



Formation of gap solitons in ferromagnetic films with a periodic metal grating

Alexey B. Ustinov and Boris A. Kalinikos

Department of Physical Electronics and Technology, St. Petersburg Electrotechnical University, 197376 St. Petersburg, Russia

Vladislav E. Demidov* and Sergej O. Demokritov

Institute for Applied Physics and Center for Nonlinear Science, University of Münster, 48149 Münster, Germany

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We have studied experimentally the formation of magnetic solitons in a periodic system constituted by a ferromagnetic film and a periodic metal grating. We show that due to the finite-size effects and the influence of dynamic magnetic losses, the wave dispersion within the stop bands caused by the periodicity allows observation of both bright and dark solitons formed via the induced and the coupled modulation instabilities. We find that the flexibility of the studied system enables a wide variation in the characteristics of the excited solitons by the proper choice of the experimental conditions.

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Management of wave dispersion through spatial periodic modulation of the parameters of the waveguiding medium is known since years as a convenient and flexible tool for tuning the characteristics of waves of different nature. This mechanism has found many technical applications, in particular, for generation, transmission, and processing of optical signals.¹ Moreover, it appears to be very important for basic studies. One of the research areas, where the possibility to tune the dispersion characteristics is especially useful, is the nonlinear physics (see, e.g., Ref. 2). Many nonlinear wave phenomena such as appearance of dynamic instabilities and formation of solitons demand particular dispersion characteristics, which often can be obtained by introducing an artificial spatial periodicity. One of the recent examples is the realization of the dispersion management of matter waves using periodic light potentials³ and its application for generation of matter-wave gap solitons.⁴

Spin waves propagating in thin ferromagnetic films represent a superb object for investigation of universal nonlinear phenomena.⁵ One of their most significant advantages is the possibility to change the type of dispersion and nonlinearity by properly choosing the direction of the external static magnetic field with respect to the direction of wave propagation.⁶ This unique flexibility allowed the observation of a wide spectrum of nonlinear phenomena, many of them being difficult to observe in other nonlinear systems.^{7–10} Using periodic spatial modulation of the parameters of magnetic films, one obtains an additional degree of freedom in manipulation of spin-wave dispersion. This mechanism is especially advantageous because it allows easy construction of the desirable dispersion by a proper choice of the modulation characteristics. In the recent years, it has been intensively studied within the paradigm of magnonic crystals, which are expected to find technical applications in microwave-frequency signal processing (see, e.g., Refs. 11–14). The large potential of magnetic periodic structures for studies of universal nonlinear phenomena, in general, and solitons, in particular, has also been discussed theoretically.^{15–17} However, up to very recently, no experimental results on this topic have been reported. In fact, the only work, where the issue of solitonic propagation in periodically structured magnetic films was addressed experimentally, is Ref. 18.

Here we report on the detailed experimental study of the formation of gap solitons in a magnetic film with periodic metal grating. We show that due to the complex spin-wave dispersion in a finite-size periodic structure characterized by nonzero dynamic losses, five frequency zones are formed within the stop bands, where the dispersion coefficient consequently changes its sign. Correspondingly, the type of the soliton changes between bright and dark as the central frequency of the soliton moves from one zone to the other. Moreover, we show that in such system, the solitons can be formed through the coupled modulation instability of two waves with essentially different dispersion. In this case, the type of the solitons appears to be mainly determined by the wave affected by the presence of the periodicity.

Figure 1 shows the sketch of the experiment. The studied waveguiding structure is 2-mm-wide and 35-mm-long stripe made from monocrystalline film of yttrium iron garnet (YIG) with the thickness of 5.2 μm . The YIG film has unpinned surface spins, a ferromagnetic resonance linewidth of 0.6 Oe at 5 GHz, and a saturation magnetization $4\pi M_s = 1750$ G. A metal grating constituted by 16 Cu stripes lithographically prepared on a quartz substrate is attached to the surface of the YIG film. The stripes have the width of 50 μm , the thickness of 10 μm , and are placed with a spatial period of 300 μm . Two 40- μm -wide and 3-mm-long microstrip antennas with a separation of 7.75 mm are used for excitation and detection of spin waves. The experimental structure is magnetized by a uniform static magnetic field $H = 1065$ Oe applied in the plane of the YIG film parallel to the antennas, which corresponds to the propagation geometry of surface spin waves.

In this work, we utilize the excitation of solitons through

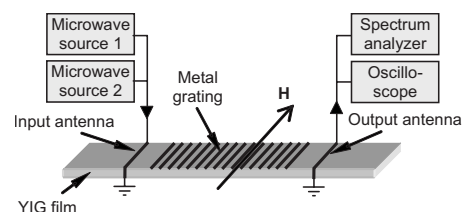


FIG. 1. Sketch of the experiment.

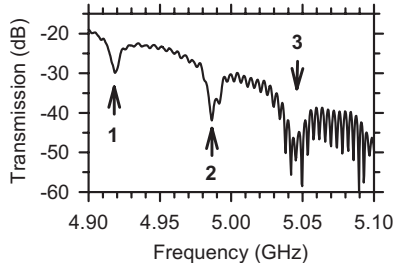


FIG. 2. Microwave transmission characteristic of the YIG film with the periodic metal grating.

the modulation instability of two copropagating continuous spin waves.^{19–21} Contrary to the method of the pulsed excitation, which was widely used in the first works on spin-wave solitons,^{22–25} the method of two waves does not predetermine the type of the excited soliton. Depending on the dispersion and the nonlinear characteristics, both bright and dark solitons can be formed as a result of the nonlinear interaction of the initial waves. In order to realize such excitation, we use two microwave sources (see Fig. 1). The continuous waves produced by the sources are simultaneously applied to the input spin-wave antenna using a resistive microwave combiner. The signal received at the output antenna is analyzed by means of a microwave spectrum analyzer and a digital oscilloscope, which allows recording of wave forms of the envelope of the microwave signal.

Figure 2 shows the measured transmission characteristic of the YIG film with the attached periodic grating. The data were obtained for the excitation power of 1 mW corresponding to the linear propagation regime. The transmission characteristic demonstrates several well-pronounced dips marked with numbers from 1 to 3, which are distinctive for magnonic crystals.^{11–14} These dips appear due to the periodicity and represent stop bands in the spin-wave spectrum corresponding to the wave numbers being a multiple of π/S (Brillouin vectors), where S is the spatial period of the metal grating. Note that these stop bands are characterized by finite transmission losses, which can be attributed to the finite number of elements constituting the grating and the nonzero spin-wave losses. The influence of these two factors on the spin-wave dispersion is demonstrated by Fig. 3, which shows a qualitative picture of the dispersion in the vicinity of the Brillouin vector k_B . The dashed-dotted line shows the dispersion of spin waves in a continuous magnetic film without the periodic structure. It is well known that adding a spatial periodicity results in a modification of the dispersion for wave numbers close to k_B leading to a formation of the frequency band, where the wave number becomes a complex value. For the case of an infinite periodic structure in the absence of losses, the dispersion curve has a well-known form shown in Fig. 3(a) by the dashed line. For such case, one would expect a total reflection of spin waves within the frequency range of the stop band marked in Fig. 3(a) as Δ . Nevertheless, if finite losses are taken into account,²⁶ the modification of the dispersion curve due the periodicity appears to be smoother, as shown in Fig. 3(a) by the solid line. In this case, the transmission should still significantly decrease around k_B due to the resonant increase in the imaginary part of the wave number k'' , as shown by Fig. 3(b) but stay finite in agreement

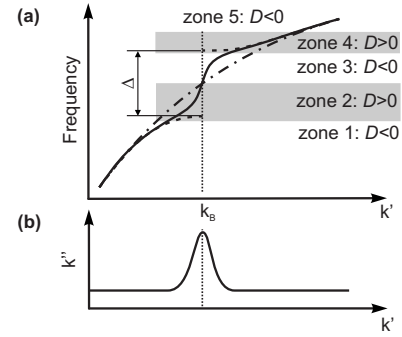


FIG. 3. (a) Qualitative picture of the spin-wave dispersion in the vicinity of the Brillouin vector k_B . Dashed-dotted line—dispersion in a continuous film, dashed line—dispersion in an infinite decay-free periodic structure, and solid line—dispersion in a periodic structure taking into account finite losses. (b) Dependence of the imaginary part of the wave number k'' on its real part k' .

with the experimental data of Fig. 2. Note that the finite number of stripes constituting the metal grating is expected to result in qualitatively similar modifications of the spin-wave dispersion.

The very interesting feature of the finite-loss dispersion in Fig. 3(a) is the consecutive variation in the curvature of the dispersion curve in the proximity of the Brillouin vector. As seen from the figure, the dispersion coefficient $D = \partial^2 \omega / \partial k^2$, characterizing this curvature, changes its sign several times. For the wave numbers distant from k_B (zones 1 and 5), the coefficient D is negative, which is typical for surface spin waves. As the wave number approaches k_B , the coefficient D becomes first positive (zone 2), then negative again (zone 3), and then changes its sign twice more (zones 4 and 5). Such changes in the sign of the dispersion coefficient represent an important feature of periodic structures from the point of view of nonlinear phenomena, in general, and soliton formation, in particular. As is well known, the formation of solitons is only possible if the nonlinearity and the dispersion can compensate each other. For bright solitons, this requires the dispersion coefficient D and the nonlinear coefficient $N = \partial \omega / \partial |u|^2$ to have opposite signs, $ND < 0$ whereas for dark solitons, these two coefficients should have the same sign, $ND > 0$. Since the nonlinear coefficient in the studied geometry is negative irrespectively of frequency,⁶ both bright and dark solitons can be formed in the considered periodic structure and the type of the soliton can be chosen by varying the central frequency of the spin-wave packet around the frequency of the transmission dips in Fig. 2.

In order to prove this hypothesis, we measured the wave forms appearing at the output of the periodic structure as a result of the nonlinear interaction of two continuous waves at closely spaced frequencies $f_1 = f_0 - \Delta f/2$ and $f_2 = f_0 + \Delta f/2$. The frequency difference between the initial waves Δf was chosen to be small enough (100–200 kHz) so that the frequencies of both waves are located within the same dispersion zone. The operation point was chosen in the vicinity of the transmission dip 2 (see Fig. 2). In the course of measurements, the central frequency f_0 was varied over the frequency range of the stop band. For each particular value of f_0 , the development of the envelope of the output signal with gradually increasing power of the initial waves $P_1 = P_2 = P$

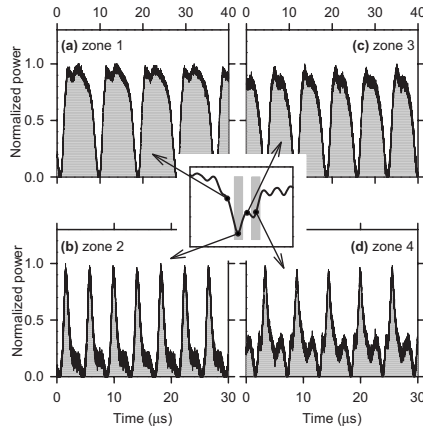


FIG. 4. Solitonic wave forms observed in the zones 1–4. Inset shows in an appropriate scale a small part of the transmission characteristic at frequencies around those of the stop band 2. Points on the curve indicate the values of f_0 used to obtain wave forms shown in panels (a)–(d).

was observed. As expected, at small values of P , providing linear propagation regime, the output wave form was found to demonstrate a harmonic beating with the frequency equal to Δf . As the power was increased, the initial waves started to interact resulting in a narrowing of positive or negative half periods of the wave form. Such narrowing is equivalent to the narrowing of a nonlinear pulse in the case of the pulsed soliton excitation regime resulting in a nonlinear transmission characteristic, which is usually considered as a signature of the beginning of the soliton formation.^{23–25} Depending on the value of f_0 , this process resulted in qualitatively different output wave forms of the spin-wave envelope showing bright or dark solitonic pulses. Typical powers, at which the soliton formation was found to start, were in the range from 5 to 10 mW depending on f_0 . Solitonic regimes were found to be stable within a certain range of P . Further increase in the input power always led to destruction of the regular pulses and to emergence of stochastic oscillations. Depending on f_0 , the destruction was observed at powers in the range 50–100 mW.

Typical results obtained in the experiment are shown in Fig. 4. The inset shows in an appropriate scale a small part of the transmission characteristic of Fig. 2 corresponding to the stop band 2. The four panels of Fig. 4 show the wave forms recorded for the frequencies f_0 marked in the inset by points. All the wave forms were obtained for the powers of initial waves $P_1 = P_2 = 20$ mW, which corresponded to the strongly nonlinear but still regular oscillation regime. As seen from Fig. 4, by varying f_0 over the frequency range of the stop band, one observes a consecutive change in the type of solitons between dark and bright. The wave form in Fig. 4(a) obtained for $f_0 = 4983$ MHz clearly shows a sequence of dark solitons. Qualitatively, similar wave forms were observed in the frequency range up to 4985 MHz. This fact agrees well with the above theoretical discussions and allows one to conclude that this frequency range corresponds to the dispersion zone 1 in Fig. 3. As f_0 was increased above 4985 MHz, the output wave forms changed from dark to bright soliton sequences [see Fig. 4(b) obtained for f_0

$= 4987$ MHz]. Bright solitons were observed in the frequency range from 4986 to 4989 MHz. In accordance with Fig. 3, this region can be associated with the dispersion zone 2. Further increase in the frequency resulted in the type of solitons changing three more times at frequencies 4989, 4992, and 4994 MHz, which can be taken as lower boundaries of the dispersion zones 3, 4, and 5, respectively. The typical wave forms observed within zones 3 and 4 are shown in Figs. 4(c) and 4(d). They were recorded for $f_0 = 4990$ MHz and 4993 MHz, respectively. Finally, at frequencies higher than 4994 MHz (dispersion zone 5), sequences of dark solitons very similar to those shown in Fig. 4(a) were detected. These results show that each zone of bright/dark soliton formation has a width of 2–3 MHz. Detailed measurements also showed that the narrow transition regions of about 0.3 MHz exist at the zone boundaries. Positioning f_0 within these transition regions corresponds to the frequencies of the initial waves lying in the neighboring dispersion zones. For such excitation regimes, formation of solitons was not observed.

Note that in the above measurements, the frequencies of the initial interacting waves were chosen to be close enough so that both waves had very similar dispersion properties. Recently, it was shown^{21,27} that soliton formation is also possible as a result of interaction of two waves with essentially different dispersion and group velocities, the so-called coupled modulation instability. Thanks to the large diversity of dispersion properties, the studied system is well suited for observation of this phenomenon. Correspondingly, we extended our experiments to the case when the frequencies of the initial waves are positioned in different dispersion zones. In this case, there are many possible combinations: the frequencies can be positioned within the stop band in different dispersion zones or they can be chosen in such a way that only one of the frequencies is positioned within the stop band whereas the other is positioned between the stop bands. The latter case is of particular interest since it corresponds to the nonlinear interaction of waves, one of which is affected and the other one is not affected by the presence of the periodicity.

Figure 5 presents the results obtained as the frequency f_1 was fixed at 4987 MHz, i.e., within the zone 2 of the stop band characterized by the positive dispersion coefficient, and the frequency f_2 was varied from 4995 to 5030 MHz, i.e., between the stop bands 2 and 3 where the dispersion coefficient is negative. As shown in Figs. 5(a) and 5(b), within the entire range of variation in f_2 bright solitons were formed as a result of interaction of the initial waves. This fact indicates that the nonlinear dynamics is mainly determined by one of the waves, in our case, by the wave at the frequency positioned within the stop band. Such domination of one of the waves appears to be typical for the coupled spin-wave modulation instability.²⁷ However, since the theory of this phenomenon is still missing, there is no rigorous explanation for the dominating role of the wave affected by the periodicity. Note here that the formation of bright solitons was also observed, as the frequency f_2 was positioned between the first and the second stop bands, which shows that the domination of the wave at the frequency of the stop band has a general character.

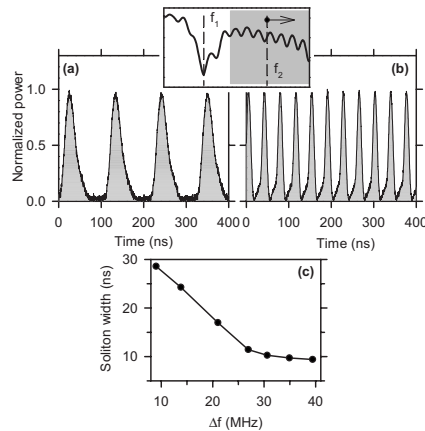


FIG. 5. (a)–(b) Typical solitonic wave forms observed as f_1 was fixed within the zone 2 and f_2 was varied, as shown in the inset. (c) Temporal width of the generated solitons as a function of the frequency separation between f_1 and f_2 (line—guide for the eyes).

Another interesting experimental observation is the variation in the temporal width of the bright solitons formed due to the coupled instability with increasing frequency difference $\Delta f = f_2 - f_1$. As seen from Fig. 5(c), characterizing this variation, the width of the solitons decreases nearly linearly for Δf increasing from 9 to 26 MHz and then saturates at the

value of about 10 ns. In a simple theoretical model,²⁸ the duration of a soliton is determined by the nonlinear and the dispersion coefficients. Since in the case of the coupled modulation instability, the dispersion coefficient is different for the two initial waves, one can suggest that the observed variation in the soliton width is connected with the change in the contribution of the waves into the nonlinear dynamics with the change in the frequency separation between them. To prove this qualitative conclusion, a rigorous theory of the coupled modulation instability is needed.

In conclusion, we have shown that the dispersion management of spin waves by introducing an artificial periodicity is a very flexible tool for investigation of solitonic phenomena. We found that the finite-size and relaxation effects significantly affect the formation of solitons. Due to the complex dispersion in such a system, both bright and dark solitons can be observed and the characteristics of the solitons can be varied in a wide range. We believe that these experimental findings will stimulate further developments in the theory of dynamic magnetic nonlinear phenomena.

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*Corresponding author; demidov@uni-muenster.de

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