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PHYSICAL REVIEW B VOLUME 1, NUMBER 11 1 JUNE 1970

Ultrasonic Attenuation in Dirty Superconductors

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Experimental support for the theory of the attenuation α of ultrasonic waves in dirty superconductors has been obtained with 10-GHz longitudinal waves in mercury-indium alloys. α varies as $H_{c2}-H$ near H_{c2} and the slope $\Delta\alpha/\Delta H$ has the predicted value.

The ultrasonic attenuation in clean second-type superconductors versus biasing magnetic field H has been studied theoretically[1] and experimentally[2] in the vicinity of H_{c2}. It varies as $(H_{c2}-H)^{1/2}$. In a dirty superconductor, the theory[3, 4] predicts a linear variation versus $\Delta H = H_{c2} - H$.

The aim of this paper is to report on experimental results which confirm this theory.

This theory has been developed within the following hypothesis: The mean free path l of the electrons is small compared to the coherence length ξ_0; the electronic Green's functions are supposed independent of H because $\omega_c l \ll v_F$, and the gap $\Delta(r)$ on which they are dependent is slowly varying on distances of the order of $(l\xi_0)^{1/2}$; the correlation functions of the electronic density are completely screened when the ultrasonic frequency ω is lower than the plasma frequency. These conditions are satisfied in our experiments.

If the two subsidiary conditions $\Delta(r) \ll \pi k T_c$ and $\hbar\omega \ll \pi k T_c$ are fulfilled, in the neighborhood of H_{c2}, the attenuation α_S for longitudinal waves in the supraconducting phase is given by

$$\frac{\alpha_S(H)}{\alpha_N} = 1 - \frac{1}{4\pi}\frac{e}{\sigma\alpha}\frac{[1+L(\rho)]}{\beta[2K_2^2(T)-1]\mu_0}(H_{c2}-H) . \quad (1)$$

In this formula, $\sigma = Ne^2\tau/m^*$ is the electrical conductivity; $\alpha = \frac{1}{3}v_F^2\tau e H_{c2}(T)\mu_0$; ρ is defined by $\rho = \alpha/2\pi k T$; $L(\rho)$ is a function tabulated in Ref. 3; $\beta = 1.16$; and $K_2(T)$ is the second parameter of Landau-Ginzburg defined by Maki (Ref. 5); the ratio $K_2(T)/K$ has been calculated by Caroli *et al.* (Ref. 6) where

$$K^2 = \left(\frac{3Ne}{2\pi^2\sigma\hbar}\right)^2 \frac{2\pi m^*}{\mu_0 k_F^5} 7\xi(3) = \frac{14\xi(3)}{\mu_0\pi^4} C(\tau)$$

with $C(\tau) = \dfrac{3m^{*3}}{4e^2\hbar^2(9\pi^7)^{1/3}N^{5/3}\tau^2}$.

Formula (1) may be written in the more compact form

$$\frac{\alpha_N - \alpha_S}{\alpha_N} = \frac{\Delta\alpha(H)}{\alpha_N} = A(T)\frac{\Delta H}{H_{c2}(T)} \quad (2)$$

with

$$A(T) = \frac{3}{4\pi\beta\mu_0}\frac{Ne}{\sigma^2 v_F^2 m^*}\frac{1+L(\rho)}{2K_2^2(T)-1} = \frac{C(\tau)}{\beta\mu_0}\frac{1+L(\rho)}{2K_2^2(T)-1} .$$

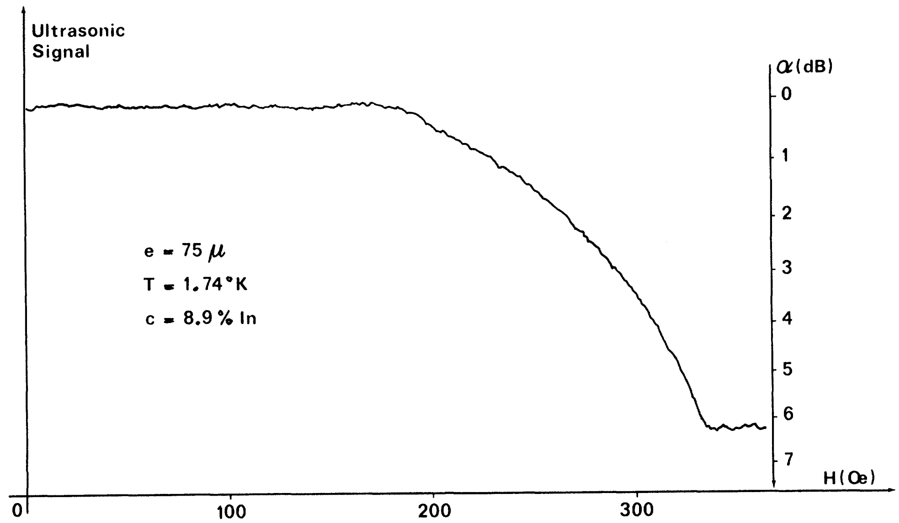

FIG. 1. Record of the intensity of the ultrasonic signal versus the applied magnetic field.

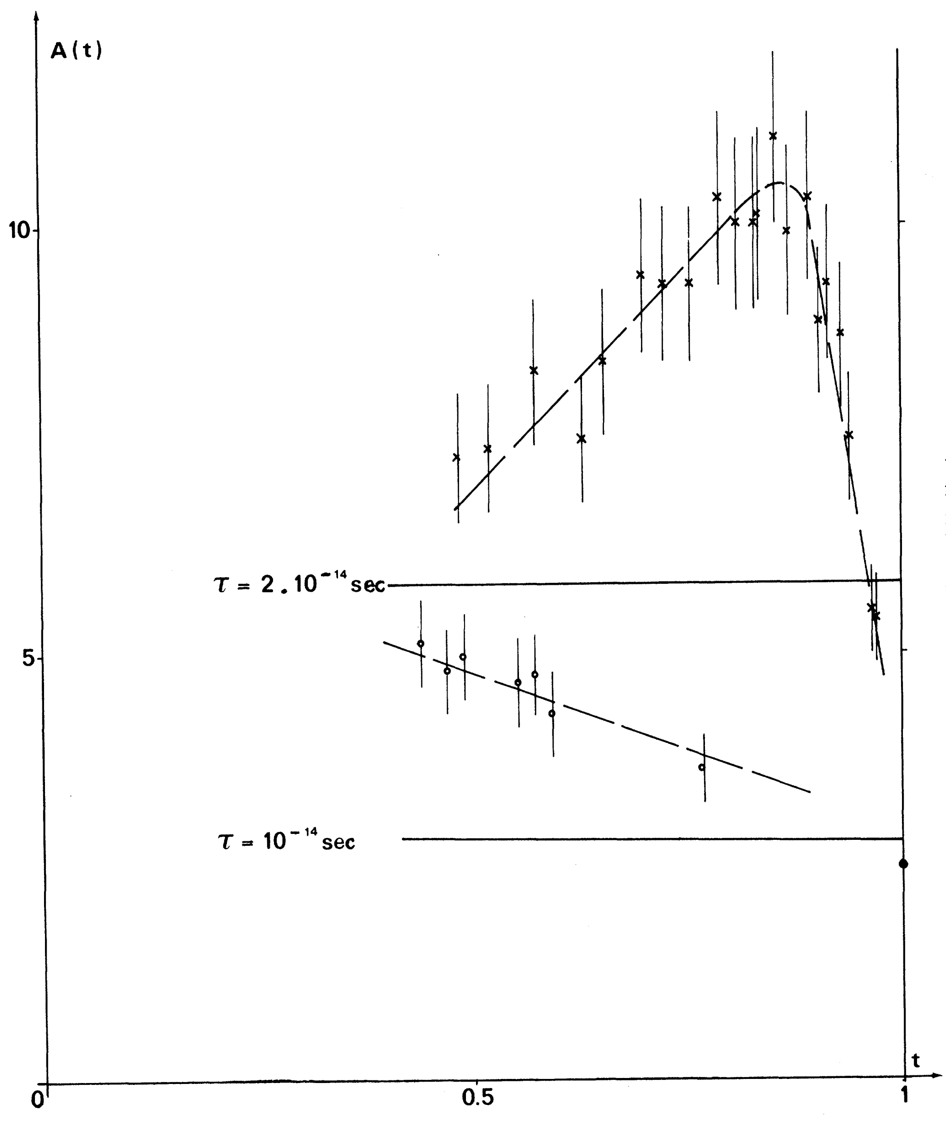

FIG. 2. Ultrasonic attenuation data showing the field dependence for a longitudinal wave propagation: $\log\Delta\alpha(H)$ versus $\log\Delta H$. (The logarithms are to the base 10.)

Experiments have been made with longitudinal ultrasonic waves of 9.4 kMHz propagating through thin polycrystalline films of mercury-indium alloys. The indium concentration varies between 2 and 10%. Under these conditions, the ultrasonic attenuation in the normal state is about 4000 dB/cm; consequently, the thickness of the samples is chosen to be of the order of 50–75 μ. Their diameter is 5 mm. The magnetic field is applied parallel to the plane of the film; with this geometry it may be supposed that the magnetization is uniform in the film.

Figure 1 shows a record of the intensity of the ultrasonic wave versus H. The linearity of the attenuation manifests itself in a $\log_{10}\Delta\alpha$ versus $\log_{10}\Delta H$ plot (Fig. 2) for a given temperature and alloy composition: The slope of the curve is equal to 1 in a domain which spreads on $\Delta H \simeq \frac{1}{8} \times (H_{c2} - H_{c1})$ near H_{c2}.

Moreover, the experimental results furnish for each sample a measure of the coefficient $A(T)$ and allow a comparison with theoretical values. With the available numerical values for mercury, formula (3) permits one to plot $A(T)$ versus $t = T/T_c$ and thus to obtain a set of curves for different relaxation times; these curves are then compared with the experimental ones (Fig. 3). With the following numerical values:

$$m^* \simeq 1.7 \text{ m}, \qquad N = 8.7\, 10^{22} \text{ cm}^{-3},$$

the results of Merriam[7] indicate that for the highest concentration of our samples, τ is of the order 10^{-14} sec. The theoretical formulas then become

$$\rho = 0.1\,[(1/t) - 1]\,,$$

$$K = 3.4\,,$$

$$A(t) = 16.3\, \frac{1 + L[\rho(t)]}{2K_2^2(t) - 1}\,.$$

The expression of $A(T)$ has no adjustable parameters and the curves calculated with the above constants fit reasonably well the experimental data for the highest alloy concentrations; $A(T)$ decreases slowly for increasing t (Fig. 3). However, with these values of m^*, N, and τ the formula is unable to predict experimental data at lower alloy concentrations; the reason is probably that the limit between type-II and type-I alloys is rapidly reached: The condition $2K^2 - 1 = 0$ is fulfilled for $\tau \sim 7 \times 10^{-14}$ sec, and the theory is only valid in the dirty limit.[8]

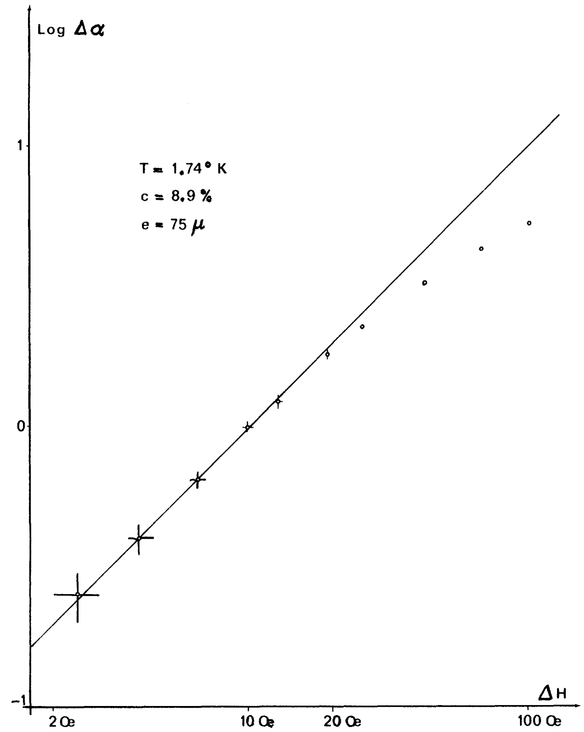

FIG. 3. Data showing the variation of the slope $A(T)$ with the reduced temperature t for two different samples (x corresponds to 8.5%, 0 corresponds to 8.9%). Two theoretical curves have been drawn for comparison and the full circle represents $A(1)$ in the dirty limit.

As a correlated phenomenon, we observed that $A(T)$ is sensitive to the annealing of the sample, that is, to the resulting value of τ.

In spite of this difference, our experiments confirm the theory that α varies linearly with H near H_{c2}, and that the order of magnitude and the shape of the curves of $A(t)$ are correct. No influence on the ultrasonic attenuation of a geometrical effect due to the lattice vortex has been observed near H_{c1}.

Note added in proof. After submission of this paper, our attention has been drawn on preliminary similar results by Gottlieb *et al.* [Phys. Letters 25A, 107 (1967)].

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PHYSICAL REVIEW B VOLUME 1, NUMBER 11 1 JUNE 1970

Fermi-Liquid Effects on Surface Impedance in the Anomalous-Skin-Effect Regime

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The reflection properties of a semi-infinite plasma in the presence of a magnetic field normal to its surface are calculated on the assumption that the response of the electrons is governed by Fermi-liquid theory and that the surface of the plasma randomizes the motion of the electrons incident upon it. A resonancelike magnetic field dependence of the surface impedance is found near that field for which propagation of the correlation-produced magnetoplasma mode first becomes possible. The size of the structure is calculated to be much enhanced over that of the corresponding structure calculated using specular scattering as a boundary condition for the electrons. Numerical and analytic results are presented and discussed for a range of parameters relevant to possible experiments in the alkali metals.

I. INTRODUCTION

The electromagnetic excitations propagated by conduction electrons of a metal immersed in a strong magnetic field[1] are becoming an increasingly useful tool for obtaining information about the many-body correlations between these electrons.[2-5] Of special interest in this regard is the wave which propagates parallel to the magnetic field at a frequency close to the cyclotron frequency. In the absence of correlations, i.e., on the assumption that each carrier is an independent particle whose motion is governed by the self-consistent field arising from the motion of all the other carriers, no such wave can exist. However, by using the Landau-Silin[6-9] form of Fermi-liquid theory (a phenomenological scheme for introducing the effects of correlation into the self-consistent-field description above), Cheng, Clarke, and Mermin[10] (CCM) predicted that a wave *can* propagate near the cyclotron frequency. Because this wave owes its very existence to the correlations, its properties should serve as a sensitive probe for measuring them. The wave should, by altering the shielding currents and electromagnetic fields at the surface, cause a change in the reflection properties of the metal. Considerations addressed to this question are presented in this paper; we calculate the change in surface admittance caused by Fermi-liquid correlation effects under the conditions ($\omega_c \approx \omega$ and anomalous-skin-effect regime) necessary to the propagation of this wave.

At first thought, such a calculation might seem to be a pointless one: Any information obtained by surface experiments could also, in principle, be obtained from a direct study of the wave in bulk. In particular, the dispersion relation for the wave in the infinite medium is available in a relatively simple form and, by fitting the observed wavelength to this dispersion relation, values of the Fermi-liquid parameters might be obtained. Reference 3 provides a beautiful example of this technique as applied to the high-frequency waves (HFW) which propagate *across* the magnetic field, and rightly points out the simplicity of an infinite-medium calculation versus the difficulty of even the simplest boundary-value problem. Why pose a difficult calculation when the same results might be obtained from a simple one?

The answer to this question lies in the observability of the phenomena. Unlike the HFW waves, the magnetoplasma mode (which we shall denote as the CCM mode) turns out *not* to be the least damped excitation which the system will support (Sec. V). The Gantmakher-Kaner (GK) oscillations,[11] single-particle excitations rather strongly coupled to the electromagnetic field, are of longer spatial range. If the sample is made thick enough to justify an analysis based on the infinite-medium situation, there is a strong chance that the GK oscillations will overwrite the CCM mode. On the other hand, if the sample is made thin, say, less than a wavelength, there is reason for serious doubt that an infinite-medium mode will even exist.