1+m_e/m_h)\hbar\omega_0$. The spectrum is shown in Fig. 1 for sample $T$. Neither sample showed any structure in the absorption over the spectral region, thus ruling out variation of absorption as a cause for oscillation. In both samples, the oscillation began to smear out if the electric field was made sufficiently large. However, the spectrum was independent of the field for $E \leq 1$ V/cm in sample $C$, and up to a higher field in sample $T$. This observation was not influenced by the resolution used. Hence the mechanism of preferential momentum loss can be ruled out.

It cannot be simply seen whether the oscillation involves the variation of carrier concentration. In the case of oscillation of intrinsic photoconductivity in $n$-type InSb, the variation of carrier concentration was ruled out as a mechanism on the ground that the photoexcitation produced a very small change in the electron concentration.[7] In the present case, the photoconductive signal corresponded also to a small change in the hole concentration which had been enhanced by background radiation. The following consideration brings out the difference between the two cases. Consider $p$-type InSb with one kind of acceptor impurity. The steady-state condition under photoexcitation is

$$G = r\Delta p(N_a^- + \Delta N_a^-) + r_0 p_0 (N_a^- + \Delta N_a^-) - g(N_a^0 + \Delta N_a^0), \quad (8)$$

where $G$ is the rate of excitation; $r$, $r_0$, and $g$ are appropriate coefficients; and $N_a^0$ and $N_a^-$ are the concentrations of neutral and ionized acceptors, respectively. In the presence of background radiation alone, we have

$$r_0 p_0 N_a^- = g N_a^0. \quad (9)$$

In view of the fact that

$$p_0 = N_a^- \quad \text{and} \quad \Delta p = \Delta N_a^- = -\Delta N_a^0,$$

we get

$$G = r\Delta p(N_a^- + \Delta N_a^-) + r_0 p_0 \Delta N_a^- - g\Delta N_a^0 \quad (10)$$

or

$$G = \Delta p r N_a^- [(1 + \Delta N_a^-/N_a^-) + (r_0/r)(1 + N_a^-/N_a^0)]. \quad (11)$$

The coefficients $r_0$ and $r$ can be expected to be of similar orders of magnitude. With $N_a^- < N_a^0$, we see that the recombination due to $\Delta p$, i.e., the first term in (10) and (11), may be important. Then $\Delta p$ depends on $r$ which can vary as the energy distribution of $\Delta p$ varies with the wavelength. In the case of intrinsic photoconductivity in $n$-type InSb at low temperatures,

[6] H. J. Stocker, H. Levenstein, and C. R. Stannard, Jr., Phys. Rev. 150, 613 (1966).

[7] V. J. Mazurczyk, G. V. Ilmenkov, and H. Y. Fan, Phys. Letters 21, 250 (1966).

we have, in place of (10),

$$G=r\Delta n(p_0+\Delta p)+r_0n_0\Delta p+g\Delta p. \qquad (12)$$

The third term gives the reduction of electron-hole generation due to the increase in hole concentration, $\Delta p$. It is obviously negligible. The hole concentration $p_0$ due to thermal generation and background radiation must be small. With

$$\Delta n \ll n_0 \quad \text{and} \quad \Delta n p_0 \ll n_0 \Delta p,$$

we get

$$G \simeq r_0 n_0 \Delta p. \qquad (13)$$

The recombination is mainly due to $n_0$. $\Delta p$, and hence $\Delta n$, do not vary with the wavelength of radiation.

When the arbitrary order numbers of the minima in the spectrum are plotted against $h\nu$, a straight line can be drawn through the series of points separated by $\Delta h\nu \simeq \hbar\omega_0$, as shown by the straight line in Fig. 1. Extrapolation of the straight line shows that $h\nu$ of the minima can be expressed by

$$h\nu=\Delta+m\hbar\omega_0, \qquad (14)$$

where $m$ is an integer, $\Delta=9.4$ meV for sample $T$, and $\Delta\sim 1.3$ meV for sample $C$. According to (4), the result implies $E_i\sim 9.4$ meV for sample $T$ and $E_i\sim\hbar\omega_0+\Delta=25.7$ meV for sample $C$. These values are somewhat different from the values, $E_i\sim 8$ meV for sample $T$ and $E_i\sim 29$ meV for sample $C$, deduced from Hall measurements. The discrepancy may be attributable to the uncertainties of the values obtained from the two types of measurements.

The mechanism of resonance recombination requires the presence of a level the ionization energy of which will be denoted by $E_r$. The smallest value of $E_r$ is $E_r=\hbar\omega_0-\epsilon_1$. Referring to (4) and (14), we get $\Delta=E_i-E_r$ if $\Delta<E_i$ and $\Delta=E_i-E_r+\hbar\omega_0$ if $\Delta>E_i$. Thus the result for sample $T$ requires a level with an ionization energy of $E_r=\hbar\omega_0+E_i-\Delta=23$ meV. Since there was no evidence for the presence of such a level, the mechanism of resonance recombination is ruled out. In the case of sample $C$, the small value of $\Delta$ in comparison with $E_i$ does not justify a special consideration of this mechanism.

## IV. EFFECTS OF MAGNETIC FIELD

The effect of magnetic field on the magnitude of photoconductivity and the photo-Hall effect were investigated. For crystals of cubic symmetry, the Hall coefficient is given by the expression

$$R=(1/B)\sigma_{xy}/(\sigma_{xx}^2+\sigma_{xy}^2), \qquad (15)$$

where $\sigma_{xx}$ and $\sigma_{xy}$ are tensor components of the conductivity under the magnetic field $B$ applied along the $z$ direction. The tensor components depend on the relaxation time and are given by integration over the distribution function $f_0$. The Hall mobility $\mu_H$ is

$$\mu_H B/c=\sigma_{xy}/\sigma_{xx}. \qquad (16)$$

Under photoexcitation, the distribution function is changed to $f=f_0+\Delta f$, and the changes $\Delta\sigma_{xx}$ and $\Delta\sigma_{xy}$ of the conductivity tensor are given by similar integration over $\Delta f$. We define $\mu_H'$ by

$$\mu_H' B/c=\Delta\sigma_{xy}/\Delta\sigma_{xx}. \qquad (17)$$

It can be shown that the change in Hall coefficient is given by

$$\frac{\Delta R}{R}=\frac{\Delta\sigma_{xx}}{\sigma_{xx}}\left[\frac{\mu_H'}{\mu_H}-2\frac{1+(\mu_H B/c)^2\mu_H'/\mu_H}{1+(\mu_H B/c)^2}\right]. \qquad (18)$$

The conductivity given by the measured current density $j_x$ and the electric field $E_x$ is

$$\sigma=j_x/E_x=\sigma_{xx}[1+(\sigma_{xy}/\sigma_{xx})^2]. \qquad (19)$$

The change $\Delta\sigma$ under photoexcitation is given by

$$\frac{\Delta\sigma}{\sigma}=\frac{\Delta\sigma_{xx}}{\sigma_{xx}}\left[1+\frac{2(\mu_H B/c)^2(\mu_H'/\mu_H-1)}{1+(\mu_H B/c)^2}\right]. \qquad (20)$$

For weak magnetic fields, we have

$$\Delta R/R\simeq(\Delta\sigma_{xx}/\sigma_{xx})(\mu_H'/\mu_H-2), \quad \Delta\sigma/\sigma\simeq\Delta\sigma_{xx}/\sigma_{xx}. \qquad (21)$$

Photoconductivity $\Delta\sigma$ may be expressed as follows:

$$\Delta\sigma=\mu'\Delta p. \qquad (22)$$

The values of $\Delta R/R$ and $\Delta\sigma/\sigma$ can be obtained from the measurements. Therefore the values of $\mu_H'/\mu_H$ or $\mu_H'$ can be obtained at various wavelengths of photoexcitation. However, the relation between $\mu'$ and $\mu_H'$ depends on the carrier scattering and the distribution function $\Delta f$. Since $\Delta f$ depends on the wavelength of excitation, $\mu'/\mu_H'$ may vary somewhat with wavelength. To this extent, using only these results to determine the roles of $\mu'$ and $\Delta p$ in the spectral variation of $\Delta\sigma$ involves some uncertainty.

### A. Sample $T$, 5°K

Figure 2 shows the photoconductivity and photo-Hall data measured at 5°K with $B=1$ kG and an electric field of a few V per cm. No changes in the oscillation was observed for smaller electric and magnetic fields. Experimentally,

$$\frac{\Delta R}{R}>0 \quad \text{and} \quad \frac{\Delta R/R}{\Delta\sigma/\sigma}\sim 70 \text{ at the maximum}, \qquad (23)$$

where $R$ and $\sigma$ are values measured with background radiation. According to (21), $\mu_H'/\mu_H\sim 72$, which indicates that the mobility of photoexcited free holes was much larger than the background mobility $\mu_H$ which was influenced strongly by impurity conduction. With

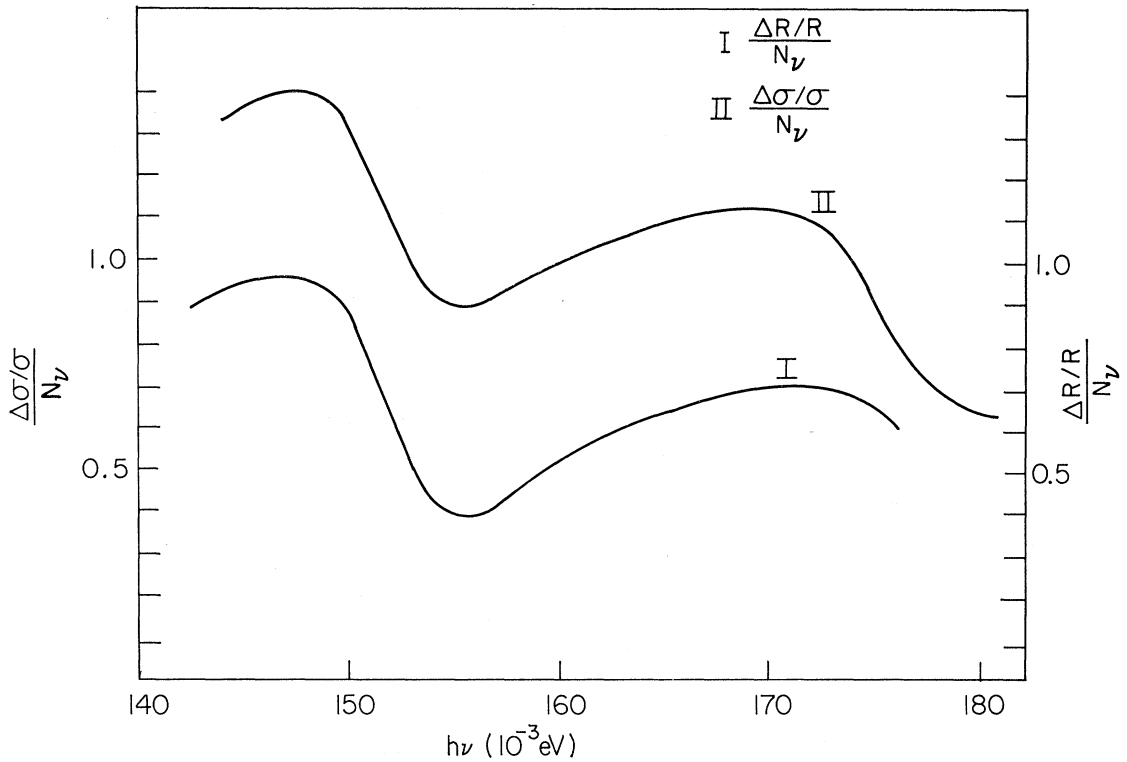


FIG. 2. Normalized photoconductivity $(\Delta\sigma/\sigma)/N_\nu$ and photo-Hall effect $(\Delta R/R)/N_\nu$ in arbitrary units. Sample $T$ at 5°K. $B=1$ kG.

respect to wavelength dependence, the data indicate that

$$\frac{\Delta R/R}{\Delta\sigma/\sigma}\propto\frac{\Delta\sigma}{N_\nu}\quad\text{or}\quad\frac{\Delta R/R}{(\Delta\sigma/\sigma)^2}N_\nu\sim\text{const}\,,\tag{24}$$

as shown by the curves in Fig. 3. $N_\nu$ denotes the number of photons absorbed per unit volume. According to (22) and (21),

$$\frac{\Delta R/R}{(\Delta\sigma/\sigma)^2}N_\nu\simeq\frac{\mu_H'}{\mu_H}\frac{N_\nu}{\mu'\Delta p/\sigma}\propto\frac{\mu_H'}{\mu'}\frac{1}{\Delta p/N_\nu}\,.\tag{25}$$

Except for an uncertainty in the wavelength dependence of $\mu_H'/\mu'$, our data indicate that $\Delta p/N_\nu$ was independent of wavelength and the oscillltion of $\Delta\sigma$ was caused by the variation of $\mu'$.

Consider now the effect of magnetic field on $\Delta\sigma$. If the oscillation of $\Delta\sigma$ is mainly caused by the variation of $\mu'$, then $\Delta\sigma$ should decrease with magnetic field more strongly at a maximum than at a minimum. This can be seen from the data in Fig. 4 measured at the maximum and at the minimum. Since the magnetic field reduced $\Delta G$ more strongly at the maximum than at the minimum, the oscillation was reduced at larger fields. According to (20),

$$\frac{\Delta\sigma}{\Delta\sigma(0)}=\frac{\Delta\sigma_{xx}}{\Delta\sigma(0)}\left[1+\left(\frac{\mu_H B}{c}\right)^2\left(\frac{2\mu_H'}{\mu_H}-1\right)\right].\tag{26}$$

For the calculation of $\Delta\sigma_{xx}/\Delta\sigma(0)$, the band structure, the distribution function, and the scattering mechanism must be known. Consider for simplicity a spherical energy band, scattering by an ionized impurity, and Maxwell-Boltzmann distribution of $f_0$ and $\Delta f$. For this case,

$$\sigma_{xx}=\sigma_0 I\,,\quad\sigma_{xy}=\sigma_0\gamma^{1/2}J\,,$$
$$\mu_H/\mu_H(0)=J/I\,,\quad\sigma_{xx}/\sigma_{xx}(0)=I\,,\tag{27}$$

where

$$I=\frac{1}{p!}\int_0^\infty\frac{x^pe^{-x}}{1+\gamma x^3}dx\,,\quad J=\frac{1}{(p+1.5)!}\times\int_0^\infty\frac{x^{p+1.5}e^{-x}}{1+\gamma x^3}dx\,,\quad\gamma=\left(\frac{\pi^{1/2}}{8}\frac{B\mu(0)}{c}\right)^2.\tag{28}$$

For scattering by an ionized impurity, $p=3$. The values of the integrals $I$ and $J$ have been tabulated for different values of $\gamma$.[8] It follows that

$$\Delta\sigma_{xx}/\Delta\sigma_{xx}(0)=I'\,,\tag{29}$$

$$\mu_H'/\mu_H'(0)=J'/I'\,,\tag{30}$$

where $I'$ and $J'$ are given by expressions similar to those of $I$ and $J$, with $\gamma$ replaced by

$$\gamma'=[\tfrac{1}{8}\pi^{1/2}B\mu'(0)/c]^2.\tag{31}$$

[8] R. Dingle, D. Arndt, and S. Roy, Appl. Sci. Res. B6, 245 (1956).

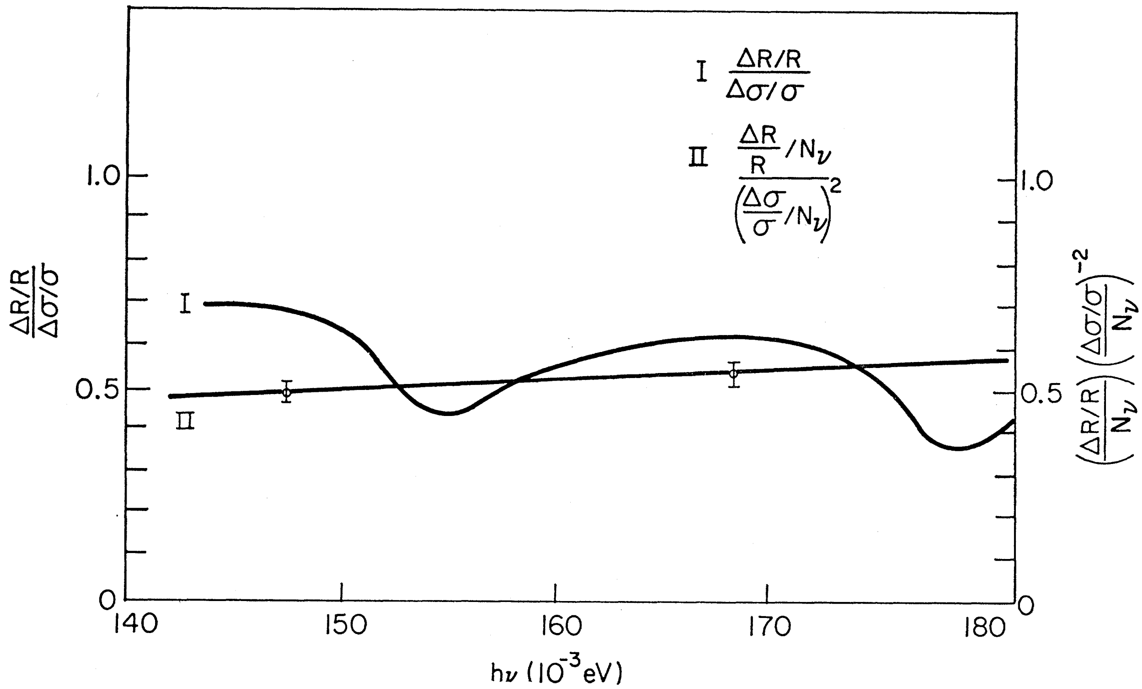


FIG. 3. Curves of $(\Delta R/R)/(\Delta\sigma/\sigma)$ and $[(\Delta R/R)/N_\nu]/[(\Delta\sigma/\sigma)/N_\nu]^2$ in arbitrary units. Sample $T$ at 5°K. $B=1$ kG.

Experimentally, the measured value of $\mu_H$ with background radiation was $\mu_H=270$ cm$^2$/V sec. $(\mu_H B/c)^2 \ll 1$ for magnetic fields, up to 7 kG, used in the experiment. We have, therefore,

$$\Delta\sigma/\Delta\sigma(0) \simeq \Delta\sigma_{xx}/\Delta\sigma(0) = \Delta\sigma_{xx}/\Delta\sigma_{xx}(0) = I'. \quad (32)$$

Using the value of $\mu_H$ and the value of $\mu_H'/\mu_H$ obtained at the maximum with low field, we get $[\mu_H'(0)]_{\max} \sim 1.94\times10^4$ cm$^2$/V sec, which gives $[\mu'(0)]_{\max} \sim 10^4$ cm$^2$/V sec for scattering by ionized impurity. The lower curve in Fig. 4 was calculated according to (32) by using this value of $[\mu'(0)]_{\max}$, and it fits well the data obtained at the maximum. The upper curve was calculated to fit the data obtained at the minimum. A value of $[\mu'(0)]_{\min}=0.58\times10^4$ cm$^2$/V sec was used. Experimentally, $\Delta\sigma_{\min}/\Delta\sigma_{\max} \simeq 0.62$, which is close to the value $[\mu'(0)]_{\min}/[\mu'(0)]_{\max}$. In these calculations, the use of Maxwell-Boltzmann type of distribution for $\Delta f$ is only an approximation. Furthermore, scattering by ionized impurities is dominant only for carriers of low energy, and it is uncertain that this type of scattering alone is a good approximation near a maximum of $\Delta\sigma$. The fit obtained and the reasonable values of $\mu'(0)$ may not be a strong justification for the correctness of the details of the model. The result does indicate that a variation of $\mu'$ was largely responsible for the oscillation of $\Delta\sigma$.

### B. Sample $T$, ~15°K

At this temperature, the conduction was dominated by free holes generated by thermal ionization. A value of $\mu_H(0) \sim 10^4$ cm$^2$/V sec was measured with low magnetic fields. The observed oscillation of $\Delta\sigma$ was somewhat smaller in amplitude than at 5°K. There was no change in the spectrum. The measurement gave $(\Delta R/R)/(\Delta\sigma/\sigma) \sim -0.8$, with very little dependence on wavelength, as shown in Fig. 5. According to (21), $\mu_H'/\mu_H \sim 1.2$. The fact that $\mu_H'$ was nearly independent of wavelength does not necessarily indicate a similar behavior for $\mu'$. Lattice scattering is more important at 15 than at 5°K, and the scattering is more significant for carriers of higher energy. The ratio $\mu_H'/\mu'$ tends to decrease with increasing importance of lattice scattering, e.g., in case of Maxwell-Boltzmann distribution $\mu_H/\mu=1.17$ for lattice scattering, whereas $\mu_H/\mu=1.93$ for scattering by ionized impurity. Since $\Delta f$ should be shifted to higher energy at a maximum of $\Delta\sigma$ than at a minimum, the ratio $\mu_H'/\mu'$ would be smaller at a maximum. Therefore $\mu'$ might have been significantly higher at a maximum than at a minimum even though $\mu_H'$ was nearly independent of wavelength.

The dependence of $\Delta\sigma$ on magnetic field was not helpful as for the lower temperature. In view of a much larger $\mu_H$ than at 5°K, (26) should be used instead of (32). At a minimum of $\Delta\sigma$, $[\mu_H']_{\min} \simeq \mu_H$ may be assumed. Taking $[\mu'(0)]_{\min}=[\mu_H']_{\min}/1.93 = \mu_H(0)/1.93$, and using Eqs. (26), (29), and (30), we calculate $[\Delta\sigma/\Delta\sigma(0)]_{\min}=0.93$ for $B=6$ kG. Since the measurement gave $(\Delta\sigma)_{\max} \simeq 1.5(\Delta\sigma)_{\min}$, we take $[\mu'(0)]_{\max}=1.5[\mu'(0)]_{\min}$ and $[\mu_H']_{\max}=1.93[\mu'(0)]_{\max}$. We get $[\Delta\sigma/\Delta\sigma(0)]_{\max}=0.88$ for $B=6$ kG. It is seen that the magnetoresistance effect is not expected to be very large although the calculation is only approxi-

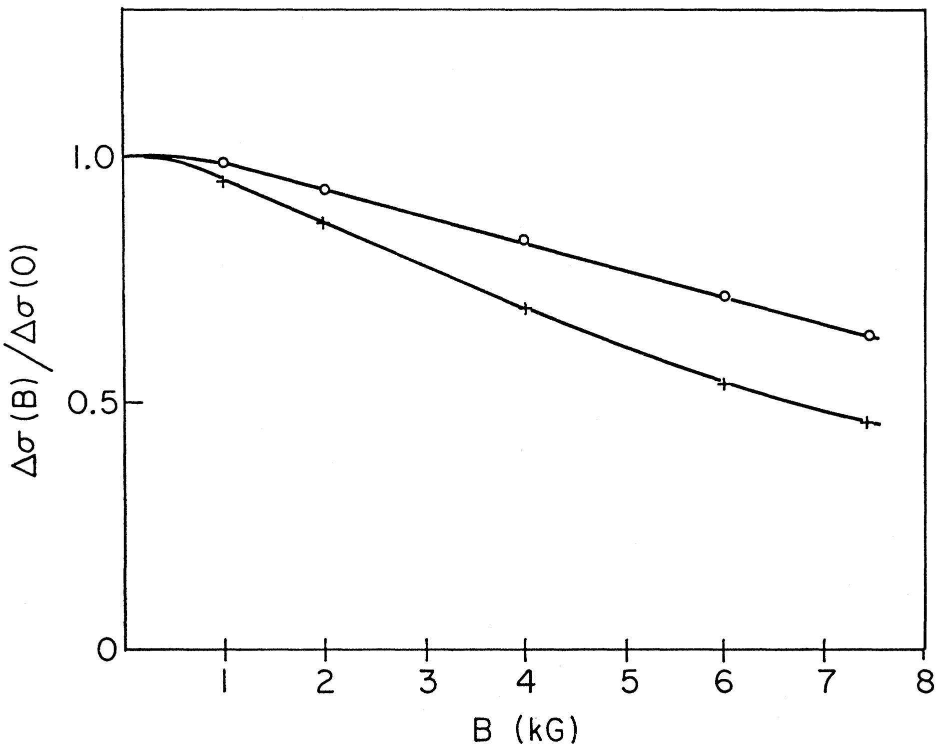


FIG. 4. Dependence of $\Delta\sigma(B)/\Delta\sigma(0)$ on the magnetic field. Sample $T$ at 5°K. The circles and the upper curve refer to a minimum in the spectrum of photoconductivity. The crosses and the lower curve refer to an adjacent maximum.

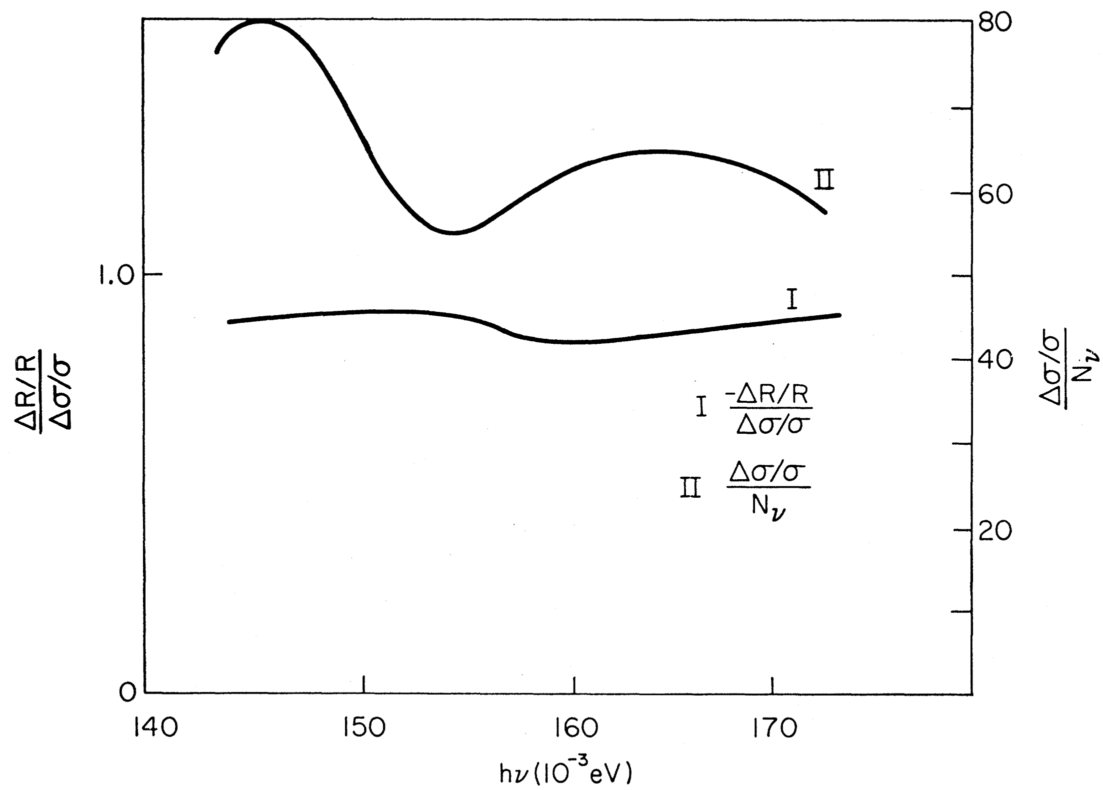


FIG. 5. Curves of $(\Delta\sigma/\sigma)/N_\nu$ in arbitrary units and $(-\Delta R/R)/(\Delta\sigma/\sigma)$. Sample $T$ at 18°K. $B=2$ kG.

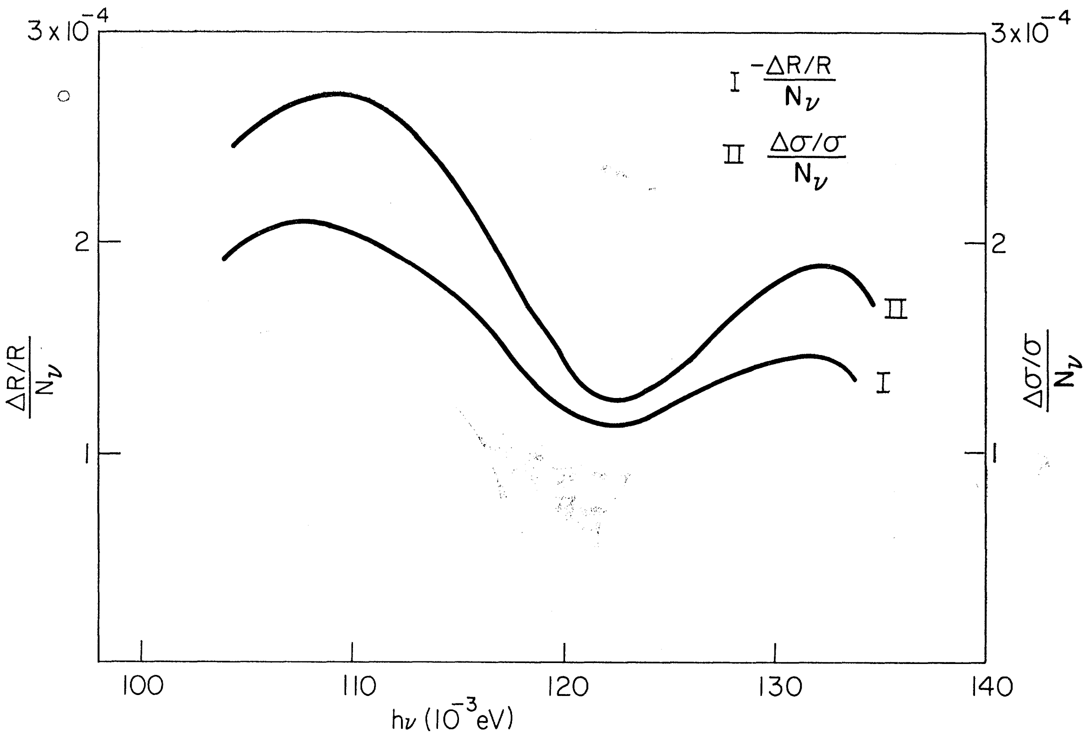


FIG. 6. Spectra of normalized photoconductivity $(\Delta\sigma/\sigma)/N_\nu$ and photo-Hall effect $(-\Delta R/R)/N_\nu$. Sample $C$ at 18°K. $B=2$ kG.

mate. In particular, the difference between the effects at a maximum and at a minimum can be expected to be small. In fact, it was not possible to detect a difference with the existing experimental uncertainty. On the whole, it is difficult to make a definite assessment of the importance of mobility variation in the oscillation of photoconductivity for this sample at $T=15$°K.

### C. Sample *C*

Measurements were made in the temperature range from 5 to 22°K. As in sample $T$ at 18°K, the conduction in this sample was dominated throughout this temperature range by free carriers generated by background radiation and thermal ionization. The measured value of $\mu_H$ was high, $\mu_H(0)=3.5\times10^4$ cm$^2$/V sec at 5°K. The measured $\Delta R$ was negative indicating that $\mu_H'/\mu_H<2$ according to (21). Figure 6 shows the results obtained at 18°K with $B=2$ kG. The variation of $-\Delta R/R$ is seen to be less than that of $\Delta\sigma/\sigma$. At the minimum, $-\Delta R/R$ was close to $\Delta\sigma/\sigma$, which may be an indication that carriers were generated near the band edge with $\mu_H'\sim\mu_H$. According to the data and (20), $(\mu_H')_{\max}/(\mu_H')_{\min}\sim1.2$. On the other hand, $(\Delta\sigma)_{\max}/(\Delta\sigma)_{\min}\sim1.7$. However, $(\mu')_{\max}/(\mu')_{\min}$ may be larger than $(\mu_H')_{\max}/(\mu_H')_{\min}$, as pointed out earlier. Although it is uncertain to what extent the oscillation of $\Delta\sigma$ was caused by the variation of $\mu'$, a variation of $\mu'$ must have played a part. If the oscillation of $\Delta\sigma$ were entirely produced by a variation of $\Delta p$, $-\Delta R/R$ should have shown an equal oscillation.

The magnetic field did not have a large effect on the spectrum of $\Delta\sigma$ for fields up to $\sim8$ kG. The oscillation seemed to be decreased by less than 10%. Intuitively, a larger decrease might be expected if the mobility $\mu'$ was indeed much larger at the maximum of $\Delta\sigma$. However, the following estimates show that the expected effect on oscillation is small. On the basis of $(\mu_H')_{\min}=\mu_H$ and $[\mu'(0)]_{\min}=[\mu_H'(0)]_{\min}/1.93$, $\Delta\sigma$ at a minimum is estimated to decrease by a factor of 0.72 at $B=7$ kG. Assuming the oscillation of $\Delta\sigma$ was entirely due to a variation $\mu'$, we have

$$[\mu'(0)]_{\max}=[\mu'(0)]_{\min}[\Delta\sigma(0)]_{\max}/[\Delta\sigma(0)]_{\min}=1.7[\mu'(0)]_{\min}=3.08\times10^4 \text{ cm}^2/\text{V sec}.$$

According to the experimental result,

$$[\mu_H'(0)]_{\max}=1.2[\mu_H'(0)]_{\min}\simeq1.2\mu_H(0)=4.2\times10^4 \text{ cm}^2/\text{V sec}.$$

The value of $[\mu_H'(0)/\mu'(0)]_{\max}$ indicates that it is not consistent to assume scattering purely by ionized impurities and that the value $p\simeq2.5$ instead of $p=3$ should be used in the integral $I$. The reduction factor for a maximum of $\Delta\sigma$ is then estimated to be very close to the reduction factor for a minimum of $\Delta\sigma$. Therefore the result does not contradict the possibility that the oscillation of $\Delta\sigma$ is mainly produced by a mobility variation.