

Density and strain dependence of $\nu=1$ energy gap in a valley-degenerate AlAs quantum well

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In an AlAs quantum well where two-dimensional electrons occupy two conduction-band valleys with anisotropic Fermi contours, we tune the valley splitting energy via the application of strain and measure the energy gaps for the quantum-Hall state at filling factor $\nu=1$. At zero valley splitting energy (strain), there is a sizeable energy gap which increases linearly with density. At a fixed density, the gap rises with increasing strain much faster than the single-particle picture predicts. Interpreting the rapid rise of the gap as a signature of valley Skyrmions, we find that their size grows as the electron density increases.

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In multicomponent two-dimensional electron systems (2DESs), electron-electron interaction significantly alters the ground-state properties and the low-lying excitations compared to the single-particle picture. For example, in a bilayer 2DES at a filling factor $\nu=1$, a remarkable quantum fluid emerges thanks to the interlayer quantum coherence.¹ This system can be described as a pseudospin ferromagnet and it is an example of the many quantum-Hall (QH) ferromagnetic ground states. More generally, when two Landau levels (LLs) simultaneously approach the chemical potential at an integer filling factor, interaction opens a gap and leads to a QH ferromagnetic ground state.² Experimentally, QH ferromagnetic phases with different pseudospin anisotropy classes have been observed at the crossing of LLs originating from levels belonging to different spin,³ confinement,^{4,5} valley,^{6–8} or layer¹ index.

In the particular case of $\nu=1$ in a system with a spin degree of freedom, in the limit of zero effective Lande g factor ($g=0$), the large Coulomb energy cost of a spin flip results in a sizeable energy gap separating the ferromagnetic ground state from its excitations. In this case, the charged excitations are Skyrmions with a nontrivial long-range spin order, a slow rotation of the spin along the radial direction.^{9–13} The size of a Skyrmion is the number of electrons participating in the excitation, and depends on the ratio of the Zeeman to Coulomb energy (E_Z/V_C). The larger the E_Z , the less preferable it is for the Skyrmion to form and, at sufficiently large E_Z , the Skyrmion size approaches one, i.e., the excitations are single spin flips. In a disorder-free system, Skyrmions have a divergent size as $E_Z \rightarrow 0$. In the presence of disorder, however, their size is limited by disorder-induced, finite-size domains.^{9–13} Although the spin Skyrmions have been studied extensively in 2DESs, this topic continues to be attractive both experimentally and theoretically in materials such as three-dimensional MnSi (Ref. 14) and bilayer graphene.¹⁵

Skyrmions are usually associated with the spin degree of freedom. However, in a 2DES with two equivalent and energetically degenerate conduction-band valleys, there is an analogy between the valley index and the electron spin. Similar to the spin Skyrmions, *valley* Skyrmions might exist in a 2DES with a valley degree of freedom.⁶ In this paper, we report energy-gap measurements of the QH state at $\nu=1$ as a function of density (n) and strain (ϵ) for a two-valley 2DES confined to an AlAs quantum well. In agreement with previ-

ous results,⁶ we observe a finite gap even at zero strain. This gap rises with strain much faster than the single-particle picture predicts and approaches the cyclotron energy at very large strains. The strain dependence of the $\nu=1$ gap at fixed density is consistent with the presence of valley Skyrmions and from the slope of the $\nu=1$ gap vs ϵ , we deduce their size. We observe that the slope decreases with decreasing n , suggesting that the Skyrmion size is limited by disorder. We also observe that the $\nu=1$ gaps at $\epsilon=0$ depend linearly on n and extrapolate to zero as $n \rightarrow 0$. This trend is inconsistent with the \sqrt{n} dependence expected for an ideal 2DES, and points to the dominance of LL mixing in our 2DES.¹⁶

We emphasize that although we can explain our results in light of valley Skyrmions, it is not obvious if this picture applies. There is a major difference between the valley indices in our samples and spin. The two valleys have anisotropic Fermi contours with their major axes rotated by 90° with respect to each other. We discuss implications of this anisotropy.

We performed experiments on a 2DES confined to a 12-nm-thick layer of AlAs, and modulation doped with Si. Our sample was grown by molecular-beam epitaxy on a (001) GaAs substrate. The electrons in this sample occupy two in-plane valleys with elliptical Fermi contours,¹⁷ each centered at an X point of the Brillouin zone, and with an anisotropic effective mass (longitudinal mass $m_l=1.05$ and transverse mass $m_t=0.205$, in units of free-electron mass). We refer to these valleys by the orientation of their major axis, [100] and [010]. To vary the occupations of the two valleys, we glue our samples to a piezoelectric actuator (piezo) and apply voltage bias to the piezo to stretch the sample in one direction and compress it in the perpendicular direction [Fig. 1(a)].^{17,18} This results in a symmetry-breaking strain $\epsilon = (\epsilon_{[100]} - \epsilon_{[010]})$, where $\epsilon_{[100]}$ and $\epsilon_{[010]}$ are the strains along [100] and [010]. For $\epsilon > 0$, electrons are transferred from the [100] valley to the [010] valley and vice versa for $\epsilon < 0$; in either case the total density remains fixed with strain. The resulting valley splitting energy is given by $E_V = \epsilon E_2$, where E_2 is the deformation potential which in AlAs has a band value of 5.8 eV.¹⁷ We use a metal-foil strain gauge glued to the opposite face of the piezo to measure the applied strain.¹⁷ Our sample had two patterned Hall-bar mesas aligned with the [100] and [110] crystal directions. The results along the two arms are similar; here we focus on the arm aligned along

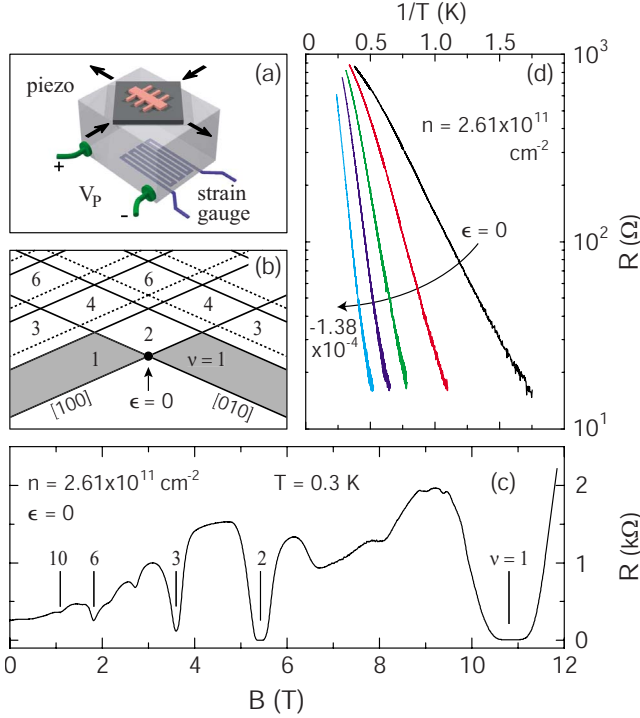


FIG. 1. (Color online) (a) Experimental setup with the sample and a strain gauge glued to opposite faces of a piezo. (b) Simple energy vs strain (ϵ) fan diagram showing the response of electron Landau levels to strain. Each Landau level is also split by the Zeeman energy and the spin-up and spin-down levels are shown by solid and dotted lines, respectively. (c) Magnetoresistance trace for balanced valleys ($\epsilon=0$). (d) Arrhenius plots of $\nu=1$ resistance minimum for five values of strain ($\epsilon=0, -0.08, -0.24, -0.45, -1.38 \times 10^{-4}$).

[100]. Using a metal gate deposited on the sample's surface, we varied n between 1.0 and $3.6 \times 10^{11} \text{ cm}^{-2}$. The magnetoresistance measurements were performed in a liquid ^3He system with a base temperature of 0.3 K .

In Fig. 1(b), we show the energy-level structure of our 2DES in a perpendicular magnetic field (B) as a function of strain. The magnetic field quantizes the orbital motion of the electrons and forces them to occupy a discrete set of LLs separated by the cyclotron energy, $\hbar\omega_c = \hbar eB/m^*$, where $m^* = \sqrt{m_t m_l}$ is the cyclotron effective mass. Moreover, B splits the energy of the two spin levels by Zeeman energy, $E_Z = g\mu_B B$. The band parameters in our system are such that for the density range under study, E_Z is greater than or comparable to $\hbar\omega_c$ (Refs. 18 and 19) as shown in Fig. 1(b). Therefore, in the absence of strain, when each of these energy levels is twofold degenerate, the QH states at $\nu = 2, 6, 10, \dots$ are expected to be strong. Consistent with this expectation, in the magnetoresistance trace taken at $\epsilon=0$ [Fig. 1(c)], resistance minima at these fillings are indeed strong. However, this simple fan diagram also predicts no QH states at $\nu=1, 3, \dots$ for zero strain. This prediction is not fulfilled experimentally. The magnetoresistance trace in Fig. 1(c) exhibits a strong resistance minimum at $\nu=1$ signaling the existence of a $\nu=1$ QH state. Furthermore, the Arrhenius plots of the $\nu=1$ resistance, shown in Fig. 1(d), reveal an

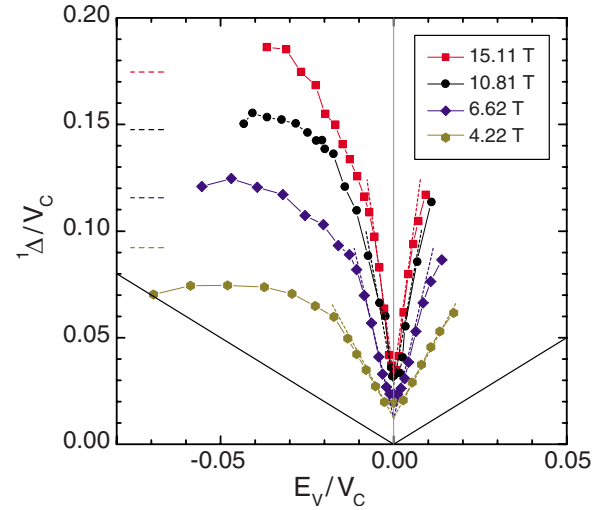


FIG. 2. (Color online) $\nu=1$ energy gap ($^1\Delta$) vs valley splitting energy (E_V) for four different n . Both x and y axes are normalized to the Coulomb energy, V_C . The single-particle prediction, $E_V = \epsilon E_2$ (with $E_2 = 5.8 \text{ eV}$) is shown as a solid line. For each n , the single-particle cyclotron energy at $\nu=1$ is marked by a dashed horizontal line on the left.

activated behavior from which energy gaps ($^1\Delta$) at different ϵ can be deduced.

The summary of our measured $^1\Delta$ as a function of strain-induced valley splitting energy ($E_V = \epsilon E_2$) for several densities is shown in Fig. 2. In this plot, the x and y axes are normalized to the Coulomb energy, $V_C = e^2/4\pi\kappa\epsilon_0 l_B$, where $\kappa=10$ is the dielectric constant of AlAs and $l_B = \sqrt{\hbar/eB}$ is the magnetic length. For our sample parameters, $V_C = 5.6\sqrt{B} \text{ meV}$ (with B in tesla). The energy gaps at $\nu=1$ exhibit three main features. First, the measured gaps are finite even at $E_V=0$ and their values depend on n . Second, as a function of E_V the gap increases. The initial rate at which the gap rises is 3 to 13 times larger than the single-particle prediction, $E_V = \epsilon E_2$ (with $E_2 = 5.8 \text{ eV}$), which is shown as a solid line in Fig. 2. Third, after this rapid rise of the gap, for large values of $|E_V|$, the rate of rise decreases and $^1\Delta$ shows a saturation, clearly visible for the lowest densities.

The simple fan diagram of Fig. 1(b) suggests that at $\epsilon=0$, the $\nu=1$ state in a noninteracting 2DES should be gapless. As the system is tuned away from $\epsilon=0$, a QH state should appear with an energy gap that linearly increases with ϵ with a slope of E_2 (solid line in Fig. 2) until a LL crossing occurs. After the crossing, $^1\Delta$ should be a constant equal to the cyclotron energy. It is clear that this simple picture is inadequate to explain our data.

We discuss here first the case of an interacting 2DES with a *spin* degree of freedom (but no valley degree of freedom), e.g., 2D electrons in GaAs. In such a system, the relevant parameter is the ratio η of the Zeeman and Coulomb energies, $\eta = E_Z/V_C$. It is well known that even in the limit of zero Zeeman energy ($\eta=0$), the ground state at $\nu=1$ is a QH ferromagnet with an energy gap (for an ideal 2DES) equal to $0.62V_C$.^{9,20} For small enough values of η , the excitations are macroscopic spin textures (Skyrmions).⁹ The rate of increase in the $\nu=1$ QH gap with η directly gives the Skyrmion size.

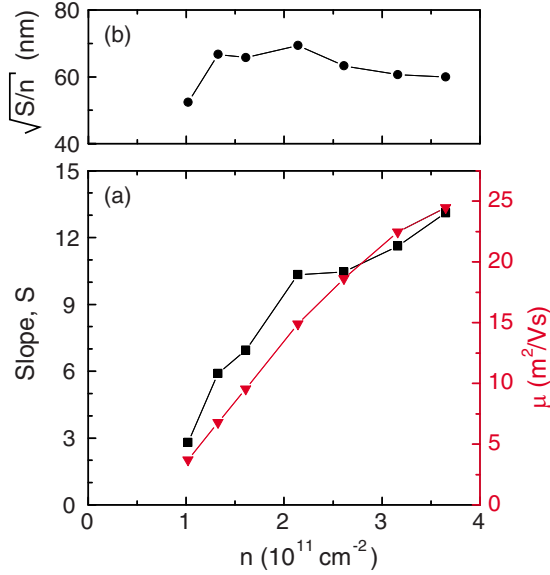


FIG. 3. (Color online) (a) Black squares and red triangles represent the initial slope (S) of $^1\Delta$ rise with E_V , and the zero-field mobility (μ) vs density, respectively. (b) Deduced length scale of the valley Skyrmion size vs density.

In an ideal 2DES, the initial slope should be infinite, then gradually decrease with increasing η , signaling finite-size Skyrmions. Beyond a critical value of η (≈ 0.054), the slope should equal one, indicating that the excitations are single spin flips.^{9,20} These behaviors have indeed been qualitatively observed in experiments on GaAs 2DESs, although there are quantitative differences.^{11–13} In particular, the size of the gaps and the gap vs η slopes are typically much smaller than what is predicted for an ideal 2DES. These discrepancies are generally attributed to the finite layer thickness, LL mixing, and disorder in real 2DESs.^{20–22}

Assuming that the valley degree of freedom in our system is equivalent to the spin degree of freedom, our data find a ready explanation in an analogous valley Skyrmion picture.⁶ In this case, E_V is equivalent to E_Z and ϵ to B . The relevant parameter is therefore $\eta = E_V/V_C$. The data of Fig. 2 can then be interpreted as evidence for a ferromagnetic QH ground state at $\eta=0$ and the slopes are a measure of the valley Skyrmion size. Similar to the spin data, our measured gap values and the slopes are smaller than predicted for the ideal case, and the difference might be attributed to the nonideal effects such as finite layer thickness, LL mixing, and disorder. We note that LL mixing is particularly important for 2D electrons in AlAs because of their large effective mass. The proximity of the second LL can be appreciated by noting that the cyclotron energy, which is marked by a dashed line for each density in Fig. 2, is a small fraction of V_C .

A remarkable aspect of our data is the density dependence of the initial slope (S) at which $^1\Delta$ rises with E_V near $\eta=0$. This dependence is shown in Fig. 3(a) with black squares, together with the dependence of the mobility (μ) of our sample (red triangles) measured at $T=0.3$ K and at zero strain when two valleys are balanced. The close correlation between S and μ might be fortuitous since mechanisms determining μ and S are different. Yet the decreases we observe

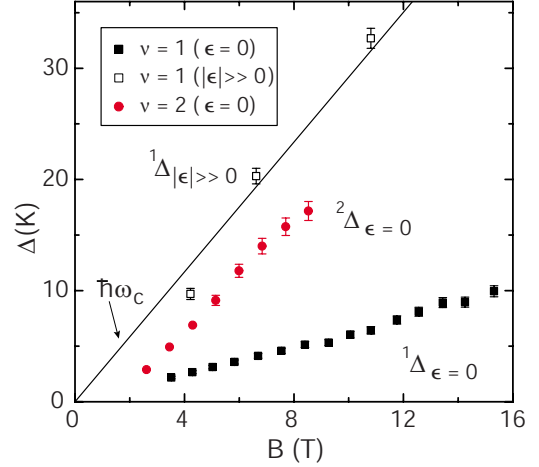


FIG. 4. (Color online) Magnetic field dependence of the energy gaps for $\nu=1$ at $\epsilon=0$ ($^1\Delta_{\epsilon=0}$) and $|\epsilon| \gg 0$ ($^1\Delta_{|\epsilon| \gg 0}$), and $\nu=2$ at $\epsilon=0$ ($^2\Delta_{\epsilon=0}$). The single-particle prediction, $\hbar\omega_c = \hbar eB/m_b$ (with $m_b = 0.46$) is shown as a solid line.

in S and μ at lower n are both likely a result of increasing disorder. The decrease in μ with decreasing n is often observed in 2DESs and stems from a loss of screening at lower densities. In the Skyrmion picture, S gives the Skyrmion size and in a disordered system, the Skyrmion size is expected to be limited by the disorder (domains). Indeed, we can estimate the length scale of the domain size as $\sqrt{S/n}$. As seen in Fig. 3(b), this length scale is roughly constant (60 ± 10 nm) and independent of n . This result suggests that the disorder landscape is fixed and the Skyrmion size is determined by the number of electrons that can fit into the domains.

Another noteworthy feature of our data is the dependence of $^1\Delta$ at zero strain ($^1\Delta_{\epsilon=0}$) on n , or equivalently on the magnetic field at $\nu=1$. The experimentally measured $^1\Delta_{\epsilon=0}$ are shown as black squares in Fig. 4. Interestingly, $^1\Delta_{\epsilon=0}$ linearly varies with B and extrapolates to zero as $B \rightarrow 0$. Similar linear B dependence of the QH state gaps when two valleys are at a coincidence has been reported experimentally for AlAs (Refs. 8 and 23) and Si (Ref. 24) 2DESs. Although it is generally believed that electron-electron interaction is responsible for the nonzero gap, the linear dependence on B is puzzling because an interaction-induced energy gap is expected to scale with $V_C \sim \sqrt{n} \sim \sqrt{B}$. In a system like ours where V_C is about ten times larger than $\hbar\omega_c$, however, there is severe LL mixing, and this mixing has been proposed to limit $^1\Delta$ to $\hbar\omega_c/2$.¹⁶ The linear dependence we observe in Fig. 4 is consistent with this theoretical expectation but the measured $^1\Delta_{\epsilon=0}$ is about two times smaller than $\hbar\omega_c/2$.

In Fig. 4, we also report the measured gaps for $\nu=1$ at $|\epsilon| \gg 0$ ($^1\Delta_{|\epsilon| \gg 0}$) and for $\nu=2$ at $\epsilon=0$ ($^2\Delta_{\epsilon=0}$). As the simple fan diagram in Fig. 1(b) suggests, both $^1\Delta_{|\epsilon| \gg 0}$ and $^2\Delta_{\epsilon=0}$ should be equal to $\hbar\omega_c$, indicated by a black line in Fig. 4. The experimentally measured $^2\Delta_{\epsilon=0}$ values are somewhat smaller than the single-particle cyclotron energy and have a negative intercept as $B \rightarrow 0$. The smaller gap values and the negative intercept might be related to the disorder broadening of the LLs. Compared to the $^2\Delta_{\epsilon=0}$ data, $^1\Delta_{|\epsilon| \gg 0}$ are closer to the single-particle values. This trend is also clear in Fig. 2

where, for large values of $|E_V|$, the gaps approach the single-particle cyclotron energies marked by a dashed line for each density. At first sight, the difference between the measured $^1\Delta_{|\epsilon|\gg 0}$ and $^2\Delta_{\epsilon=0}$ is puzzling. We emphasize, however, that at $\nu=1$ and $|\epsilon|\gg 0$, we have a *single-valley*, single-spin 2DES (i.e., a fully spin- and valley-polarized 2DES) whereas at $\nu=2$ and $\epsilon=0$, we have a *two-valley*, single-spin 2DES. It has been reported^{25,26} that the effective mass of an interacting 2DES depends on the spin and valley occupations and that the mass is suppressed in a fully spin- and valley-polarized system. It is possible that for $\nu=1$ at $|\epsilon|\gg 0$, the LL broadening and effective-mass suppression, which have the opposite influence on the gap values, cancel each other and bring $^1\Delta_{|\epsilon|\gg 0}$ close to the single-particle values.

We close with a few words of caution regarding the applicability of the Skyrmions to our system. The Skyrmion excitations should exist in a system whose Hamiltonian has an SU(2) symmetry. For an SU(2) symmetric case, the ground state has identical energies for all directions of the

pseudospin orientation with a long-range order. However, thanks to the rotation of the major axes of the two valleys, SU(2) symmetry is broken in our system and it should have an easy-axis anisotropy. The energy-gap measurements as a function of the pseudospin Zeeman energy of an easy-axis ferromagnetic system at $\nu=4$ have been reported previously,⁴ and the measured gaps are qualitatively consistent with ours. It is possible then that our system at $\nu=1$ is an easy-axis ferromagnet. We also speculate that in our samples, the long-range order might be destroyed because of the random potential from the strain inhomogeneity, resulting in a paramagnetic ground state.²⁷ The strain and density dependence of the energy gaps for these proposed scenarios are currently unknown.

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