where

$$E_{k_f c k_f v} = \left(\frac{\hbar^2 k_f^2}{2m_{cv}} + E_g\right).$$

In deriving Eqs. (16) and (36), we have used the property of the Kronecker δ, $(\delta_{ij})^2 = \delta_{ij}$. If we assume that the polarization vector $\vec{\alpha}$ of the electromagnetic wave points in the z axis and make the approximation that the matrix element $H'_{k_f c k_f v}$ is independent of the vector $\vec{k}_f$, we obtain, after carrying out the integration, the result given in Eq. (18).

*Work performed under the auspices of the U. S. Atomic Energy Commission.

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Carrier Repopulation from Infrared Faraday Rotation under Hot-Carrier Conditions

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The magnitude of the Faraday rotation in Ge and Si under hot-electron conditions has been estimated for different orientations of the magnetic and the hot-electron dc field. The possibility of the evaluation of carrier repopulation from the analysis of Faraday-rotation data is discussed. The conditions required for a successful experiment for the application of the method are examined. It is found that the experiment is possible at room temperature, but the conditions are more favorable at liquid-nitrogen or lower temperatures.

I. INTRODUCTION

The Faraday rotation of infrared signals is produced in semiconductors mainly by the free carriers if the wavelength of the signal is beyond the absorption edge. The specific angle of rotation is determined by the frequency of the signal, the dielectric constant of the material, the free-carrier concentration, and the effective mass of the carriers. It is independent of the momentum relaxation time of the carriers if the frequency is such that the product of the frequency and relaxation time is much larger than unity. Therefore, one may determine one of the above-mentioned three parameters of the material, i.e., the dielectric constant, the carrier concentration, and the effective mass of carriers, since we know two of them from other experiments. This method has been applied to obtain the effective mass of carriers in III-V compounds and in Ge.[1-3]

The free carriers in a semiconductor, except in a few cases, normally occupy one valley or equivalent valleys in equal numbers in a many-valley semiconductor. The carrier population in the different valleys may be altered when a high electric field is applied to produce hot-electron conditions. Such population transfer affects significantly the conductivity and galvanomagnetic properties of semiconductors under hot-electron conditions and

is also believed to be the cause of bulk negative differential conductivity (BNDC) observed in GaAs[4] and in Ge.[5,6] The view that the population transfer is the underlying mechanism of BNDC in GaAs is now generally accepted, but for germanium it is still being discussed in the literature.[7] Experimental evidence for population transfer in Ge comes from the magnitudes of conductivity and Hall-mobility anisotropies, the Sasaki effect, and pressure effects in hot-electron characteristic, in all of which the change of relaxation time plays a more significant role than carrier repopulation. There are no direct experimental results giving the carrier population in the different valleys under the above-mentioned conditions. Recently, it has been suggested that the carrier concentrations may be directly obtained by measuring the plasma frequency provided the product of plasma frequency and momentum relaxation time is much larger than unity.[8] However, for Ge and Si with the usual carrier concentrations the plasma frequency is such that the above condition is not satisfied and the method is inapplicable. The infrared Faraday rotation as discussed earlier provides, on the other hand, an alternative method for the direct determination of carrier concentration and is applicable to these materials. In an earlier note the authors discussed the suitability of this method in estimating the population transfer in GaAs.[9] The purpose of the present paper is to examine the expected electric field dependence of Faraday rotation in Ge and Si and the possibility of applying this method for the estimation of carrier repopulation in these materials. It may be mentioned that Faraday rotation in n-type Ge and Si was studied earlier by Ipatova *et al.*[10] confining their attention to the saturation region in the hot-electron characteristics.

II. INFRARED FARADAY ROTATION IN MANY-VALLEY SEMICONDUCTORS

The specific angle of Faraday rotation for a many-valley semiconductor at infrared frequencies in the presence of a hot-electron dc field F is given by[10]

$$\Theta(F) = j\omega(\mu\epsilon)^{1/2}\,\epsilon_{xy}/2\epsilon\,, \tag{1}$$

where ω is the signal frequency and μ and ϵ are, respectively, the permeability and permittivity of the material. The magnetic field has been assumed to be applied in the z direction, and ϵ_{xy} is the xy component of the dielectric tensor. It is given by

$$\epsilon_{xy} = \epsilon \sum_v j \frac{N_v e^2}{\omega^2 m_c \epsilon}\,\frac{\omega_0}{\omega}\,\beta_{xy}\,, \tag{2}$$

where N_v is the carrier population in a valley, and m_c is the conductivity effective mass of the material; $\omega_0 = eB/m_c$, B being the magnetic induction, $\beta_{xy} = \alpha_{xx}\,\alpha_{yy} - \alpha_{xy}^2$, and α_{ij} is a component of the reciprocal effective-mass tensor α normalized by m_c.

It should be noted that in deriving the above expression for Θ it is assumed that $\omega^2 \gg \sum_v (N_v e^2/m_c\epsilon)\times\alpha_{ii}$. It is also assumed that the time period of the signal is much smaller than the relaxation time, i.e., $\omega\tau \gg 1$.

We may calculate the Faraday rotation angle for germanium and silicon under hot-electron conditions using the above expression for Θ provided N_v corresponding to the applied hot-electron field is known. Ipatova *et al.*[10] calculated the values of N_v assuming that the carriers are scattered by intravalley optical phonons and have energies much larger than the optical phonon energy. The Faraday rotation angle is found to be independent of the applied field and lattice temperature. However, this assumption may be valid only at very high fields where the mobility is found to vary as $1/F$.

The results of Ipatova *et al.*[10] do not give any idea of the expected change in the angle of rotation at fields below the saturation region or at high fields where the nonequivalent intervalley scattering is important. Therefore it is not possible to assess on the basis of these results whether the Faraday rotation data would be suitable for the determination of carrier repopulation at all fields. We present in Sec. III the estimated values of Θ at different electric fields for two lattice temperatures, 77 and 300 °K. These have been calculated using values of $N_v(F)$, obtained either from the analysis of experimental anisotropy characteristics or estimated from elaborate theoretical analysis.[11–15]

III. FARADAY ROTATION IN Ge AND Si UNDER HOT-ELECTRON CONDITIONS

We consider first the case of silicon for which the situation is somewhat simpler. The satellite valleys along the ⟨111⟩ directions, being separated by large energy gaps from the main ⟨100⟩ valleys, do not play any role. Carrier repopulation may be produced in this case when the electric field is applied along the [100] direction, since the valleys along [100] and [$\bar{1}$00] are differently heated compared to the other valleys. The specific angle of Faraday rotation for this case is given by

$$\Theta_{100}^{010}(F) = \frac{\pi}{\lambda}\,K^{1/2}\left(\frac{\omega_p}{\omega}\right)^2\left(\frac{\omega_0}{\omega}\right) \times\left[\frac{N_1(F)}{N}\,\alpha_l\alpha_t + \tfrac{1}{2}\left(1-\frac{N_1(F)}{N}\right)(\alpha_l\alpha_t+\alpha_t^2)\right], \tag{3}$$

$$\Theta_{100}^{100}(F) = \frac{\pi}{\lambda}\,K^{1/2}\left(\frac{\omega_p}{\omega}\right)^2\left(\frac{\omega_0}{\omega}\right) \times\left[\frac{N_1(F)}{N}\,\alpha_t^2 + \left(1-\frac{N_1(F)}{N}\right)\alpha_l\alpha_t\right], \tag{4}$$

where K is the dielectric constant of the material, $\omega_p = (N_v e^2/m_c \epsilon)^{1/2}$ is the plasma frequency, $N_1(F)$ is the carrier concentration in the [100] valley, and α_l and α_t are the longitudinal and transverse components of the reciprocal effective-mass tensor normalized by m_c

In the above equations the superscript on Θ indicates the direction of the magnetic field and of the propagation of infrared fields and the subscript, that of the high electric field. The values of Θ at different electric fields for the two orientations obtained by using the estimated values of $N_1(F)$ [11,12] and the mass values are shown in Fig. 1. The rotation angle varies by a large amount with the electric field and the curves for the two orientations of the magnetic field are also different. The difference is much enhanced for 77 °K in comparison to that for 300 °K. We find from Eq. (3) that if we study the value of Θ at any field normalized by the low-field value we may obtain $N_1(F)/N$ using the known value of the mass anisotropy factor K_m. On the other hand, if we perform experiments with the two orientations of the magnetic field we may obtain $N_1(F)/N$ and in addition K_m. It is interesting to note that the value of $\Theta(F)/\Theta(0)$ approaches the same field-independent value for both the lattice temperatures at very high fields. The value for $\Theta^{100}_{100}(F)/\Theta(0)$ for very high fields is also the same as that given by Ipatova *et al.* [10]

In case of germanium the situation is complicated by the possibility of carrier transfer to ⟨100⟩ valleys from the main ⟨111⟩ valleys. However, it is this suspected transfer which leads to interesting phenomena and needs investigation. There are two important directions of the field to be considered, the [111] and the [100] directions.

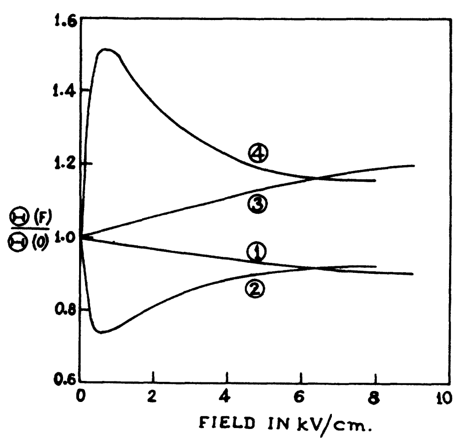

FIG. 1. Variation of Faraday-rotation angle in silicon with dc electric field. Curves 1 and 2 show the variation of Θ^{010}_{100} at 300 and 77 °K, respectively. Curves 3 and 4 are for Θ^{100}_{100} at 300 and 77 °K, respectively.

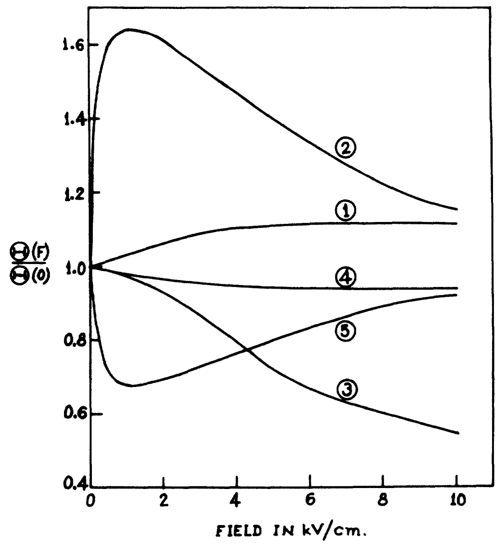

FIG. 2. Variation of Faraday-rotation angle in germanium with dc electric field. Curves 1 and 2 show the variation of Θ^{111}_{111} at 300 and 77 °K, respectively. Curves 4 and 5 show the variation of $\Theta^{1\bar{1}0}_{111}$ at 300 and 77 °K, respectively. Curve 3 is for Θ^{010}_{100} at 77 °K.

When the electric field is applied in the [100] direction the carriers in the ⟨111⟩ valleys are all equally heated and would have the same carrier concentration. If there is transfer of carriers to the ⟨100⟩ satellite valleys, we can also assume that the transferred carriers are equally distributed among these valleys, since heating of carriers in these valleys would be quite small in the field range of interest. The specific angle of rotation in this case is thus given by

$$\Theta^{010}_{100}(F) = \frac{\pi}{\lambda} K^{1/2} \left(\frac{\omega_p}{\omega}\right)^2 \left(\frac{\omega_0}{\omega}\right) \left(\frac{N_{111}(F)}{N} \left(\tfrac{8}{3}\alpha_l \alpha_t + \tfrac{4}{3}\alpha_t^2\right) + \frac{N_{100}(F)}{N} \left(4\alpha_l\alpha_t + 2\alpha_t^2\right)\right) , \qquad (5)$$

where $N_{111}(F)$ and $N_{100}(F)$ are, respectively, the carrier concentrations in each of the ⟨111⟩ and ⟨100⟩ valleys.

It is evident that any variation of Θ with the applied electric field would indicate transfer of population from ⟨111⟩ valleys to ⟨100⟩ valleys. We show in Fig. 2 the expected variation of Θ with the field obtained by using the estimated carrier population quoted by Heinrich. [13] The expected change is quite large at lower temperatures and should be easily observable.

Next, let us assume that the electric field is applied in the [111] direction. In this case, of the four

valleys in the ⟨111⟩ directions, three are equally heated, but the fourth one is much less heated. Hence, under these conditions there are three classes of valleys: the hot ⟨111⟩, cool [111], and the ⟨100⟩ valleys. The specific angles of rotation in this case are given by

$$\Theta_{111}^{1\bar{1}0}(F)=\frac{\pi}{\lambda}\, K^{1/2}\left(\frac{\omega_p}{\omega}\right)^2\left(\frac{\omega_0}{\omega}\right)\left(\frac{N_{c111}(F)}{N}\,\alpha_l\alpha_t + \frac{N_{h111}(F)}{N}\left(\tfrac{5}{3}\alpha_l\alpha_t+\tfrac{4}{3}\alpha_t^2\right) + \frac{N_{100}(F)}{N}\left(4\alpha_l\alpha_t+2\alpha_t^2\right)\right) . \quad (6)$$

$$\Theta_{111}^{111}(F)=\frac{\pi}{\lambda}\, K^{1/2}\left(\frac{\omega_p}{\omega}\right)^2\left(\frac{\omega_0}{\omega}\right)\left(\frac{N_{c111}(F)}{N}\,\alpha_t^2 + \frac{N_{h111}(F)}{N}\left(\tfrac{8}{3}\alpha_l\alpha_t+\tfrac{1}{3}\alpha_t^2\right) + \frac{N_{100}(F)}{N}\left(4\alpha_l\alpha_t+2\alpha_t^2\right)\right). \quad (7)$$

The subscripts c and h refer to cold and hot valleys, respectively.

We show in Fig. 2 the expected change in Θ with electric field for the temperatures of 300 and 77 °K assuming that there is no transfer of population to ⟨100⟩ valleys. The carrier population in the ⟨111⟩ valleys have been taken from the earlier estimates.[14,15] Since there are three values of carrier concentrations, these can, in principle, be determined using in addition to the above equations the number-conservation equation, provided all the effective masses are known.

We note that, as in silicon, the value of $\Theta(F)/\Theta(0)$ becomes field independent at high fields, and the field-independent value is nearly the same for 77 and 300 °K. But this value is much smaller than that estimated by Ipatova *et al.*[10] The difference originates from the fact that the theoretically estimated values[16] of N_c/N_h are larger than that obtained from the analysis of experiments.

We see from the above results that in principle a Faraday-rotation experiment at infrared frequencies in the presence of a high electric field across the sample would enable one to obtain the electron population in the equivalent and nonequivalent valleys for different values of the field. The anisotropy factor for the effective mass in Si may also be estimated from such data. Thus, an experiment of this nature would provide direct evidence to support or reject the electron-transfer theory which has been suggested as an explanation of BNDC in Ge. In addition, knowledge of the electron population in the different valleys at various fields would allow one to estimate the intervalley relaxation rates and shed significant light on the role of intervalley scattering in determining the hot-electron conduction characteristics of Ge and Si.

IV. FEASIBILITY OF EXPERIMENT

We show in Table I the estimated low-field Faraday-rotation angle and the values of relevant parameters of Ge and Si for room temperature (300 °K). It is evident from Table I that at room temperature the frequency of the infrared signal is required to be chosen greater than 5×10^{13} rad/sec to make $\omega\tau$ larger than 10 so that the effect of τ on Θ is negligible. We also find that for this frequency the other two assumptions, namely, that the plasma frequency and the cyclotron-resonance frequency are much smaller than the infrared frequency, are also valid. Faraday rotation of 1–5 °/cm for a magnetic field of 1 W/m^2 may be obtained at this frequency using samples of resistivities 2–5 Ω cm as used in hot-electron experiments. Faraday-rotation angles of this order have been measured.[17] However, the angle appears to be too small for studying the field dependence with sufficient accuracy to elicit information with regard to the carrier repopulation as discussed earlier. To increase the rotation angle the carrier concentration may be enhanced by higher doping. But then impurity scattering would become important and the hot-electron characteristics will be modified from their normal form. Alternatively, the carrier concentration in the samples may be

TABLE I. Estimated Faraday rotation at room temperature.

Material	Collision frequency c/s	Plasma frequency (rad/sec) for room-temperature resistivities of 5 Ω cm	2 Ω cm	Highest cyclotron-resonance frequency (eB/m_t) at $B=1$ W/m^2	Estimated low-field Faraday rotation at $\omega=5\times10^{13}$ rad/sec ($\lambda=37\ \mu$) [deg cm^{-1} (W/m^2)$^{-1}$] 5 Ω cm	2 Ω cm
Ge	3.6×10^{12}	7.25×10^{11}	1.16×10^{12}	2.2×10^{12}	1.5	4.0
Si	5×10^{12}	9.65×10^{11}	1.54×10^{12}	8.8×10^{11}	1	2.5

enhanced either by shining light or external injection. However, hot-electron experiments using these techniques are not yet reported. The conditions at room temperature may thus be considered marginal for a successful experiment as suggested in this paper.

At 77 °K, on the other hand, the collision frequency is increased about ten times. The condition $\omega\tau \gg 1$ may be satisfied by choosing the signal frequency one order lower. The wavelength for the signal is then required to be smaller than 370 μ. Since the Faraday rotation increases as λ^2, we find that the rotation angle is increased by a factor of 10^2. Hence, experimental requirements will be more easily satisfied at 77 °K and the suggested experiment should be considered feasible at this temperature. It is also at this temperature that BNDC has been obtained in germanium, and the results of the experiment would be of particular interest.

ACKNOWLEDGMENT

The authors are thankful to P. K. Basu for helpful discussions.

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Phonon Dispersion Relations in Ge at 80 °K

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Phonon dispersion relations in Ge have been measured at 80 °K for all principal symmetry directions and on some lines at the boundary of the Brillouin zone. The measurements were performed by neutron inelastic scattering using a three-axis crystal spectrometer. In order to achieve maximum accuracy, care was taken to reduce resolution widths as far as possible. In general, the estimated uncertainties of the measured phonon frequencies range from 0.3% to 0.5% for optical and from 0.3% to 1% for acoustic phonons. Frequencies of 238 phonons are tabulated and phonon linewidths are given for the symmetry directions.

INTRODUCTION

Ge is an obvious material for phonon spectrometry with neutrons since it is a good representative for the covalent bond and diamond structure and is available in excellent large crystals with quite good coherent neutron scattering properties. Moreover, its technical importance lends it a special interest, particularly as regards thermodynamic properties related to the phonon frequency distribution. In fact, it was among the first substances to be studied in the early days of neutron measurements. In 1958 Brockhouse and Iyengar[1] published phonon dispersion curves for the [100] and [110] directions obtained by crystal spectrometry, and in 1959 Ghose *et al.*[2] presented time-of-flight measurements for the [100] and [110] directions. The development of the neutron method now allows us to