$$H_{1c} = |\rho_k|^{-1}[(\tfrac{1}{2}\omega - \omega_k)^2 + \eta_k^2]^{1/2} . \quad (9)$$

In the case of Madsen and Tanaka[3] there is not a perfect agreement with the result of other theories, whereas in our case Eq. (9) is exact. The instability criterion (8) indicates the intensity of the excitation for which the Green's-function series does not converge. This criterion should also be applicable to determine the threshold condition in other nonlinear effects, such as the stimulated Raman or Brillouin scattering.

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[1]S. V. Tyablikov, *Methods in the Quantum Theory of Magnetism* (Plenum, New York, 1967).
[2]T. Tanaka, K. Moorjani, and T. Morita, Phys. Rev. 155, 388 (1967).
[3]E. Madsen and T. Tanaka, Phys. Rev. 184, 527 (1969).
[4]F. R. Morgenthaler, J. Appl. Phys. Suppl. 31, 95 (1960).
[5]E. Schlömann, J. J. Green, and U. Milano, J. Appl. Phys. Suppl. 31, 386 (1960).
[6]E. Schlömann, J. Appl. Phys. 34, 1998 (1963).
[7]R. I. Joseph and E. Schlömann, J. Appl. Phys. 38, 1915 (1967).

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Note on Gadzuk's Theory of Field-Induced Tunneling

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Recently, Gadzuk[1,2] presented theoretical studies concerning band-structure effects on field-induced tunneling from metals and resonant tunneling through atoms absorbed on metal surfaces. The calculation of the matrix elements necessary for comparison with experiment in these papers appears to be seriously in error and, at least in the second paper, to cast doubts on the usefulness of these calculations for comparison with the experimental work of Plummer and Young.[3] The basic error is the statement $j_l(ikr) = i^l j_l(kr)$. That this is incorrect can be seen clearly by considering the case $l = 0$ where $j_0(x) = x^{-1}\sin x$. The matrix elements are easily recalculated by replacing $j_l(x)$ by $(\pi/2x)^{1/2} I_{l+1/2}(x)$ wherever it occurs. Thus, for example, Eqs. (35a) and (35b) of Ref. 2 should read

$$I_1 = \frac{\Gamma(7)}{k^8(\beta_s^2 - 1)^7}(7\beta_s^6 + 35\beta_s^4 + 21\beta_s^2 + 1) ,$$

$$I_2 = \frac{3\Gamma(8)}{k^{10}(\beta_s + 1)^8} + \frac{\Gamma(9)}{2k^{10}}[(\beta_s - 1)^{-9} - (\beta_s + 1)^{-9}] .$$

Hence, for $k \approx 1.2$ Å^{-1}, $a_s \approx 1.5$ Å^{-1}, $I_1(\text{corr})/I_1(35a) \simeq -25$, $I_2(\text{corr})/I_2(35b) \simeq 30\,000$.

[1]J. W. Gadzuk, Phys. Rev. 182, 416 (1969).
[2]J. W. Gadzuk, Phys. Rev. B 1, 2110 (1970).
[3]E. W. Plummer and R. D. Young, Phys. Rev. B 1, 2088 (1970).

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Reply to Comments on a Theory of Field-Induced Tunneling

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In the preceding paper,[1] Glasser has pointed out an oversight in some manipulative details leading to approximate expressions in this writer's theoretical studies of field-induced tunneling.[2,3] However, the mishap is not as disastrous as it might first appear. In all cases in which the tunneling matrix elements were calculated, the physically significant results were presented in the form of ratios of ma-