Fiske-amplified superfluid interferometry

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(Received 22 February 2010; published 17 May 2010)

We report an experimental demonstration of signal amplification for a superfluid ^{4}He interferometer by allowing the matter waves from two Josephson mass current sources to interact with the resonances within the experimental cell. We characterize the interferometer using an external phase shift and demonstrate a 30-fold enhancement in intrinsic phase sensitivity. We also demonstrate a truly continuous operation of this interference device.

DOI: 10.1103/PhysRevB.81.172502 PACS number(s): 67.25.dg, 07.60.Ly, 85.25.Dq, 03.75.Lm

An important dynamical phenomenon observed in superconducting weak links[1] is the Fiske effect,[2] where interaction of the Josephson oscillation with internal electrodynamic resonances in the junction causes current spikes. A similar phenomenon exists in superfluid weak links[3,4] (formed by an array of nanoscale apertures connecting two superfluid reservoirs), where the Josephson mass current oscillation can interact with acoustic resonances within the experimental cavity causing mass current spikes. Moreover, since the mass current oscillation is typically detected by a flexible diaphragm hydraulically coupled to the oscillating superfluid, the Fiske-type interaction at natural resonances can lead to a substantial signal amplification at particular frequencies. This amplification was reported by Hoskinson *et al.* for a superfluid ^{4}He single weak link system.[5] A unique technique[6] to produce a constant chemical-potential difference was used to drive Josephson oscillation at various frequencies, and remarkable stability and amplitude magnification were observed when the oscillation was driven to join a few resonant features. It has been suggested since then that this Fiske phenomenon (combined with a constant chemical-potential "battery" technique) could be an ideal method for operating superfluid quantum interference devices,[7] analogous to dc-superconducting quantum interference devices (SQUIDs).[8] Here we demonstrate the application of these techniques to such a device. We show that a superfluid interferometer can now be operated continuously without interruption and the interferometer sensitivity can be enhanced significantly (by as much as a factor of ~34). These results show promise in applications for superfluid-based matter wave interferometry.

Our apparatus is schematically shown in Fig. 1. A small volume is bounded on one side by two arrays of nanoscale apertures (indicated by crosses) and the other side by a flexible diaphragm. Each array consists of 75×75 nominally 60 nm apertures spaced on 2 μm square lattice in a 60-nm-thick silicon nitride membrane. We place a heater (R1) in the small bounded volume. The diaphragm and electrode below it form an electrostatic pressure pump. The inside is filled with superfluid ^{4}He, and we configure the apparatus to form a loop of superfluid interrupted by two junctions as in the case of a dc-SQUID. The entire apparatus is immersed in a bath of temperature-regulated superfluid helium. A SQUID-based displacement sensor[9] is used to monitor the position of the diaphragm and hence the fluid motion in aperture arrays. An equivalent electrical circuit for this neutral matter interference device and a SEM picture of apertures are shown in Figs. 2(a) and 2(b).

Well below the superfluid transition temperature, T_λ, superfluid within the aperture arrays is characterized by a linear current-phase relation with discrete 2π phase slips. Closer to T_λ, it follows a sinelike dc-Josephson current phase relation. In both regimes, when a constant dc chemical-potential difference $\Delta\mu$ is applied across the aperture arrays, fluid within each array exhibits ac mass current oscillation[3,4,10] at a Josephson frequency $\Delta\mu/h$. Imposing the condition $\oint\vec{\nabla}\phi\cdot d\vec{l}=0$ (for the closed loop of superfluid through two Josephson junctions), one can show that the combined oscillation amplitude from two arrays exhibits interference depending on the external phase shift $\Delta\phi_{ext}$ injected into the system.[7] In a general case where each array exhibits different oscillation amplitudes I_1 and I_2, the overall amplitude can be written as

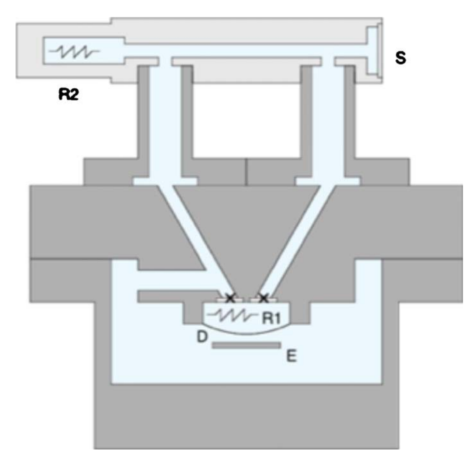

FIG. 1. (Color online) Experimental apparatus. The X's indicate the positions of the two aperture arrays. (d) A flexible metallized diaphragm and (e) a fixed electrode are used to create pressure difference across the aperture arrays by the application of an electrostatic force between them. The diaphragm also forms the input element for a sensitive microphone based on superconducting electronics (not shown) to detect the mass current oscillation. A heater (R1) is used to induce temperature difference across the aperture arrays. A heater (R2) and a thin Cu sheet(s) serve as a heat source and a temperature sink for the top arm, respectively. The height of the apparatus is ~7 cm.

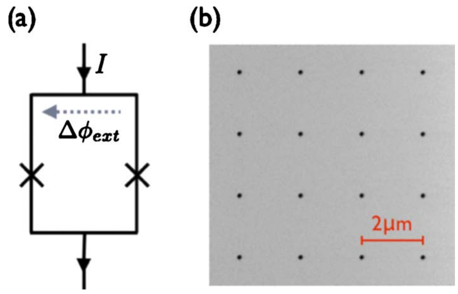

FIG. 2. (Color online) (a) A sketch of the equivalent interferometer topology, emphasizing the analogy with a dc-SQUID. (b) A SEM image of aperture arrays. 2 μm spacing between the apertures.

$$(I_1 + I_2)\sqrt{\cos^2\left(\frac{\Delta\phi_{ext}}{2}\right) + \gamma^2 \sin^2\left(\frac{\Delta\phi_{ext}}{2}\right)}, \qquad (1)$$

where $\gamma \equiv (I_1 - I_2)/(I_1 + I_2)$ is the asymmetry factor. Phase shift $\Delta\phi_{ext}$ can be induced in variety of ways including rotation through Sagnac effect,[7,11,12] which makes this type of interferometer an interesting candidate for applications such as geodesy, seismology, inertial navigation, etc. as well as a unique tool for fundamental physics.[13,14] Here, to characterize the interferometer performance, we introduce $\Delta\phi_{ext}$ by means of superfluid heat current. Since superfluid velocity is associated with quantum-mechanical phase gradient,[15] one can inject phase bias along an interferometer loop by locally creating thermally driven superfluid current with a heater (R2 in Fig. 1) and a thermal sink (S) placed in a sense arm loop. The underlying physics behind this technique is described in more detail elsewhere.[16]

Chemical-potential difference $\Delta\mu$ for superfluid helium (per unit mass) is given by $\Delta P/\rho - s\Delta T$, where ρ is the fluid density, s is the entropy per unit mass, and ΔP and ΔT are pressure and temperature differences, respectively.[15] This means that we can apply ΔP and/or ΔT across the aperture arrays to drive Josephson mass current oscillation. One complication is that there is a finite dc component to fluid flow driven by $\Delta\mu$, which causes the applied $\Delta\mu$ to decay over time (lowering the Josephson frequency and eventually ending the oscillation as $f_J \to 0$). One can drive the oscillation, Fourier transform the response in real time to measure its frequency and amplitude, keep pulling on the diaphragm (while adjusting the rate) to maintain the oscillation at a particular frequency. Although we have successfully used this feedback technique in our prior work, there is a range to how far one can pull the diaphragm in this measurement scheme. This range is on the order of $\sim$10 nm, limited by the transducer dynamic range for a given sensitivity required to detect much smaller Josephson oscillation. We have typically been able to sustain the oscillation for tens of seconds before we have to let go of the diaphragm to let it relax back to its initial equilibrium position (and start the pull again). This corresponds to, for example, a few 10 000 oscillation cycles for a $\sim$1 kHz signal for one pull on the diaphragm (or a few thousand Fourier spectra to average over if transforms are taken from $\sim$30 cycles of oscillation in time domain). This is more than sufficient for many controlled experiments where one can induce a phase shift of interest and then subsequently measure it. However it is clearly not ideal for practical applications. This is where a constant $\Delta\mu$ battery method[6] becomes extremely useful.

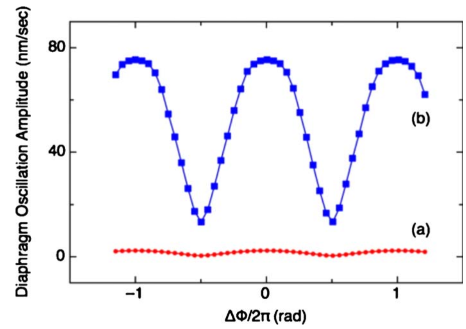

FIG. 3. (Color online) (a) Quantum interference pattern from two Josephson junctions. (b) The same pattern probed on resonance with Fiske amplification. Without amplification, the diaphragm velocity amplitude $\dot{x}$ is related to mass current oscillation amplitude I from aperture arrays by the relation $I = \rho A\dot{x}$, where ρ is the fluid density and A is the diaphragm area.

When power is applied to the heater placed in the small inner volume (R1 in Fig. 1), superfluid fraction of the fluid flows toward that inner volume while the normal fraction flows out. The entropy is carried by the normal flow and thermal conduction through the wall. If the applied heat is small enough, a steady state after a short transient behavior (which can be Josephson oscillation) is established such that $\Delta\mu = 0$. However as the applied power is raised further, superfluid flow eventually reaches the so-called critical current. A new steady state is then established where $\Delta\mu$ is now nonzero and constant, determined by the applied power.[6] This constant $\Delta\mu$ drives Josephson oscillation continuously within the aperture arrays, and the oscillation frequency (proportional to $\Delta\mu$) can be adjusted easily by turning the heater knob. This is essentially equivalent to having a battery to drive Josephson oscillation in superconducting weak-link systems. In the work reported here, we apply this technique to our interferometer to achieve truly continuous operation and then make an "amplifier" out of the Fiske phenomenon by allowing the matter waves from two Josephson oscillation sources to interact with the resonances within the experimental cell.

The diaphragm velocity oscillation amplitude $\dot{x}$ as a function of $\Delta\phi_{ext}$ at $f_J = 5.2$ kHz is shown and marked as curve (a) in Fig. 3. A 2π periodic modulation is due to interference from two Josephson mass current sources as described earlier. The modulation depth seen in this particular data is representative of those observed in previous experiments.[7,16] This should now be compared with curve (b) taken on resonance (5.9 kHz). A significant signal amplification is clearly visible. Both curves are taken at $T_\lambda - T = 6$ mK. The sensitivity of the superfluid quantum interference device is the slope of the modulation pattern at the steepest point, and the sensitivity enhancement is $\sim$34 comparing the curves (a) and (b) shown in Fig. 3.

There were several difficulties to overcome to achieve

these results. One was the lack of a simple and quiet way to inject phase bias to investigate the interference. For example, with the first superfluid ^{4}He interferometer,[7] the "flux" in the interferometer loop could only be varied by reorienting the whole apparatus with respect to earth's rotation axis (taking advantage of the Sagnac effect). It was not possible to do this while maintaining the Josephson oscillation due to large mechanical disturbances associated with physical reorientation. This hurdle was overcome by demonstrating a vibration-free nonmechanical method to inject phase variations using superfluid heat current.[16] The second problem to employing the Fiske amplification in the past was the vibrational as well as rotational noise from the environment. There were cases where excess noise would constantly ring up the diaphragm oscillation at resonances rendering the reliable observation of Josephson amplitude modulation difficult when driven at those particular frequencies. We have cleared this hurdle by designing a shorter and stiffer cryostat system on a 1.5 ton granite slab and by applying stringent vibration isolation to eliminate noise coupling from the environment.

The intrinsic sensitivity for this particular device (with Fiske amplification) can be estimated by multiplying the inverse slope $(d\dot{x}/d\Delta\phi)^{-1}$ of a modulation curve at its steepest point by the smallest detectable velocity $\delta\dot{x}_{min}=\omega_J\delta x_{min}$, where $\omega_J=2\pi f_J$, and $\delta x_{min}\approx 1$ fm is the smallest detectable diaphragm displacement in 1 Hz bandwidth. For the curve (b) shown in Fig. 3, the phase resolution is 8×10^{-4} rad/$\sqrt{\text{Hz}}$. If used as a gyroscope (with ≈ 10 cm^2 enclosed area of the device reported here), this translates to a rotational resolution of 8×10^{-9} rad s^{-1}/$\sqrt{\text{Hz}}$, which surpasses typical sensitivities of conventional atom interferometers[17–19] and comes within an order of magnitude away from the short-term sensitivity of the world's best dual atom interferometer[20] (6×10^{-10} rad s^{-1}/$\sqrt{\text{Hz}}$). Considering that this enhancement is done without any improvement on other key parameters of the device such as the sensing area, the number of sense loop turns, the number of apertures and their sizes in junctions etc., it shows great promise for the superfluid interferometer. However a sensitive device is only as good as the environment. We currently find the phase noise from the environment [measured at the steepest part of the modulation curve (b) in Fig. 3] to be 3×10^{-3} rad/$\sqrt{\text{Hz}}$. This noise level is almost 20 times smaller than what has been seen in earlier work but about four times higher than the smallest phase change the device is capable of detecting (with Fiske amplification) if only limited by electronic noise. One may be able to use an astatic sense loop configuration to make the device more independent of phase noise from the environment. Work leading to such configuration is currently under way. Some more significant improvement might be made by suspending the experimental cell from the so-called 1 K pot inside of a vacuum can. One could also use active vibration isolation. The ultimate sensitivity without any environmental noise could be limited by phase fluctuation across (or within) the aperture arrays.[21] However our device still seems vibration limited and far from any fundamental limit.

We note that there are many frequencies at which the Josephson mass current oscillation exhibits the kind of amplification shown in Fig. 3. As we slowly vary the heater power to change the oscillation frequency, we see that f_J tends to jump and adhere to resonant features nearby. The oscillation amplitude increases considerably (as seen in Fig. 3) and the locking is strong enough that small change in heater power to attempt to change f_J will not drive the signal off resonance. This leads to considerable hysteresis in exactly what heater power is needed to lock the oscillation on a particular resonance and also to shake it loose from it, and those values can vary as much as $\sim$40%. We suspect that standing pressure waves are set up and are feeding back on the fluid oscillation in aperture arrays. The strong natural locking observed at resonance is far superior to conventional frequency feedback used in the past, and we find that its stability contributes to suppression of phase noise as well. Comparing the data taken on resonance and off resonance, the phase noise when on resonance is consistently smaller by a factor of $\sim$3. Some of the observed resonant features can be identified with a simple frequency sweep on the diaphragm. However there are many modes that do not respond at all to such excitation, making the investigation of resonant features themselves intriguing but somewhat nontrivial with the current apparatus. Resonances that are present in the system include diaphragm drum-head modes, those that arise from superfluid weak link system (plasma modes[10,22]), resonances in various cell cavities associated with pressure waves (superfluid first sound[15]), modes associated with temperature waves (superfluid second sound[15]), higher harmonics of all these different modes, and those caused by their mixing. It may make sense to explore these in a single junction configuration where the geometry is much simpler and iterative runs can be carried out smoothly with modifications to cell parameters. Investigation with ac pressure and thermal drives in conjunction with modeling and simulation may allow one to engineer a particular resonance into apparatus to further enhance the sensitivity in the future.

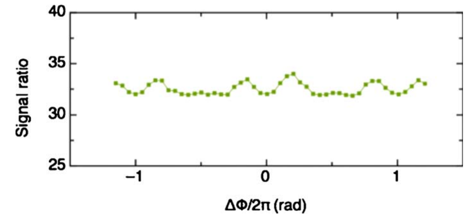

FIG. 4. (Color online) Signal ratio for curves (a) and (b) from Fig. 3.

Figure 4 shows the amplified signal divided by the nonamplified signal for the pair shown in Fig. 3. The signal amplification factor is relatively flat for different $\Delta\phi_{ext}$ values although the enhancement seems to be consistently higher for points surrounding (but not at) the crest of the modulation pattern. This results in an interference pattern slightly flattened near the crest when amplified. We emphasize that the modulation pattern is repeatable for a given temperature and for a given resonant feature, making the reported sensitivity enhancement extremely robust. Combined with the uninterrupted phase sensing mode of operation achieved here, this significant sensitivity enhancement should open a door for applications beyond superfluid research.

It is certainly fascinating to take a step back and realize that we are directly coupling the fluid motion governed by quantum mechanics to a rather "classical" oscillator or cavity to harness signal amplification reported here. The capability to employ this type of technique is remarkable and it is rooted in the fact that, with $\sim 10^{23}$ atoms involved in superfluidity, quantum mechanics results in fluid flow that is macroscopic. This is an element in superfluid interferometry that is quite unique compared to atom and laser interferometers[18,23–25] and should be used to its advantage for the development of more sensitive interferometers for both applied and fundamental scientific research.

In conclusion, we have demonstrated the application of Fiske phenomenon to amplify the signal for a superfluid quantum interference device. We have achieved a significant sensitivity enhancement (as much as a factor of ~ 34). We have also shown (using the constant $\Delta\mu$ method) that the interferometer can now be operated in a continuous mode which would be a key element in further developing this device for studies such as geophysics, inertial navigation, etc. Further investigation of various resonant features should allow one to improve on the signal enhancement reported here.

The author thanks D. Rogers for machining and metal working, S. Bevis, J. Chervinsky, and C. Colby for infrastructure support and assistance, and J. VanDelden for nanofabrication work. The author acknowledges valuable discussion with K. Penanen, O. Sahin, F. Vollmer, A. Joshi, R. Packard, and E. Hoskinson. The author thanks R. Packard for support during transition from UC Berkeley to the Rowland Institute at Harvard. This research was supported by the Rowland Institute at Harvard University.

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