Photomagnetoelectric Effect in Graded Band-Gap Semiconductors

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The equation of continuity has been written for a graded band-gap semiconductor taking into account the effect of band-edge gradients and the variation of the radiative recombination lifetime with position. The equation is then solved to give the value of excess minority carriers at a point in such a specimen. The effect of surface states is included through the appropriate boundary conditions. These expressions are deduced for a semiconductor which is space-charge free as a result of inhomogeneous doping, i.e., the doping being position dependent in the direction of band-edge gradients. The resulting expression shows that the I_y^{sc} consists of the usual photodiffusion term plus a component arising from a "quasifield" setup due to band-edge gradients. Simplified expressions are obtained under the following conditions: (a) The generation of carriers takes place only on the front surface, and (b) the variation of the radiative lifetime with position is neglected. In the latter case, the expressions for I_y^{sc} are further analyzed under the conditions of small and large band-edge gradients. In this case, the conditions for sign reversal of I_y^{sc} are discussed and it is found that spectral response of I_y^{sc} depends on whether the band-edge gradient helps or opposes the concentration gradient. A few curves are plotted to exhibit the spectral response and the sign reversal of I_y^{sc}.

I. INTRODUCTION

In recent years the study of graded mixed semiconductors has assumed greater importance on account of successful attempts to grow single crystals with graded composition.[1-4] Cohen-Solal *et al.* developed a method of growing single-crystal films employing an epitaxial technique which enabled them to prepare well-controlled mixed crystals of CdTe and HgTe of graded composition.[1] Van Ruyven and Dev[3] reported results of optical absorption and emission studies[4] on graded zinc-cadmium-sulfide crystals which they prepared. With the practical realization of such crystals, having slowly graded composition along a particular direction, it is thus possible to exploit the unique properties of these semiconductors characterized by a position-dependent band gap.

Kroemer[5] suggested that the quasielectric field proportional to band-edge gradients in these semiconductors could be profitably utilized to improve the high-frequency characteristics of transistor structures. This has been supported by the recent analysis of Martin and Stratton.[6] However, theoretical interest in these graded band-gap semiconductors stems mainly from their possible applications in solar-energy conversion on account of high quantum efficiency in a wide spectral range. Photovoltaic effects associated with a gradient in the band gap have, therefore, been studied by several investigators.[7-9] Of particular interest is the study of $Cd_xHg_{1-x}Te$ graded band-gap structures as photoelectromagnetic detectors for near and middle infrared by Cohen-Solal and co-workers.[2,9] In addition, studies in electric-field-dependent luminescence spectra of graded zinc-cadmium-sulphide crystals by Hill and Williams[10] have evoked greater interest in this field.

Theoretically the band gap in a graded mixed semiconductor can be calculated by the virtual-crystal approximation, which consists in replacing the actual potential centered on each lattice site by a weighted average. The applicability of the virtual-crystal approximation, which was earlier applied to metallic alloys, to mixed semiconductors is discussed by Parmenter.[11] In a graded mixed semiconductor, therefore, it is reasonable to expect a position-dependent band gap varying along its length corresponding to the varying composition.[12,13] Theoretical consideration of the electronic transport through such a graded band-gap semiconductor indicates that the parameters such as mobility, effective mass, and recombination lifetime of minority carriers would, in general, be position dependent. Gora and Williams,[13] in particular, pointed out that in writing an expression for the carrier current, one has to consider not only the quasifield arising from the band-edge gradients, but also the contribution to this field by the gradient of effective mass. The term in the effective-mass gradient arises, physically speaking, from the effect of the density of states on diffusion. The effect of the band-edge gradients on the diffusion of carriers is discussed in a fundamental way by Van Ruyven and Williams.[14] They have also remarked that apart from the above gradients, which affect the electronic conduction of the graded band-gap semiconductors, there would be a space-charge field in these semiconductors. However, it has been pointed out that by doping such a graded band-

gap semiconductor inhomogeneously, it is possible to make the semiconductor space-charge free, if the following condition is satisfied:

$$\frac{d\chi(x)}{dx} = kT\frac{d}{dx}\,[\ln N_D(x)], \tag{1}$$

where χ is the position-dependent electron affinity and N_D is the position-dependent density of shallow donors. These workers have identified the electronic band edge for an inhomogeneously mixed crystal as the classical turning point of the plane-wave part of ψ. This turning point occurs at different positions for different states within a band, and therefore the band edge is position dependent. Electronic transport in these graded band-gap semiconductors is considered as the intraband transition between the eigenstates of the complete Hamiltonian. Thus they have shown that the term in the gradient of the band edge in the transport equation arises from asymmetric diffusion of carriers.

The authors[15] have recently discussed the photovoltage between the illuminated and the dark surface of a graded band-gap semiconductor due to the diffusion of electrons and holes down the concentration gradient of photogenerated carriers. In the present paper an attempt is made to investigate the photoelectromagnetic effect in a graded band-gap semiconductor taking into consideration the surface effects.

II. THEORETICAL

We consider a nondegenerate strongly n-type graded mixed semiconductor sample with one face exposed to a monochromatic radiation as shown in Fig. 1. In order to derive a minority-carrier transport equation, we follow the argument of Gora and Williams[13] to consider the usual relationship for finding the hole and electron concentration in the graded band-gap semiconductor,

$$\begin{aligned} p &= \text{const}\times(m_p^*)^{3/2}\exp[(E_v - E_f)/kT], \\ n &= \text{const}\times(m_n^*)^{3/2}\exp[(E_f - E_c)/kT], \end{aligned} \tag{2}$$

where E_f is the quasi-Fermi level. Here $m_{p,n}^*$ and $E_{v,c}$ now designate position-dependent effective masses and band edges, respectively. Similarly, the usual expressions for the hole and electron currents can be written as

$$J_{px} = ep\mu_p F_p - kT\mu_p\frac{dp}{dx}\ , \tag{3}$$

where the effective field is now given by

$$F_p = E + \frac{1}{e}\left(\frac{dE_v}{dx} + \frac{3}{2}\frac{kT}{m_p^*}\frac{dm_p^*}{dx}\right)\ . \tag{4}$$

We also have

$$J_{nx} = en\mu_n F_n + kT\mu_n\frac{dn}{dx}\ , \tag{5}$$

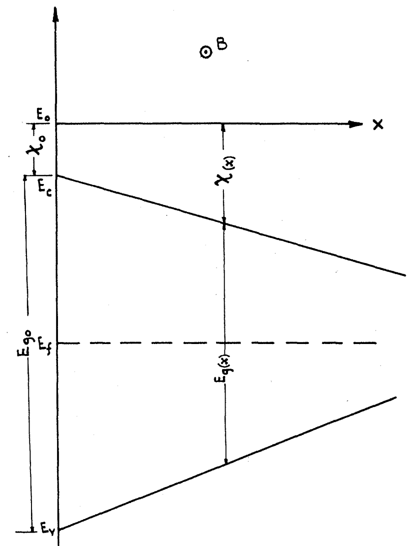

FIG. 1. Energy-level diagram.

where

$$F_n = E + \frac{1}{e}\left(\frac{dE_c}{dx} - \frac{3}{2}\frac{kT}{m_n^*}\frac{dm_n^*}{dx}\right)\ .$$

In the normal equation of continuity,

$$\frac{\partial p}{\partial t} = g_n - r_n - \frac{1}{e}\,\text{div}J_{px} = k_1 I e^{-k_1 x} - \frac{(p - p_0)}{\tau_p} - \frac{1}{e}\text{div}J_{px}, \tag{6}$$

we have to remember that p, J_p, in the case of graded band-gap semiconductors, are given by Eqs. (2) and (3), and further, that τ_p, the radiative recombination lifetime, is also position dependent. g_n and r_n represent generation and recombination rates, respectively, and I is the intensity of monochromatic radiation at the front surface ($x = 0$).

Combining Eqs. (3) and (6), and assuming that $P = p - p_0 = n - n_0$, we obtain the following equation for excess minority-carrier concentration as a function of position in the direction of composition gradient:

$$\frac{\partial P}{\partial t} + P\left(\frac{1}{\tau_p} + \mu_p\frac{dF_p}{dx} + F_p\frac{d\mu_p}{dx}\right) + \frac{dP}{dx}\left(\mu_p F_p - \frac{kT}{e}\frac{d\mu_p}{dx}\right) + \frac{kT}{e}\,\mu_p\frac{d^2P}{dx^2} = k_1 I e^{-k_1 x}\ . \tag{7}$$

It may be mentioned here that absorption coefficient k_1 would vary with x as the band gap is varying in this direction. However, if we restrict ourselves only to the case of strong absorption, k_1, mentioned in Eq. (7), may be assumed to be the suitable average absorption coefficient independent of position.

A. PME Short-Circuit Current I_y^{sc}

When the semiconductor is illuminated in the x direction from the large band-gap side (see Fig. 1) and a magnetic field is applied in the z direction, photomagnetoelectric (PME) short-circuit current I_y^{sc} in the y direction may be obtained under the following simplifying assumptions: (a) The position dependence of effective mass and mobility is neglected, i.e.,

$$\frac{d\mu_p}{dx}=\frac{dm_p^*}{dx}=0.$$

(b) The variation of band gap with position is linear; one may write (see Fig. 1)

$$E_c=-\chi_0-\alpha x,\quad E_v=-\chi_0-E_{g0}+\alpha x,\quad \chi=-\chi_0-\alpha x\ ,$$

$$\frac{dE_g}{dx}=-2\alpha\,,\qquad \frac{d\chi}{dx}=-\alpha=\frac{1}{2}\frac{dE_g}{dx}\ .$$

(c) Band-to-band recombination dominates over other mechanisms; and hence, one may write the following expression for recombination lifetime as suggested by Gora and Williams[13]:

$$\tau_p=\text{const}\times(m_p^*)^{3/2}\exp[(E_c-E_f)/kT]=ae^{bx},$$

where

$$a=\text{const}\times(m_p^*)^{3/2}\exp[(-\chi_0-E_f)/kT]=\tau_{p0}(\text{say}),$$

$$b=-\alpha/kT=-2\gamma.$$

(d) Hall angle is small, i.e., $\tan\theta\approx\theta$ and $\mu B\ll 1$.

In steady state $J_{px}+J_{nx}=0$, and hence for weak illumination one obtains from Eqs. (3) and (5), following Cohen-Solal and Marfaing,[2] the excess carrier current

$$J_{nx}=-J_{px}$$

$$=eD\left[\frac{dP}{dx}+P\left(\frac{1}{2kT}\frac{dE_g}{dx}+\frac{(\delta-1)}{2(\delta+1)}\frac{d}{dx}(\ln\delta)\right)\right]\ ,$$

where $\delta=n_0/p_0$ is the doping ratio and

$$D=(kT/e)\,[\mu_n\mu_p(n_0+p_0)/(n\mu_n+p\mu_p)]$$

is the ambipolar diffusion coefficient.

In the present case of a strongly n-type graded band-gap semiconductor, i.e., $n_0\gg p_0$ and $n_0\simeq N_D$, the density of shallow donors, the above equation may be further simplified by introducing the condition for a space-charge-free graded band-gap semiconductor as given by Eq. (1) and assuming that $\ln n_0\gg\ln p_0$ (within an accuracy of 10%) as follows:

$$J_{nx}=-J_{px}\simeq eD_p\left[\frac{dP}{dx}+P\left(\frac{1}{2kT}\frac{dE_g}{dx}+\frac{1}{2}\frac{d}{dx}(\ln N_D)\right)\right]$$

$$=eD_p\left(\frac{dP}{dx}+\frac{3}{2}\frac{P}{kT}\frac{dE_g}{dx}\right)\ . \qquad (8)$$

Thus it is observed from Eq. (8) that the carrier current is now made up of two parts: the usual diffusion current, denoted by the first term in the Eq. (8), and an additional current due to the quasified setup due to the band-edge gradient, denoted by the second term in Eq. (8).

The PME short-circuit current I_y^{sc} may be written as

$$I_y^{sc}=\int_0^\omega(J_{px}\theta_p+J_{nx}\theta_n)\,dx\ .$$

Now substituting for $J_{px}=-J_{nx}$ from Eq. (8) and remembering that $\tan\theta_p=\mu_pB$ and $\tan\theta_n=-\mu_nB$, we have

$$I_y^{sc}=(\mu_n+\mu_p)BeD_p\left(P(0)-P(\omega)+\frac{3\alpha}{kT}\int_0^\omega P\,dx\right), \qquad (9)$$

where ω is the thickness of the specimen in the x direction.

B. PME Open-Circuit Field E_y

PME open-circuit field E_y may be obtained easily from the equations for hole and electron currents in the y direction as written below:

$$J_{py}=\mu_p(epE_y+BJ_{px}),\quad J_{ny}=\mu_n(enE_y-BJ_{nx}), \qquad (10)$$

where J_{px} and J_{nx} are given by Eq. (8), subject to the condition $\mu_pB\ll 1$. To obtain E_y for a steady magnetic induction B, one makes use of the condition that the total current in the y direction is zero,[16] which is expressed as follows:

$$\int_0^\omega(J_{ny}+J_{py})\,dx=0.$$

Substituting from J_{ny} and J_{py} from Eq. (10), one obtains

$$E_y=\frac{B(\mu_n+\mu_p)\int_0^\omega J_{nx}\,dx}{\int_0^\omega\sigma(x)dx+e(\mu_p+\mu_n)\int_0^\omega P\,dx}$$

$$=\frac{eBD_p(\mu_p+\mu_n)[P(\omega)-P(0)-3\alpha/kT\int_0^\omega P\,dx]}{\int_0^\omega\sigma(x)\,dx+e(\mu_p+\mu_n)\int_0^\omega P\,dx}\ . \qquad (11)$$

Thus the PME short-circuit current I_y^{sc} and open-circuit field E_y may be obtained from Eqs. (9) and (11), provided appropriate values of excess carrier density P, subject to the boundary conditions, are substituted in these expressions. In order to obtain the excess carrier density, the equation of continuity [i.e., Eq. (7)] has to be solved under steady-state condition and simplifying assumptions mentioned earlier. In the absence of the external field the equation of continuity may thus be reduced to the form

$$\frac{P}{\tau_p}+\left(\frac{\mu_p\alpha}{e}\right)\frac{dP}{dx}-\frac{kT\mu_p}{e}\frac{d^2P}{dx^2}=k_1Ie^{-k_1x}\ , \qquad (12)$$

where

$$\tau_p=\tau_{p0}e^{bx}\ .$$

This may also be written as

$$\frac{d^2P}{dx^2}+A\frac{dP}{dx}+Be^{-bx}P=Ce^{-k_1x}\ ,$$

where

$$A=b=-\alpha/kT,\quad B=-e/akT\mu_p,\quad C=-k_1Ie/kT\mu_p.$$

The above equation may be put into a recognizable form of the Bessel equation by changing variable x to z, where $z=e^{-bx}$,

$$\frac{d^2P}{dz^2}-\left(\frac{kTe}{a\mu_p\alpha^2}\right)\frac{P}{z}=\frac{C}{b^2}z^{(k_1/b-2)}=h(z). \qquad (13)$$

The complete solution of this equation may be obtained in terms of modified Bessel functions as given below,[17]

$$P=z^{1/2}\left\{C_1I_1\left[\frac{2}{\alpha}\left(\frac{kTe}{a\mu_p}\right)^{1/2}\cdot z^{1/2}\right]+C_2K_1\left[\frac{2}{\alpha}\left(\frac{kTe}{a\mu_p}\right)^{1/2}\cdot z^{1/2}\right]\right\} \qquad (14)$$

or

$$P=C_1u_1+C_2u_2\ ,$$

where

$$u_1=z^{1/2}I_1\left[\frac{2}{\alpha}\left(\frac{kTe}{a\mu_p}\right)^{1/2}\cdot z^{1/2}\right]\ ,$$

$$u_2=z^{1/2}K_1\left[\frac{2}{\alpha}\left(\frac{kTe}{a\mu_p}\right)^{1/2}\cdot z^{1/2}\right]\ ,$$

and C_1 and C_2 are functions of z, to be evaluated following the method given below[18]:

$$C_1=-\int\frac{h(z)u_2(z)}{W(u_1,u_2)}\,dz+P_1=-\frac{4C}{b^2}\beta^{-(2k_1/b-1)}\times\int z'^mK_1(z')dz'+P_1,$$

where

$$P_1=\text{const},\quad z'=\beta z^{1/2},\quad m=2(k_1/b-1),$$

$$\beta=\frac{2}{\alpha}\left(\frac{kTe}{a\mu_p}\right)^{1/2}=\frac{1}{\gamma(D_p\tau_{p0})^{1/2}}=\frac{1}{\gamma L_p}\ ,$$

and making use of the property

$$I_1(z')K_1'(z')-I_1'(z')K_1(z')=1/z',$$

the Wronskian $W(u_1,u_2)$ is evaluated, which comes out to be equal to $\frac{1}{2}$. Similarly,

$$C_2=\int\frac{h(z)u_1(z)}{W(u_1,u_2)}dz+P_2=\frac{4C}{b^2}\beta^{-(2k_1/b-1)}\times\int z'^mI_1(z')dz'+P_2.$$

Substituting these values of C_1 and C_2 in Eq. (14) and making use of the relation $I_0K_1+I_1K_0=1/z'$, we obtain the following expression for P:

$$P=(4C/b^2)\beta^{-2k_1/b}\left[z'^m+z'(P_1I_1+P_2K_1)-mz'\left(I_1\int z'^{m-1}K_0dz'+K_1\int z'^{m-1}I_0dz'\right)\right]. \qquad (15)$$

The two integrals may be evaluated for given values of m, as shown in Appendix A.

C. Effect of Surface States

P_1 and P_2 in Eq. (15) can be obtained from the boundary conditions. If s_1 and s_2 are the front- and back-surface recombination velocities, respectively, then the following boundary conditions may be written. At $x=0$, $z'=\beta$:

$$J_{px}(0)=-eD_p\left(\frac{dP(0)}{dx}+\frac{3}{2}\frac{P(0)}{kT}\frac{dE_g}{dx}\right)=-P(0)s_1e$$

or

$$D_p\frac{dP(0)}{dx}=P(0)\left(s_1+\frac{3\mu_p\alpha}{e}\right)\ . \qquad (16)$$

At $x=\omega$, $z'=\beta e^{-b\omega/2}$:

$$D_p\frac{dP(\omega)}{dx}=-P(\omega)\left(s_2-\frac{3\mu_p\alpha}{e}\right)\ . \qquad (17)$$

Introducing these boundary conditions, we can easily obtain the following values of P_1 and P_2 (see Appendix B): When m is a positive odd integer,

$$P_1=\frac{1}{\lambda}\left(\left\{\beta^m\left[m-\left(6+\frac{S_1}{\gamma L_pW}\right)\right]+\Sigma(0)\right\}\left[z'(\omega)^2K_0(z'(\omega))-\left(\frac{S_2}{\gamma L_pW}-6\right)z'(\omega)K_1(z'(\omega))\right]-\left[\beta^2K_0(\beta)+\left(6+\frac{S_1}{\gamma L_pW}\right)\beta K_1(\beta)\right]\left\{z'(\omega)^m\left[m+\left(\frac{S_2}{\gamma L_pW}-6\right)\right]+\Sigma(\omega)\right\}\right)\ , \qquad (18)$$

$$P_2=\frac{1}{\lambda}\left(\left[\beta^2 I_0(\beta)-\left(6+\frac{S_1}{\gamma L_p W}\right)\beta I_1(\beta)\right]\left\{z'(\omega)^m\left[m+\left(\frac{S_2}{\gamma L_p W}-6\right)\right]+\Sigma(\omega)\right\}\right.$$

$$\left.-\left\{\beta^m\left[m-\left(6+\frac{S_1}{\gamma L_p W}\right)\right]+\Sigma(0)\right\}\left[z'(\omega)^2 I_0(z'(\omega))+\left(\frac{S_2}{\gamma L_p W}-6\right)z'(\omega)I_1(z'(\omega))\right]\right), \qquad (19)$$

where

$$\lambda=\left[z'(\omega)^2 I_0(z'(\omega))+\left(\frac{S_2}{\gamma L_p W}-6\right)z'(\omega)I_1(z'(\omega))\right]\left[\beta^2 K_0(\beta)+\left(6+\frac{S_1}{\gamma L_p W}\right)\beta K_1(\beta)\right]$$

$$-\left[\beta^2 I_0(\beta)-\left(6+\frac{S_1}{\gamma L_p W}\right)\beta I_1(\beta)\right]\left[z'(\omega)^2 K_0(z'(\omega))-\left(\frac{S_2}{\gamma L_p W}-6\right)z'(\omega)K_1(z'(\omega))\right].$$

When m is a negative odd integer, $\Sigma(0-)$ and $\Sigma(\omega-)$ will appear in the expressions. Here $S_1=s_1\omega/D_p$, $S_2=s_2\omega/D_p$, and $W=\omega/L_p$.

D. Position Dependence of σ

It has to be remembered that in the case of the graded band-gap semiconductor, σ is also position dependent in the x direction as shown below:

$$\sigma=e(p_0\mu_p+n_0\mu_n).$$

Substituting for p_0 and n_0 from Eq. (2) and remembering that E_c and E_v are position dependent in the x direction, one may write

$$\sigma(x)=\sigma(0)\,e^{2\gamma x}. \qquad (20)$$

III. RESULTS AND DISCUSSIONS

It may be pointed out at this stage that the study of short-circuit current due to the PME effect on the basis of Eq. (9) involves knowledge of P_1 and P_2 which in turn depend upon the properties of the series $\Sigma(0)$, $\Sigma(0-)$ and $\Sigma(\omega)$, $\Sigma(\omega-)$. The appropriate values of the series may, however, be obtained only if sufficient information is available to enable us to calculate the range of values of m suitable for a practical semiconductor with a graded band gap. The mathematical complexities involved in the above general case may, however, be avoided if the following conditions hold good.

(a) The generation of carriers takes place along a plane at the front surface only, generation in the bulk of the semiconductor being negligible. This case corresponds to a very strong absorption at the front surface so that the generation term in the continuity equation may be neglected at all points in volume of the specimen except at the front surface. Thus Eq. (12) reduces to

$$\frac{d^2P}{dx^2}+A\frac{dP}{dx}+Be^{-bx}P=0.$$

Complete solution of this equation is given by

$$P=z^{1/2}P_1I_1(\beta z^{1/2})+z^{1/2}P_2K_1(\beta z^{1/2}). \qquad (21)$$

The boundary conditions are (assuming quantum efficiency to be unity) at $x=0$,

$$J_{px}(0)=eI-eD_p\left(\frac{dP(0)}{dx}+\frac{3}{2}\frac{P(0)}{kT}\frac{dE_g}{dx}\right)=-P(0)s_1e$$

or

$$D_p\frac{dP(0)}{dx}=P(0)\left(s_1+\frac{3\mu_p\alpha}{e}\right)+I;$$

at $x=\omega$,

$$D_p\frac{dP(\omega)}{dx}=P(\omega)\left(s_2-\frac{3\mu_p\alpha}{e}\right),$$

which yields the following values of P_1 and P_2:

$$P_1=(I/\lambda)\left[We^{\gamma L_pW}K_0(z'(\omega))+K_1(z'(\omega))(S_2-6\gamma L_pW)\right]$$

and

$$P_2=(I/\lambda)\left[We^{\gamma L_pW}I_0(z'(\omega))+I_1(z'(\omega))(S_2-6\gamma L_pW)\right],$$

where

$$\lambda=[WI_0(\beta)-(S_1+6\gamma L_pW)I_1(\beta)]$$
$$\times[We^{\gamma L_pW}K_0(z'(\omega))+(S_2-6\gamma L_pW)K_1(z'(\omega))]$$
$$-[WK_0(\beta)-(S_1+6\gamma L_pW)K_1(\beta)]$$
$$\times[We^{\gamma L_pW}I_0(z'(\omega))+(S_2-6\gamma L_pW)I_1(z'(\omega))].$$

The short-circuit current is given by

$$I_y^{\mathrm{sc}}=(\mu_n+\mu_p)BeD_p[P_1\{I_1(\beta)-e^{\gamma L_pW}I_1(z'(\omega))$$
$$+6\gamma L_p[I_0(z'(\omega))-I_0(\beta)]\}$$
$$+P_2\{K_1(\beta)-e^{\gamma L_pW}K_1(z'(\omega))+6\gamma L_p$$
$$\times[K_0(\beta)-K_0(z'(\omega))]\}].$$

(b) The recombination lifetime τ_p is assumed to be independent of position. The effect of position

dependence of τ_p may then be considered qualitatively as the recombination rate for a nondegenerate semiconductor which may be assumed to increase with decrease in band gap ($R \sim e^{-\Delta E/kT}$). Thus, for the case shown in Fig. 1, this will result in a steeper density gradient of minority carriers.

The complete solution of Eq. (12) is then

$$P = P_1 e^{(\gamma-\delta)x} + P_2 e^{(\gamma+\delta)x} - \frac{\omega^2 k_1 I e^{-k_1 x}}{D_p(K^2 + 2KW\gamma L_p - W^2)} , \tag{22}$$

where $K = k_1\omega$ and

$$\delta = -\frac{(\mu_p^2\alpha^2 + 4kTe\mu_p/\tau_p)^{1/2}}{2kT\mu_p} .$$

Introducing the boundary conditions [Eqs. (16) and (17)] we can easily obtain the following values of P_1 and P_2 from Eq. (22):

$$P_1 = \frac{k_1 I}{\beta_1\lambda}\{(S_1 + 6\gamma\omega + K)[6\gamma\omega - S_2 - (\gamma+\delta)\omega]e^{(\gamma+\delta)\omega} - (K - S_2 + 6\gamma\omega)[S_1 + 6\gamma\omega - (\gamma+\delta)\omega]e^{-K}\} , \tag{23}$$

$$P_2 = \frac{k_1 I}{\beta_1\lambda}\{(K - S_2 + 6\gamma\omega)[S_1 + 6\gamma\omega - (\gamma-\delta)\omega]e^{-K} - (S_1 + 6\gamma\omega + K)[6\gamma\omega - S_2 - (\gamma-\delta)\omega]e^{(\gamma-\delta)\omega}\} , \tag{24}$$

where

$$\lambda = [S_1 + 6\gamma\omega - (\gamma+\delta)\omega][6\gamma\omega - S_2 - (\gamma-\delta)\omega]e^{(\gamma-\delta)\omega} - [S_1 + 6\gamma\omega - (\gamma-\delta)\omega][6\gamma\omega - S_2 - (\gamma+\delta)\omega]e^{(\gamma+\delta)\omega}$$

and

$$\beta_1 = -(D_p/\omega^2)(K^2 + 2\gamma L_p K W - W^2) .$$

Now the study of the PME effect may be divided into the following two sections according to the magnitude of band-edge gradient. In what follows we study, analytically, the effects of various factors, namely, band-edge gradient, front- and back-surface recombinations, bulk recombination, etc., on the spectral response of the PME short-circuit current. As the photogenerated carriers in a graded band-gap semiconductor have to move under the action of various fields, the possibility of observing sign reversal in the direction of flow of the PME short-circuit current, as in the case of homogeneous semiconductors,[19,20] exists. Conditions under which sign reversal in I_y^{sc} may be obtained are, therefore, discussed.

1. *Band-Edge Gradient Small* $[(\gamma L_p)^2 \ll 1]$

Thus we have

$$\delta = -(1/L_p)(1+\gamma^2 L_p^2)^{1/2} \approx -(1/L_p)(1 + \tfrac{1}{2}\gamma^2 L_p^2) ,$$

therefore,

$$\gamma + \delta \simeq -(1/L_p)(1 - \gamma L_p), \quad \gamma - \delta \simeq (1/L_p)(1 + \gamma L_p) .$$

The expression for I_y^{sc} may be obtained from Eq. (9) and (22) as shown below:

$$\frac{I_y^{sc}}{(\mu_p + \mu_n)BeD_p} = P_1(1 - e^{(\gamma-\delta)\omega})\left(1 - \frac{6\gamma}{\gamma-\delta}\right) + P_2(1 - e^{(\gamma+\delta)\omega})\left(1 - \frac{6\gamma}{\gamma+\delta}\right) - \frac{I\omega^2(1 - e^{-K})(k_1 + 6\gamma)}{D_p(K^2 + 2KW\gamma L_p - W^2)} . \tag{25}$$

We now consider the following two cases:

(a) Under the assumption of strong absorption, if bulk recombination in the semiconductor specimen is found to be dominant over surface recombination, we may write $K \gg 1$, W and S_1, $S_2 \ll W$. On the basis of these simplifying conditions, the expression for I_y^{sc}, as given in Eq. (25), may be reduced to

$$\frac{I_y^{sc}}{(\mu_p + \mu_n)BeD_p} \simeq -\frac{k_1 I(K + 6\gamma L_p W)}{\beta_1 W}\left(1 - \frac{2e^{\gamma L_p W}}{e^W - e^{-W}}\right) \approx -\frac{k_1 I(K + 6\gamma L_p W)}{\beta_1 W} ,$$

since W is likely to be more than unity (e.g., $W \approx 4$ or more) as bulk recombination has been assumed to be high in most cases. Thus, sign reversal in I_y^{sc} will occur when

$$K = -6\gamma L_p W. \tag{26}$$

Physically, this means that sign reversal takes place when the diffusion of photogenerated carriers in the positive x direction is balanced by the carrier motion due to band-edge gradients in the opposite direction, effects of surface recombination velocities being negligible. This condition may also prove to be a simple tool for the measurement of (dE_g/dx) in a semiconductor specimen.

(b) If surface recombination is dominant over bulk recombination, then, under the assumption of strong absorption, we have $K \gg 1$, W and S_1, $S_2 \gg W$. Thus Eq. (25) reduces to

$$\frac{I_y^{sc}}{(\mu_n + \mu_p)BeD_p} \simeq \frac{k_1 I}{\beta_1}\left(1 - \frac{(K + S_1)}{S_1}[1 + 6\gamma L_p(1 - e^{\gamma L_p W}\operatorname{sech} W)]\right) .$$

When $W \ll 1$, sign reversal occurs if

$$-6\gamma L_p = WS_1/(K + S_1) . \tag{27}$$

If, however, W is greater than unity, so that

$e^W \gg e^{-W}$, condition for sign reversal will be given by

$$-6\gamma L_p = (K - S_1)/K \ . \tag{28}$$

2. *Band-Edge Gradient Large* ($\gamma L_p \gg 1$)

We then have

$$\delta \simeq -\gamma[1 + 1/2(\gamma L_p)^2].$$

Therefore,

$$\gamma + \delta \approx -\gamma/2(\gamma L_p)^2 \approx 0, \quad \gamma - \delta \approx 2\gamma.$$

(a) For strong absorption and bulk recombination dominant over surface recombination, i.e., $K \gg 1$, W and $W \gg S_1, S_2$, we can reduce Eq. (25) to the following simplified forms if another assumption $\gamma L_p W \gg S_1, S_2$ is allowed to be made. Thus, if γ is positive,

$$\frac{I_y^{sc}}{(\mu_p + \mu_n)BeD_p} \approx \frac{k_1 I(K + 6\gamma L_p W)}{\beta_1}\left(\frac{1}{K} + \frac{1}{2\gamma L_p W} - 2\right)$$

$$\approx -\frac{2k_1 I(K + 6\gamma L_p W)}{\beta_1} \quad \text{if } \gamma L_p W \gg 1,$$

which is normally satisfied. In this case, there is no possibility of sign reversal. If, however, γ is negative, so that $2e^{2\gamma L_p W} \ll 1/2\gamma L_p W$ and $(1 + W/\gamma L_p)e^{-K} \ll 1/K$, the expression for I_y^{sc} may be written as

$$\frac{I_y^{sc}}{(\mu_p + \mu_n)BeD_p} \approx -\frac{I}{2\gamma K D_p}(K + 6\gamma L_p W);$$

the condition for sign reversal for this case may, therefore, be obtained as

$$K = -6\gamma L_p W. \tag{29}$$

(b) Again for strong absorption and surface recombination dominant over bulk recombination we have $K \gg 1$, W and S_1, $S_2 \gg W$. If, in addition, we assume that S_1, $S_2 \gg \gamma L_p W$, Eq. (25) may be written as

$$\frac{I_y^{sc}}{(\mu_p + \mu_n)BeD_p} = -\frac{k_1 I}{\beta_1}\left(1 + 12\gamma L_p W + \frac{2K}{S_1}(1 + 6\gamma L_p W)\right).$$

Thus, sign reversal takes place when

$$-12\gamma L_p W = 1 + \frac{(K/S_1)}{(1 + K/S_1)} \ . \tag{30}$$

Results given above may now be summarized:

(i) An expression for the current due to photogenerated carriers in a space-charge-free graded band-gap semiconductor has been derived. A simplified expression for the PME short-circuit current thus obtained may be conveniently used to study the effects of front- and back-surface recombination of the semiconductor specimen. This may be found to be especially suitable in the study of spectral response of the short-circuit current and its dependence on the type of substrate material and the conditions under which the semiconductor specimen is grown.

(ii) It has been pointed out that under certain conditions sign reversal may be observed in the PME short-circuit current as in the case of a homogeneous semiconductor.[19] The condition of sign reversal ($k_1 = -6\gamma$) as shown in Eqs. (26) and

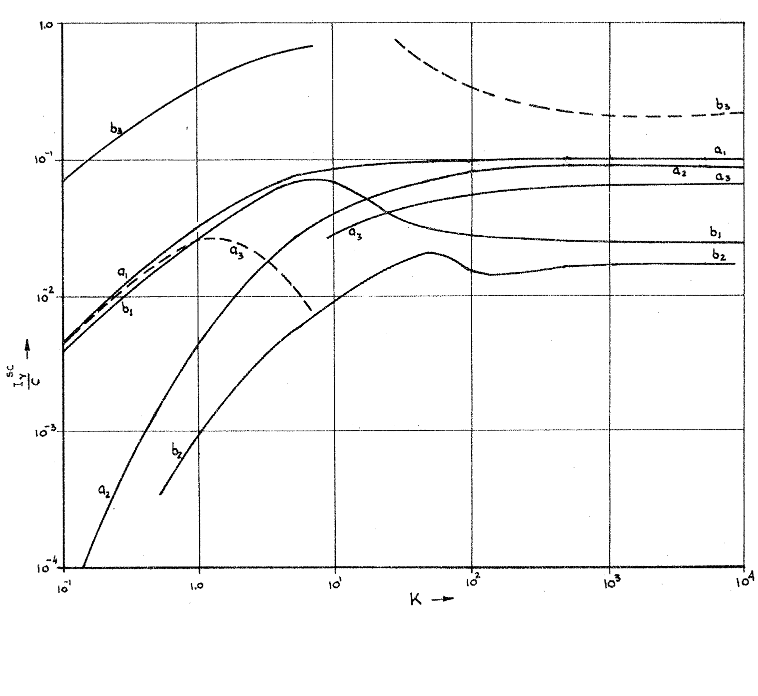

FIG. 2. Spectral response of PME short-circuit current I_y^{sc} for small band-edge gradients. In curve a_1: $W = 10$, $S_1 = 1$, $S_2 = 1$, $\gamma L_p = +0.1$; curve a_2: $W = 10$, $S_1 = 1$, $S_2 = 1$, $\gamma L_p = 0$; curve a_3: $W = 10$, $S_1 = 1$, $S_2 = 1$, $\gamma L_p = -0.1$; curve b_1: $W = 10$, $S_1 = 50$, $S_2 = 50$, $\gamma L_p = +0.1$; curve b_2: $W = 10$, $S_1 = 50$, $S_2 = 50$, $\gamma L_p = 0$; curve b_3: $W = 10$, $S_1 = 50$, $S_2 = 50$, $\gamma L_p = -0.1$.

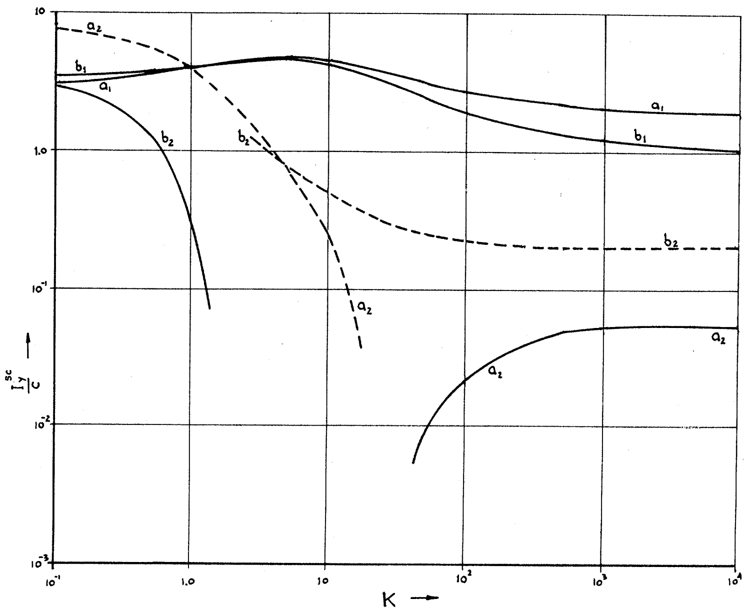

FIG. 3. Spectral response of PME short-circuit current I_y^{sc} for large band-edge gradients. In curve a_1: $W=0.1$, $S_1=1$, $S_2=1$, $\gamma L_p=+100$; curve a_2: $W=0.1$, $S_1=1$, $S_2=1$, $\gamma L_p=-100$; curve b_1: $W=0.1$, $S_1=50$, $S_2=50$, $\gamma L_p=+100$; curve b_2: $W=0.1$, $S_1=50$, $S_2=50$, $\gamma L_p=-100$.

(29) may prove to be a very useful tool in the determination of band-edge gradients as this involves measurement of only one parameter, namely, the absorption coefficient k_1.

A few typical curves for large and small band-edge gradients are shown in Figs. 2 and 3. For small band-edge gradients the value of γ was taken to be 10^2/cm, while for large band-edge gradients γ was taken to be $\approx 10^5$/cm. It may be pointed out here that even such large band-edge gradients do not violate the condition mentioned by Gora and Williams[13] for continuous variation of band edge. According to them if there is a change of 1% in composition over 100 atomic spacing, the band edge can be considered to be continuous. In practice $\gamma\approx 10^5$/cm have been realized by Cohen-Solal *et al.*[1] in thin films of CdHgTe of graded composition.

APPENDIX A

(a) When m is a positive odd integer, we have

$$\int z'^m I_0(z')dz' = I_1(z')z'^m - (m-1)z'^{m-1}I_0(z') + (m-1)^2\int z'^{m-2}I_0(z')dz' .$$

This is a recurring formula, hence we obtain

$$\int z'^m I_0(z')dz' = I_1(z')[z'^m + (m-1)^2 z'^{m-2} + (m-1)^2(m-3)^2 z'^{m-4} + \cdots] - I_0(z')[(m-1)z'^{m-1} + (m-1)^2(m-3)z'^{m-3} + (m-1)^2(m-3)^2(m-5)z'^{m-5} + \cdots] .$$

If m is even the series will utlimately consist of the integral $\int I_0(z')dz'$, values of which are tabulated.

Similarly, we have

$$\int z'^m K_0(z')dz' = -K_1(z')[z'^m + (m-1)^2 z'^{m-2} + (m-1)^2(m-3)^2 z'^{m-4} + \cdots] - K_0(z')[(m-1)z'^{m-1} + (m-1)^2(m-3)z'^{m-3} + (m-1)^2(m-3)^2(m-5)z'^{m-5} + \cdots].$$

In this case also, if m is even, the last term in the series will consist of $\int K_0(z')dz'$, values of which are tabulated.

(b) When m is a negative odd integer, we have

$$\int z'^m I_0(z')dz' = \frac{I_0(z')z'^{m+1}}{(m+1)} - \frac{I_1(z')z'^{m+2}}{(m+2)} + \frac{1}{(m+1)^2}\int z'^{m+2} I_0(z')dz'$$

$$= I_0(z')\left(\frac{z'^{m+1}}{(m+1)} + \frac{z'^{m+3}}{(m+1)^2(m+3)} + \cdots\right) - I_1(z')\left(\frac{z'^{m+2}}{(m+1)^2} + \frac{z'^{m+4}}{(m+3)^2(m+1)^2} + \cdots\right) .$$

Similarly, we have

$$\int z'^m K_0(z')dz' = \frac{K_0(z')z'^{m+1}}{(m+1)} + \frac{K_1(z')z^{m+2}}{(m+1)^2} + \frac{1}{(m+1)^2} \int z'^{m+2} K_0(z')dz'$$

$$= K_0(z')\left(\frac{z'^{m+1}}{(m+1)} + \frac{z'^{m+3}}{(m+1)^2(m+3)} + \cdots\right) + K_1(z')\left(\frac{z'^{m+2}}{(m+1)^2} + \frac{z'^{m+4}}{(m+3)^2(m+1)^2} + \cdots\right) .$$

APPENDIX B

(a) When m is a positive odd integer, we have

$$I_1(z')\int z'^{m-1}K_0(z')dz'$$
$$= I_1(z')\{K_1(z')[z'^{m-1} + (m-2)^2 z'^{m-3} + \cdots]$$
$$- K_0(z')[(m-2)z'^{m-2} + (m-2)^2(m-4)z'^{m-4} + \cdots]\},$$

$$K_1(z')\int z'^{m-1}I_0(z')dz'$$
$$= K_1(z')\{I_1(z')[z'^{m-1} + (m-2)^2 z'^{m-3} + \cdots]$$
$$- I_0(z')[(m-2)z'^{m-2} + (m-2)^2(m-4)z'^{m-4} + \cdots]\} .$$

Adding, we have

$$I_1(z')\int z'^{m-1}K_0(z')dz' + K_1(z')\int z'^{m-1}I_0(z')dz'$$
$$= -\frac{1}{z'}[(m-2)z'^{m-2} + (m-2)^2(m-4)z'^{m-4} + \cdots] .$$

Therefore, from Eq. (15), we have

$$P = \frac{4C}{b^2}\beta^{-2k_1/b}\{z'^m + [m(m-2)z'^{m-2}$$
$$+ m(m-2)^2(m-4)z'^{m-4} + \cdots]$$
$$+ z'(P_1 I_1(z') + P_2 K_1(z'))\} .$$

Substituting the boundary conditions, at $x=0$, we have

$$P_1\left[\beta^2 I_0(\beta) - \left(6 + \frac{S_1}{\gamma L_p W}\right)\beta I_1(\beta)\right]$$
$$- P_2\left[\beta^2 K_0(\beta) + \left(6 + \frac{S_1}{\gamma L_p W}\right)\beta K_1(\beta)\right]$$
$$+ \left\{\beta^m\left[m - \left(6 + \frac{S_1}{\gamma L_p W}\right)\right] + \Sigma(0)\right\} = 0, \quad (31)$$

where

$$\Sigma(0) = [m(m-2)^2\beta^{m-2} + m(m-2)^2(m-4)^2\beta^{m-4} + \cdots]$$
$$+ \left(6 + \frac{S_1}{\gamma L_p W}\right)[m(m-2)\beta^{m-2}$$
$$+ m(m-2)^2(m-4)\beta^{m-4} + \cdots] .$$

At $x=\omega$,

$$z' = \beta e^{-b\omega/2} = z'(\omega),$$

say,

$$P_1\left[z'(\omega)^2 I_0(z'(\omega)) + \left(\frac{S_2}{\gamma L_p W} - 6\right)z'(\omega)I_1(z'(\omega))\right]$$
$$- P_2\left[z'(\omega)^2 K_0(z'(\omega)) - \left(\frac{S_2}{\gamma L_p W} - 6\right)z'(\omega)K_1(z'(\omega))\right]$$
$$+ \left\{z'(\omega)^m\left[m + \left(\frac{S_2}{\gamma L_p W} - 6\right)\right] + \Sigma(\omega)\right\} = 0, \quad (32)$$

where

$$\Sigma(\omega) = [m(m-2)^2 z'(\omega)^{m-2}$$
$$+ m(m-2)^2(m-4)^2 z'(\omega)^{m-4} + \cdots]$$
$$+ \left(\frac{S_2}{\gamma L_p W} - 6\right)[m(m-2)z'(\omega)^{m-2}$$
$$+ m(m-2)^2(m-4)z'(\omega)^{m-4} + \cdots] .$$

Thus, values of P_1 and P_2 may be found from Eqs. (31) and (32) following the usual method.

(b) When m is a negative odd integer, we have

$$I_1(z')\int z'^{m-1}K_0(z')dz' = I_1(z')\left[K_0(z')\left(\frac{z'^m}{m} + \frac{z'^{m+2}}{m^2(m+2)} + \cdots\right) + K_1(z')\left(\frac{z'^{m+1}}{m^2} + \frac{z'^{m+3}}{m^2(m+2)^2} + \cdots\right)\right],$$

$$K_1(z')\int z'^{m-1}I_0(z')dz' = K_1(z')\left[I_0(z')\left(\frac{z'^m}{m} + \frac{z'^{m+2}}{m^2(m+2)} + \cdots\right) - I_1(z')\left(\frac{z'^{m+1}}{m^2} + \frac{z'^{m+3}}{m^2(m+2)^2} + \cdots\right)\right].$$

Adding, we have

$$I_1(z')\int z'^{m-1}K_0(z')dz'+K_1(z')\int z'^{m-1}I_0(z')dz' = \frac{1}{z'}\left(\frac{z'^m}{m}+\frac{z'^{m+2}}{m^2(m+2)}+\cdots\right).$$

Therefore, from Eq. (15), we have

$$P=\frac{4C}{b^2}\,\beta^{-2k_1/b}\left[z'^m - m\left(\frac{z'^m}{m}+\frac{z'^{m+2}}{m^2(m+2)}+\cdots\right) + z'[P_1I_1(z')+P_2K_1(z')]\right].$$

Substituting in the boundary condition, at $x=0$, we have

$$P_1\left[\beta^2 I_0(\beta)-\left(6+\frac{S_1}{\gamma L_p W}\right)\beta I_1(\beta)\right] - P_2\left[\beta^2K_0(\beta)+\left(6+\frac{S_1}{\gamma L_p W}\right)\beta K_1(\beta)\right] + \left\{\beta^m\left[m-\left(6+\frac{S_1}{\gamma L_p W}\right)\right]+\Sigma(0-)\right\}=0, \quad (33)$$

where

$$\Sigma(0-)=\left[-m\left(\beta^m+\frac{\beta^{m+2}}{m^2}+\cdots\right)+m\left(6+\frac{S_1}{\gamma L_p W}\right)\times\left(\frac{\beta^m}{m}+\frac{\beta^{m+2}}{m^2(m+2)}+\cdots\right)\right].$$

At $x=\omega$, we have

$$P_1\left[z'(\omega)^2I_0(z'(\omega))+\left(\frac{S_2}{\gamma L_p W}-6\right)z'(\omega)I_1(z'(\omega))\right] - P_2\left[z'(\omega)^2K_0(z'(\omega))-\left(\frac{S_2}{\gamma L_p W}-6\right)z'(\omega)K_1(z'(\omega))\right] + \left\{z'(\omega)^m\left[m+\left(\frac{S_2}{\gamma L_p W}-6\right)\right]+\Sigma(\omega-)\right\}=0, \quad (34)$$

where

$$\Sigma(\omega-)=\left[-m\left(z'(\omega)^m+\frac{z'(\omega)^{m+2}}{m^2}+\cdots\right) - m\left(\frac{S_2}{\gamma L_p W}-6\right)\left(\frac{z'(\omega)^m}{m}+\frac{z'(\omega)^{m+2}}{m^2(m+2)}+\cdots\right)\right].$$

Values of P_1 and P_2 may be obtained by solving Eqs. (33) and (34) given above.

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