Quantum coherence in spin-torque nano-oscillators

Sergio M. Rezende
Departamento de Física, Universidade Federal de Pernambuco, Recife, Pernambuco 50670-901, PE, Brazil
(Received 13 December 2009; revised manuscript received 3 February 2010; published 2 March 2010)

We study the quantum coherence properties of spin waves in a ferromagnetic thin film driven by a dc current in a spin-torque nano-oscillator. We show that if the current barely exceeds the threshold value the driven magnon states are not coherent and exhibit pronounced thermal noise. Only when the current exceeds the threshold value by a certain amount that depends on the material parameters and temperature of the nanostructure the microwave oscillation will exhibit quantum coherence.

DOI: 10.1103/PhysRevB.81.092401 PACS number(s): 75.30.Ds, 75.40.Gb, 75.70.−i, 76.50.+g

It has been well-established experimentally[1–5] that spin waves can be excited by a direct current traversing a magnetic multilayer when the current density exceeds a critical value. The origin of this effect lies on the spin-transfer-induced (STI) torque created by spin-polarized electrons in a current injected into a ferromagnetic thin film that acts on the magnetization opposing the process of relaxation.[6–8] Since the STI torque is proportional to the current density this effect becomes important when the lateral dimensions of the contacts are on the order of a few hundred nanometers. In a spin-transfer nano-oscillator (STNO), illustrated in Fig. 1, a microwave signal is generated by the oscillation in the giant magnetoresistance produced by the precessing magnetization of the spin waves in the ferromagnetic free layer.[9,10]

Although the operation of STNOs is based on phenomena of quantum origin almost all theoretical studies of the spin dynamics utilize the classical Landau-Lifshitz-Gilbert equation of motion. This is justified by the fact that the number of electrons involved in the process is very large so one can work in the classical limit. In previous papers,[11,12] we presented a spin-wave theory for the dc current excitation showing that nonlinear effects due to magnon interactions account for the stabilization of the magnetization precession and the frequency shifts observed with increasing current. The theory explains the now well established[2–5,9–13] downward frequency shifts (redshifts) with increasing current when the magnetic field is applied in the film plane and upward frequency shifts (blueshifts) when the field is perpendicular to the film. In this Brief Report, we study in more detail the quantum nature of the spin waves excited by the dc current near threshold. The formalism used is the same employed to study quantum coherence in spin waves driven by microwave fields[14] and in laser systems.[15]

We consider a ferromagnetic film of thickness d traversed by a direct current with spin-polarized electrons. The theory is based on a quantum spin-wave formalism to treat the magnetic excitations described by the following system Hamiltonian,

$$H = H_S + H_R + H_{RS}, \tag{1}$$

where H_S, H_R, and H_{RS} are, respectively, the Hamiltonians for the spin-wave system, the heath bath reservoir and the reservoir-system interaction. For the objectives of this paper the system Hamiltonian can be written as $H_S = H_0 + H^{(4)}$, where

$$H_0 = \hbar \sum_k \omega_k c_k^+ c_k, \tag{2}$$

is the unperturbed Hamiltonian for free magnons with frequency ω_k and wave vector $\vec{k}$ assumed in the film plane, described by creation and annihilation operators c_k^+ and c_k, containing Zeeman, volume anisotropy of crystalline or shape origin, volume exchange, interlayer exchange, surface anisotropy, and dipolar contributions.[12] $H^{(4)}$ represents the four-magnon interaction that has contributions conserving energy and momentum given by[16–18]

$$H^{(4)} = \hbar \sum_{k,k'} \left(\frac{1}{2} S_{kk'} c_k^+ c_{-k}^+ c_{k'} c_{-k'} + T_{kk'} c_k^+ c_{k'}^+ c_k c_{k'} \right), \tag{3}$$

where the interaction coefficients are determined mainly by the dipolar and exchange energies. For the k values relevant to the experiments the exchange contribution is negligible so that the coefficients in Eq. (3) are given approximately by the surface and bulk values for the dipolar interaction and depend on the direction of the magnetization.

The Hamiltonian for the heat bath can be written as[19]

$$H_R = \hbar \sum_\alpha \omega_\alpha R_\alpha^+ R_\alpha, \tag{4}$$

where ω_α, R_α^+, and R_α represent, respectively, the frequency, the creation, and the annihilation operators of the heat bath modes. Finally,

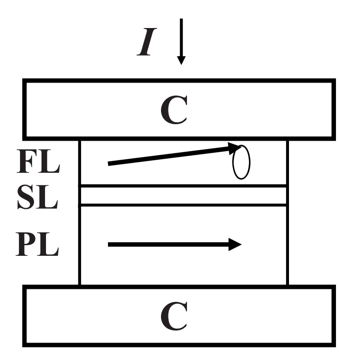

FIG. 1. Schematic structure of a STNO. Current is injected through contacts C. Electrons flowing upwards are spin polarized by the magnetization of the thick ferromagnetic pinned layer, traverse the thin metallic or insulating nonmagnetic spacer layer, and drive the magnetization of the ferromagnetic free layer.

1098-0121/2010/81(9)/092401(4)

$$H_{RS} = \hbar \sum_{k,\alpha} g^*_{k,\alpha} R^+_\alpha c_k + g_{k,\alpha} R_\alpha c^+_k, \tag{5}$$

represents the interaction between the magnons k and the heat bath modes, characterized by a two-mode interaction with coupling coefficient $g_{k,\alpha}$. In order to obtain the equations of motion for the magnon operators in the film traversed by spin-polarized electrons we consider that the spin angular momentum $\hbar\vec{S}$ is driven by a STI torque as given by Slonczewski,[7]

$$\tau_{STI} = \frac{\beta J}{\gamma M_S} \vec{M} \times (\hat{z} \times \vec{M}), \tag{6}$$

with

$$\beta = \varepsilon \hbar \gamma / 2 d M_s e, \tag{7}$$

where J is the electric current density traversing the film in the perpendicular direction and assumed to be uniform, $\gamma = g\mu_B/\hbar$ is the gyromagnetic ratio, g is the spectroscopic factor, μ_B is the Bohr magneton, e is the electron charge, M_s is the saturation magnetization, $\hat{z}$ is the direction of the spin polarization, determined by the applied field that magnetizes the films, and ε is a spin transfer efficiency parameter that depends on the materials of the multilayer.[7] The essential feature of the STI torque is that it acts on the magnetization deviating it away from equilibrium, producing an effect opposite to that of the relaxation and effectively driving its motion. Assuming that only one standing wave mode supported by the bound film is excited by the polarized current, one can express the torque (6) in terms of the magnon operators[12] and using the Heisenberg equation one can obtain the equation of motion for the magnon destruction operator,

$$\frac{dc_k}{dt} = - i\omega_k c_k - (\eta_k - \beta J) c_k - \beta J \frac{\delta}{SN} c^+_k c_k c_k - i \frac{T_k}{SN} c^+_k c_k c_k + F_k(t) \tag{8}$$

where

$$\eta_k = \pi D(\omega_k) |g_{k,\alpha}|^2, \tag{9}$$

$$F_k(t) = - i \sum_\alpha g_{k,\alpha} R_\alpha e^{-i\omega_\alpha t}, \tag{10}$$

where η_k represents the magnetic relaxation rate expressed in terms of the interaction between magnon k and the heat reservoir, $D(\omega)$ is the magnon density of states, and $F_k(t)$ is a Langevin random force with correlators of Markoffian systems type.[14,15] The other parameters are $\delta = 3/2(u_k^2 + v_k^2)$, u_k and v_k are the coefficients of the Bogoliubov transformation relating the collective spin deviation operators to the normal mode spin-wave operators, N is the number of spins S in the sample, and T_k is the coefficient arising from the surface dipolar and anisotropy contributions to the four-magnon interaction. For the magnetization perpendicular to the film plane one has[12] $u_k = 1$, $v_k = 0$, and

$$T_k = \gamma 6 \pi M_{eff}, \tag{11}$$

whereas for the magnetization parallel to the film[12] $u_k > 1$, $v_k^2 = u_k^2 - 1$, and

$$T_k = - \gamma 3 \pi M_{eff} \alpha, \tag{12}$$

where M_{eff} is the effective magnetization of the film including the effect of surface anisotropy and α is a factor given by

$$\alpha = u_k^4 - 3u_k^3 v_k + 4u_k^2 v_k^2 - 3u_k v_k^3 + v_k^4. \tag{13}$$

For the material parameters of Ref. 5 $u_k = 1.32$, $v_k = 0.87$ so that $\alpha = 2.01$. Note that Eqs. (8), (11), and (12) have factors that were obtained assuming that the mode driven by the dc current is a standing wave, as appropriate for a nanopillar structure. Note also that except for the term in T_k, Eq. (8) has the same form as the equation for the electromagnetic field operator for a laser.[15] The classical form of Eq. (8) with some approximations in the coefficients has been widely used to study several properties of STNOs.[10]

Initially we consider that the expectation values of the operators c_k^+ and c_k can be treated as classical variables, denoted by c_k^* and c_k and neglect the fluctuating force. Equation (8) then gives,

$$\frac{dc_k}{dt} = - i(\omega_k + \Delta\omega_k) c_k - \left[\eta_k - \beta J \left(1 - \frac{n_k \delta}{SN} \right) \right] c_k, \tag{14}$$

where

$$\Delta\omega_k = T_k n_k / SN, \tag{15}$$

is the frequency shift due to the nonlinear interaction and $n_k = c_k^* c_k$ *is* the number of magnons.

Equation (14) reveals that when a current with density above the critical value $J_c = \eta_k / \beta$ traverses the film, the spin-wave amplitude initially with thermal values grows exponentially in time while the number of magnons n_k is small. However, as n_k increases and approaches SN the STI driving decreases due to its negative nonlinear term and its effect is balanced by the relaxation, so that the spin-wave amplitude saturates. With Eq. (14) and its complex conjugate one can obtain a simple equation for the magnon number in the steady state at times $t \gg \eta_k^{-1}$,

$$n_{ss} = SN \frac{I - I_c}{I\delta}, \tag{16}$$

where $I_c = J_c a$ is the critical current, a being area of the current cross section. With Eqs. (15) and (16) we obtain the frequency shift as a function of current for $I \geq I_c$,

$$\Delta\omega_k = \gamma T_k \frac{I - I_c}{I\delta}. \tag{17}$$

This result together with Eqs. (11) and (12) shows the well known fact that frequency shifts upwards with increasing current (blueshift) for films magnetized perpendicular to the plane whereas for films magnetized in the plane it shifts downwards (redshift).[2–5,9–13]

Equation (14) has been employed in many studies of STNOs, isolated or coupled to one another. However Eq. (14) does not bear all of the quantum features contained in Eq. (8). For the study of the full quantum nature of the spin excitations in STNOs it is convenient to work in the representation of coherent magnon states. As is well known the eigenstates $|n_k\rangle$ of the Hamiltonian H_0 and of the number

operator $n_k = c_k^+ c_k$ can be obtained by applying integral powers of the creation operator to the vacuum. These stationary states have precisely defined number of magnons n_k and uncertain phase. They are used in nearly all quantum treatments of thermodynamic properties, relaxation mechanisms, and other phenomena involving magnons. However, they have zero expectation value for the small-signal transverse magnetization operators m_x and m_y and consequently cannot generate microwave radiation. The states that correspond to classical spin waves are the coherent magnon states,[20,21] defined in analogy to the coherent photon states introduced by Glauber.[22] A coherent magnon state is the eigenket of the circularly polarized magnetization operator $m^+ = m_x + i m_y$. It can be written as the direct product of single-mode coherent states, defined as the eigenstates of the annihilation operator, $c_k|\alpha_k\rangle = \alpha_k|\alpha_k\rangle$, where the eigenvalue α_k is a complex number. Although the coherent states are not eigenstates of the unperturbed Hamiltonian and as such do not have a well defined number of magnons, they have nonzero expectation values for the magnetization m^+ with a well-defined phase. The coherent state $|\alpha_k\rangle$ can be expanded in terms of the eigenstates $|n_k\rangle$ and has an expectation value for the number operator given by $\langle n_k\rangle = |\alpha_k|^2$. The coherent states are not orthogonal to one another, but they form a complete set, so they can be used as a basis for the expansion of an arbitrary state. In order to study the coherence properties of a magnon system, it is convenient to use the density matrix operator ρ and its representation as a statistical mixture of coherent states,[22]

$$\rho = \int P(\alpha_k)|\alpha_k\rangle\langle\alpha_k| d^2\alpha_k, \tag{18}$$

where $P(\alpha_k)$ is a probability density, called P representation, satisfying the normalization condition $\int P(\alpha_k) d^2\alpha_k = 1$ and $d^2\alpha_k = d(\mathrm{Re}\ \alpha_k) d(\mathrm{Im}\ \alpha_k)$. As shown by Glauber,[22] if ρ represents a thermal Bose-Einstein distribution, $P(\alpha_k)$ is a Gaussian function. On the other hand, if ρ corresponds to a coherent state, it is described by a Poisson distribution and $P(\alpha_k)$ is a Dirac δ function.

In order to study the coherence properties of the spin-wave mode driven by the STI torque one has to use methods of statistical mechanics appropriate for boson systems interacting with a heat bath. We follow here a procedure similar to that used to study the microwave excitation of spin waves by the so-called subsidiary resonance process[14] and lasers.[15] First we expand the quantum states in the representation of coherent magnon states $|\alpha_k\rangle$. From Eq. (8) we obtain for the coherent state eigenvalue in a frame rotating with frequency ω_k, defined by $c_k'|\alpha_k\rangle = \alpha_k(t) e^{-i\omega_k t}|\alpha_k\rangle$, where $c_k' = c_k/(SN)^{1/2}$ is the normalized destruction operator, the following equation of motion

$$\frac{d\alpha_k(t)}{dt} + iT_k|\alpha_k|^2\alpha_k(t) - \eta R\delta\left(\frac{R-1}{R\delta} - |\alpha_k|^2\right)\alpha_k(t) = F_k(t)e^{i\omega_k t} \tag{19}$$

where $R = I/I_c$ is a driving parameter. Equation (19) contains all the information carried by the equations of motion for the magnon operators. It is similar to the nonlinear Langevin equation which appears in Brownian motion studies, microwave spin-wave pumping[14] and laser theory.[15] However, Eq. (19) contains the nonlinear frequency shift term in T_k which plays very important role in the properties of STNOs.[10–13,23,24] Equation (19) reveals that the magnon modes with amplitude α_k are driven thermally by the heat bath modes and also by the STI torque produced by the spin-polarized dc current. The solutions of Eq. (19) confirm the previous analysis. For $I < I_c$ the driving parameter R is less than 1 and the magnon amplitudes are essentially the ones of the thermal reservoir. For $I > I_c$, $R > 1$ and the magnon amplitudes grow exponentially and are limited by the effect of the nonlinear interaction. Well above threshold the steady-state solution of Eq. (19) gives for the normalized number of magnons $n_k' = n_k/SN = |\alpha_k|^2$ an expression identical to Eq. (16). The final step to obtain information about the coherence of the excited mode is to find an equation for the probability density $P(\alpha_k)$, defined in Eq. (18), that is stochastically equivalent to the Langevin equation. Using $\alpha_k = a_k \exp(i\phi_k)$ we obtain a Fokker-Plank equation in the form[14,15,24]

$$\frac{\partial P}{\partial t'} + \frac{1}{x}\frac{\partial}{\partial x}[(A - x^2)x^2 P] = \frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial P}{\partial x}\right) + T_k' x^2 \frac{\partial P}{\partial \phi_k} + \frac{1}{x^2}\frac{\partial^2 P}{\partial \phi_k^2}, \tag{20}$$

where

$$t' = (R\delta\bar{n}_k')^{1/2}\eta_k t, \tag{21}$$

$$x = (R\delta/\bar{n}_k')^{1/4} a_k, \tag{22}$$

$$T_k' = T_k/(R\delta\eta_k), \tag{23}$$

represent normalized time, magnon amplitude and nonlinear coefficient, and A is related to the driving parameter R by,

$$A = \left(\frac{1}{R\delta\bar{n}_k'}\right)^{1/2}(R-1). \tag{24}$$

Equation (20) describes the full dynamics of the magnon excitation in the film by the dc spin-polarized current. Here we are interested in the stationary solution of Eq. (20) independent of ϕ_k which has the form,

$$P(x) = C\exp\left(\tfrac{1}{2}Ax^2 - \tfrac{1}{4}x^4\right). \tag{25}$$

where C is a normalization constant such that the integral of $P(x)$ in the range of x from zero to infinity is equal to unity. Note that for obtaining Eq. (25) all integration constants were set to zero to satisfy this condition.

Figure 2 shows plots of $P(x)$ for four values of $R = I/I_c$ for material parameters as in the experiments of Ref. 5, $\delta = 3.7$, $\omega_k/2\pi = 4$ GHz, and $T = 300$ K. For $I = 0.8 I_c$, the coefficient A is negative (−2.92) and $P(x)$ behaves as a Gaussian distribution, characteristic of systems in thermal equilibrium and described by incoherent magnon states.[22] For $I = I_c$, $A = 0$, and $P(x)$ also looks a broad Gaussian function indicating a noisy oscillation. For $I = 1.2\ I_c$, A is positive but small (2.38), and the stationary state consists of two components, a coherent one convoluted with a smaller fluctuation with Gaussian dis-

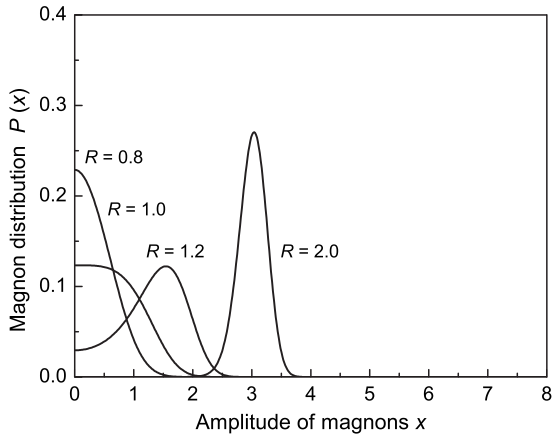

FIG. 2. Plots of the probability density $P(x)$ as a function of the normalized magnon amplitude x for four values of the driving parameter $R=I/I_c$ for material parameters as in the experiments of Ref. 5, $\delta=3.7$, $\omega_k/2\pi=4$ GHz, and $T=300$ K.

tribution. Finally for $I=2.0\ I_c$, A is positive and large (9.24) and $P(x)$ becomes similar to a Poisson distribution, characterizing the dominance of a coherent state. Since the variance of $P(x)$ is proportional to A^{-1}, for $A \gg 1$ the function $P(x)$ becomes a deltalike distribution, characteristic of a pure coherent magnon state.[22] For $A>0$ it is easy to show that $P(x)$ given by Eq. (25) has a peak at $x_0=A^{1/2}$. Thus for large values of A $P(x)$ represents a coherent state with an average number of magnons given by $x_0^2=A$. This gives for the magnon number an expression identical to Eq. (16).

The results in Fig. 2 reveal that it if the current is only slightly above threshold one does not have a pure coherent state. The quantum state is a coherent state with a random phase characteristic of thermal noise. From Fig. 2 one can infer that the magnons approach coherent states for a driving coefficient $A \geq 2$. Using this condition and considering $\bar{n}_k \approx k_B T/\hbar\omega_k$ (T is the temperature and k_B the Boltzmann constant), $N=ad/a_0^3$ (a_0 is the lattice parameter), with Eq. (24), considering $R \approx 1$ we obtain the current increment above threshold necessary to attain coherence for the conditions of Ref. 5,

$$\left(\frac{I-I_c}{I_c}\right) \geq 2\left(\frac{k_B T a_0^3 \delta}{\hbar\omega Sad}\right)^{1/2}. \tag{26}$$

For a film with dimensions $d=5$ nm and $a=100 \times 100$ nm^2 and with parameters appropriate for the experiments of Ref. 5, $\delta=3.7$, $T=300$ K, $a_0=0.35$ nm, and $\omega/2\pi=4$ GHz, condition (26) gives $(I-I_c)/I_c \geq 0.1$. Equation (26) shows that the microwave signal generated by the STNO becomes coherent when the current exceeds the threshold value by an amount that increases with increasing temperature and decreasing frequency or film volume. If the driving current is above threshold but below the value given by Eq. (26) the generated microwave is a noisy signal with a broad spectrum, as observed experimentally[25] and demonstrated theoretically.[24] At larger driving currents the coherence increases so that one expects a narrower spectrum of the generated microwave signal. This has been confirmed by detailed calculations showing that the linewidth of the signal decreases with increasing oscillation power.[23,24]

This work was supported by the Brazilian Ministry of Science and Technology.

[1] M. Tsoi, J. Appl. Phys. **91**, 6801 (2002).
[2] S. I. Kiselev, J. C. Sankey, I. N. Krivorotov, N. C. Emley, R. J. Schoelkopf, R. A. Buhrman, and D. C. Ralph, Nature (London) **425**, 380 (2003).
[3] W. H. Rippard, M. R. Pufall, S. Kaka, S. E. Russek, and T. J. Silva, Phys. Rev. Lett. **92**, 027201 (2004); S. Kaka, M. R. Pufall, W. H. Rippard, T. J. Silva, S. E. Russek, and J. A. Katine, Nature (London) **437**, 389 (2005).
[4] S. I. Kiselev, J. C. Sankey, I. N. Krivorotov, N. C. Emley, M. Rinkoski, C. Perez, R. A. Buhrman, and D. C. Ralph, Phys. Rev. Lett. **93**, 036601 (2004).
[5] I. N. Krivorotov, N. C. Emley, J. C. Sankey, S. I. Kiselev, D. C. Ralph, and R. A. Buhrman, Science **307**, 228 (2005).
[6] L. Berger, Phys. Rev. B **54**, 9353 (1996).
[7] J. C. Slonczewski, J. Magn. Magn. Mater. **159**, L1 (1996); **195**, 261 (1999).
[8] For a review of the theory see D. C. Ralph and M. D. Stiles, J. Magn. Magn. Mater. **320**, 1190 (2008).
[9] J. Grollier, V. Cros, and A. Fert, Phys. Rev. B **73**, 060409(R) (2006).
[10] For an excellent review on STNO see A. Slavin and V. Tiberkevich, IEEE Trans. Magn. **45**, 1875 (2009).
[11] S. M. Rezende, F. M. de Aguiar, and A. Azevedo, Phys. Rev. Lett. **94**, 037202 (2005).
[12] S. M. Rezende, F. M. de Aguiar, and A. Azevedo, Phys. Rev. B **73**, 094402 (2006).
[13] A. Slavin and P. Kabos, IEEE Trans. Magn. **41**, 1264 (2005).
[14] C. B. de Araújo and S. M. Rezende, Phys. Rev. B **9**, 3074 (1974).
[15] See for example, H. Haken, Rev. Mod. Phys. **47**, 67 (1975); P. Meystre and M. Sargent III, *Elements of Quantum Optics* (Springer-Verlag, Berlin, 1992).
[16] A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, *Spin Waves* (North-Holland, Amsterdam, 1968).
[17] V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiziol. Nauk **114**, 609 (1974); Sov. Phys. Usp. **17**, 896 (1975).
[18] R. M. White, *Quantum Theory of Magnetism*, 3rd ed. (Springer-Verlag, Berlin, 2007).
[19] U. Balucani, F. Barocchi, and V. Tognetti, Phys. Rev. A **5**, 442 (1972).
[20] S. M. Rezende and N. Zagury, Phys. Lett. A **29**, 47 (1969).
[21] N. Zagury and S. M. Rezende, Phys. Rev. B **4**, 201 (1971).
[22] R. J. Glauber, Phys. Rev. **131**, 2766 (1963).
[23] J. V. Kim, V. Tiberkevich, and A. N. Slavin, Phys. Rev. Lett. **100**, 017207 (2008).
[24] J. V. Kim, Q. Mistral, C. Chappert, V. S. Tiberkevich, and A. N. Slavin, Phys. Rev. Lett. **100**, 167201 (2008).
[25] Q. Mistral, J.-V. Kim, T. Devolder, P. Crozat, C. Chappert, J. A. Katine, M. J. Carey, and K. Ito, Appl. Phys. Lett. **88**, 192507 (2006).