

perhaps interesting to note that the H_{hf} contribution of a Fe nn equals roughly that determined in pure iron by adding small nonmagnetic impurities (e.g., 27 kOe for Al impurities).⁶

This interpretation also explains the additional resonances found in the ordered Ni₃Fe at 145 and 186 kOe, and attributed by Burch *et al.*² to Ni with three and five Fe nn, respectively. The above figures show that replacement of one nn Ni by a nn Fe alters the field by 21.5 kOe, which then explains the additional resonances at 186 and 145 kOe with respect to the main resonance at 167 kOe. These results may also be used

to interpret the somewhat complicated spectrum of the disordered sample of Ni₃Fe. The broad maximum at 55 MHz (which corresponds to 146 kOe) is certainly due to the resonance of Ni nuclei in a perfectly disordered environment; its field value of 146 kOe is in agreement with that obtained from the Mössbauer measurements (147 kOe). In the spectrum there are two additional sharp resonances, which can be attributed to Ni nuclei, namely at 155 and 177 kOe. We suggest that the 155-kOe resonance comes from Ni nuclei, which have three nn Fe, nine nn Ni, and six nnn Fe, and the 177-kOe resonance from Ni nuclei with four nn Fe, eight nn Ni, and six nnn Fe. We cannot as yet explain why these configurations are more stable than others.

⁶ G. K. Wertheim, V. Jaccarino, J. H. Wernick, and D. N. E. Buchanan, Phys. Rev. Letters **12**, 24 (1964).

Diamagnetic Susceptibility at the Transition to the Superconducting State*

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An expression of Schmid for the “extra” diamagnetic susceptibility is evaluated to facilitate comparison with the experiments of Gollub *et al.*

IN a paper of the same title, Schmid¹ has obtained an expression for the contribution to the diamagnetic susceptibility at zero field arising from fluctuations in the superconducting order parameter. The same result has been obtained by Schmidt.² We here generalize Schmid's result to the experimentally realized case of finite fields. Instead of the susceptibility, we calculate directly the magnetization, which is the measured quantity. The experiment has been carried out by Gollub *et al.*³

Our starting point is the expression for the free energy of Schmid:

$$\mathcal{F} = -\frac{VkT}{4\pi\bar{B}} \int_0^\infty \frac{dk}{2\pi} \sum_{n=0}^\infty \ln \frac{2m^*\pi kT/\hbar^2}{k^2 + \bar{B}(n + \frac{1}{2}) + \zeta^{-2}}, \quad (1)$$

where V is the volume of the system, m^* is the mass of the electron pair, ζ is the coherence length at zero field, and $\bar{B} = 4eB/\hbar c$ is proportional to the magnetic field. The expression for \mathcal{F} is divergent, but the part of it which depends on field is not.

We now introduce the function $G(y) = [y] - y$, where $[y]$ is the integer part of y . Then we may write

$$\mathcal{F} = -\frac{VkT}{4\pi} \int_0^\infty dx \int \frac{dk}{2\pi} \mathcal{L}(k, x) \left[G'\left(\frac{x}{\bar{B}}\right) + 1 \right]. \quad (2)$$

Here $\mathcal{L} = \ln[2m^*\pi kT\hbar^{-2}/(k^2 + x + \zeta^{-2})]$ and G' is the derivative of G , which has δ -function contributions at $y = n + \frac{1}{2}$.

Thus we obtain

$$\begin{aligned} \mathfrak{M} &= -\frac{\partial \mathcal{F}}{\partial B} = -\frac{VkT}{4\pi} \frac{4e}{\hbar c} \int_0^\infty dx \int \frac{dk}{2\pi} \frac{x}{\bar{B}^2} \mathcal{L}(k, x) G''\left(\frac{x}{\bar{B}}\right) \\ &= -\frac{VkT}{4\pi} \frac{4e}{\hbar c} \int_0^\infty dx \int \frac{dk}{2\pi} G\left(\frac{x}{\bar{B}}\right) \frac{d^2}{dx^2} x \mathcal{L}(k, x). \end{aligned} \quad (3)$$

Carrying out the differentiation, and the integral over k , replacing x by $y\bar{B}$ and inserting the explicit expression for G , we obtain

$$-\frac{\mathfrak{M}}{\sqrt{B}} = \frac{VkT}{4\pi} \left(\frac{4e}{\hbar c}\right)^{3/2} f(\gamma), \quad (4)$$

where

$$\gamma = (\bar{B}\zeta^2)^{-1} = \frac{1}{2} \frac{T - T_{c0}}{B} \left| \frac{\partial H_{c2}}{\partial T} \right| \quad (5)$$

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¹ Albert Schmid, Phys. Rev. (to be published).

² H. Schmidt, Z. Physik **216**, 336 (1968). (There is a misprint of a factor of 4 in this result.)

³ J. P. Gollub, M. R. Beasley, R. S. Newbower, and M. Tinkham, Phys. Rev. Letters **22**, 1288 (1969). We thank Professor Tinkham for communicating his results prior to publication.

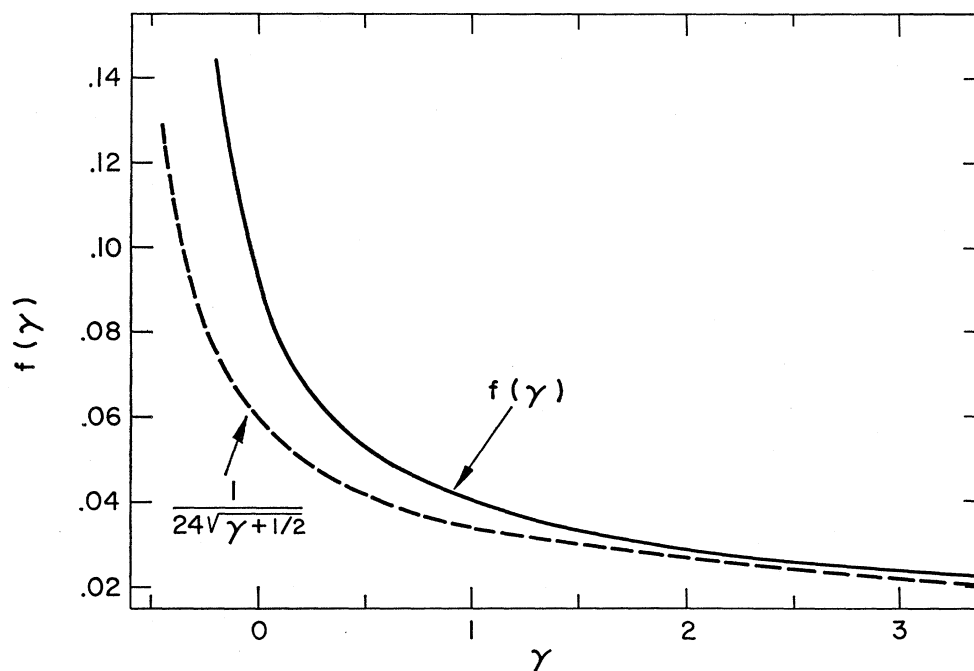


FIG. 1. Function $f(\gamma)$ defined in the text. Also plotted is $1/24(\gamma+\frac{1}{2})^{1/2}$, which is the form first used by Gollub *et al.* (Ref. 3) to fit their experimental results. It was heuristically obtained by modifying Schmid's result by simply shifting the transition temperature T_c .

and

$$f(\gamma) = \int_0^{1/2} dy y \left(\frac{3}{4} \frac{1}{(y+\gamma)^{1/2}} + \frac{1}{4} \frac{\gamma}{(y+\gamma)^{3/2}} \right) + \sum_{n=1}^{\infty} \int_{-1/2}^{1/2} dy y \left(\frac{3}{4} \frac{1}{(y+n+\gamma)^{1/2}} + \frac{1}{4} \frac{\gamma}{(y+n+\gamma)^{3/2}} \right). \quad (6)$$

The integrals may be performed, and f may be written

$$f(\gamma) = \frac{1}{8(\gamma+\frac{1}{2})^{1/2}} - \frac{1}{32} \sum_n b_n^3 \left(1 + \frac{\gamma}{c_n} \right), \quad (7)$$

where $c_n = [(n+\gamma)^2 - \frac{1}{4}]^{1/2}$ and $b_n = [\frac{1}{2}(n+\gamma) + \frac{1}{2}c_n]^{-1/2}$. The series is related to the generalized Riemann ζ function. The low-field limit, $\gamma \rightarrow \infty$, is in agreement with the result of Schmid, giving the result $f(\gamma) \rightarrow 1/24\gamma^{1/2}$. Since the field dependence enters through γ , it does not suffice to consider just the low-field limit, however. In particular, the transition is shifted from T_{c0} to $T_c(H)$,³ for which $\gamma = -\frac{1}{2}$. In this limit $f(\gamma) \rightarrow 1/4(\gamma+\frac{1}{2})^{1/2}$. Figure 1 gives the result of a numerical computation of this function.