with the field gradient induced in the electronic shells by the external charge. Both terms are equal if $u_1$ and $\bar{u}_1$ satisfy the Schrödinger equations (A3) and (A4).

If now $u_1$ and $\bar{u}_1$ are not exact solutions, the energy (A6) exceeds the true energy but it is to be minimized. It is not, however, the total energy which is minimized and not even the term in $Q/R^3$, but the term in $1/R^6$, which is essentially the quadrupole polarizability of the ion. This procedure can be defended by noticing that minimizing the $1/R^6$ energy should ultimately lead to a trial function $u_1$ which is negligibly different from the true solution $u_1$ of (A3). If $u_1$ satisfies (A3), then for any integrable function $\bar{u}_1$ orthogonal to $u_0$,

$$\langle u_1^* | H_0 - E_0 | \bar{u}_1^* \rangle = \langle u_0 | P_2 r^2 | \bar{u}_1^* \rangle.$$

The term proportional to $Q/R^3$ in (A6) then can be written entirely in terms of the true $u_1$ and is therefore obtained correctly.

PHYSICAL REVIEW VOLUME 102, NUMBER 3 MAY 1, 1956

# Paramagnetic Resonance Hyperfine Structure of $Co^{56}$†

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(Received January 3, 1956)

The microwave paramagnetic resonance hyperfine structure of 72-day $Co^{56}$ has been observed in a magnetically dilute single crystal of cobalt potassium sulfate at 20°K. From the number of hfs components and their spacing relative to $Co^{59}$, the spin $I(Co^{56})=4$ and the magnetic moment $|\mu(Co^{56})|=3.855\pm0.007$ nm are directly determined. These results are discussed in relation to those of nuclear alignment experiments and the nuclear shell model.

## I. INTRODUCTION

IN a recent paper,[1] referred to as I, the direct measurement of the spin and magnetic moment of $Co^{60}$ by microwave paramagnetic resonance was described. The present paper presents the results of a similar measurement of the spin and magnetic moment of 72-day $Co^{56}$. These values, as those of $Co^{60}$, are of particular interest in connection with previous investigations of these nuclei by nuclear alignment and $\beta$, $\gamma$ spectroscopy. The paramagnetic resonance of divalent cobalt ions in a Tutton salt single crystal is observed at 20°K in a conventional paramagnetic resonance spectrometer. As described in I, from the number of hyperfine components and their spacing relative to that of $Co^{59}$ we can determine respectively the spin and magnetic moment. For details on the crystallography of the Tutton salts, the Hamiltonian determining the line spacings, and the experimental apparatus we refer to I.

## II. EXPERIMENTAL PROCEDURE

The radiocobalt was produced by the $Fe^{56}(p,n)Co^{56}$ reaction with 12-Mev protons from the 60-in. Crocker cyclotron using an iron-plated water-cooled copper target. Two bombardments were made, the first using iron plated from a bath of reagent grade purity. Preliminary examination of the cobalt paramagnetic resonance hfs from this sample indicated that stable $Co^{59}$ was more abundant than $Co^{56}$ by a factor 200, undoubtedly because of a cobalt impurity in the iron. Since the intense $Co^{59}$ hfs lines obscured some of the $Co^{56}$ hfs lines, a second bombardment was made using iron plated from a bath obtained by ether extraction to remove the trace impurity of cobalt. The radiocobalt was extracted from this bombarded target by a procedure similar to that of Maxwell *et al.*,[2] and was added in the form of $CoSO_4$ to a heavy-water solution of $ZnK_2(SO_4)_2$. A single crystal of the Tutton salt $(Co, Zn)K_2(SO_4)_2 \cdot 6D_2O$ was grown, weighing about 70 mg and containing about 4 mC of $Co^{56}$. The activity was identified as that of $Co^{56}$ by observing its $\gamma$-ray spectrum with a recording scintillation spectrometer. This crystal was mounted in the cavity of the paramagnetic resonance spectrometer with $z_1$, the symmetry axis of the crystalline electric field, oriented parallel to the external magnetic field $H$, and the susceptibility axis $K_2$ perpendicular to the field $H$. The observed spectrum at 20°K is shown in Fig. 1. The eight hfs lines labeled 1, 2, ···, 8 have the $g$ factor and spacing of $Co^{59}$ and are due to a residual trace impurity of cobalt in the iron target. The nine lines labeled 1′, 2′, ···, 9′ have the same $g$ factor and are thus also due to cobalt; in fact they are the hfs lines of $Co^{56}$, the only other cobalt isotope present in the sample. For the paramagnetic resonance transitions observed there are $2I+1$ hfs components for a nuclear spin $I$; thus we conclude

$$I(Co^{56})=4. \quad (1)$$

This interpretation is verified by rotating the crystal about the $K_2$ axis: the two sets of hfs lines move to-

† This research has been supported in part by the U. S. Atomic Energy Commission.

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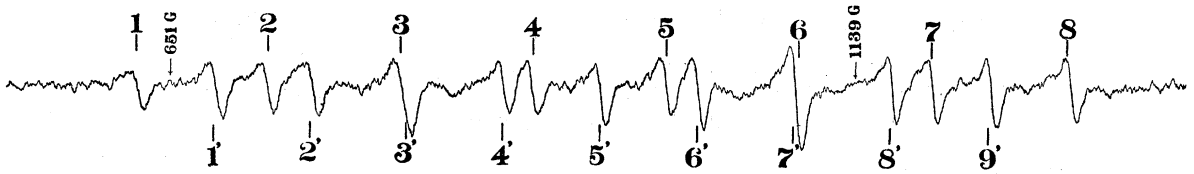


FIG. 1. Paramagnetic resonance hfs of $Co^{59}$ and 72-day $Co^{56}$ in magnetically dilute $CoK_2(SO_4)_2 \cdot 6D_2O$ with the magnetic field $H$ parallel to $z$, the axis of the crystalline electric field. The derivative of the absorption lines is displayed as a function of the field $H$ (in gauss) at a frequency of approximately 9200 Mc/sec. The lines 1, 2, $\cdots$, 8 are the hfs of $Co^{59}(I=7/2)$, and the lines 1′, 2′, $\cdots$, 9′ are those of $Co^{56}$, showing $I(Co^{56})=4$. The lines 3 and 3′ and also 6 and 7′ overlap each other. There is another similar spectrum, less well resolved, at approximately twice the $H$ field value due to the other magnetically unequivalent $Co^{++}$ ions in the crystal.

gether, always with the same $g$ factor and the same relative spacing. This excludes the possibility that the primed lines are due to spurious crystalline effects.

As shown in I, the splitting between each component is proportional to $\mu/I$ and hence the splitting $\Delta H$ between the outside lines is proportional to $\mu$. For a series of thirteen runs taken at varying angles, the ratio $\Delta H(Co^{59})/\Delta H(Co^{56})$ and hence the ratio of the magnetic moments yields

$$\Delta H(Co^{59})/\Delta H(Co^{56})=\mu(Co^{59}/\mu(Co^{56}) =1.205\pm0.002. \quad (2)$$

Using Proctor and Yu's[3] value (without diamagnetic correction) of 4.6399 nm for the magnetic moment of $Co^{59}$, we thus obtain for the magnetic moment of $Co^{56}$

$$|\mu(Co^{56})|=3.855\pm0.007 \text{ nm}. \quad (3)$$

Unfortunately this technique does not allow a determination of the sign of the magnetic moment.

## III. DISCUSSION

Gallaher, Poppema, and associates,[4] combining the results of the $\beta$, $\gamma$ spectroscopy experiments of Sahai *et al.*[5] and of their own nuclear alignment experiments, have formulated a decay scheme for the $Mn^{56}$-$Co^{56}$-$Fe^{56}$ system. In the alignment experiments of the Leiden group, the spin and parity of the initial and final states in a radiative transition and the magnetic moment of the originally aligned nucleus can be inferred from the anisotropy of the $\gamma$ radiation and its defendence on temperature. In this scheme, the most probable value of the spin was found to be $I(Co^{56})=4$; however, $I=5$ was not definitely excluded. They also found a magnetic moment $\mu(Co^{56})=2.8\pm0.9$ nm. The accuracy of the moment determination is limited, among other things, by the inability to establish the exact relationship of the magnetic temperature scale $T^*$ to the thermodynamical temperature scale $T$. Perhaps our direct accurate value of the moment will help to establish this relationship.

We have also compared our results to those expected from a simple nuclear shell model.[6] The fact that $Co^{56}$ has 27 protons means that we have a single nucleon vacancy in the $f_{7/2}$ shell which is completed with 28 nucleons, and the fact that it has 29 neutrons means that we have a single nucleon in the $p_{3/2}$ shell. The spin of the nucleus compounded from these two configurations should have, according to Nordheim,[7] a value of 2, 3, 4, or 5, and probably $I\geq3$. As in I, we calculate the nuclear $g$-factor for $jj$ coupling on the following simple assumptions.

(a) Schmidt-limit $g$ values:

$$g_p=1.655 \quad \text{and} \quad g_n=-1.275.$$

(b) Dirac-limit $g$ values:

$$g_p=1.143 \quad \text{and} \quad g_n=0.$$

(c) Empirical $g$ values: Using the $g$-factors of neighboring $Co^{59}$ and $Cr^{53}$, we find

$$g_p=1.3257 \quad \text{and} \quad g_n=-0.3157.$$

The magnetic moments calculated in this way are compared to the experimentally measured values in Table I. The reasonably good agreement indicates that

TABLE I. Comparison of $\mu(Co^{56})$ values.

| Experimental moments | | Calculated moments | | |
|---|---|---|---|---|
| Nuclear alignment | Hyperfine structure | Schmidt limit | Dirac limit | Empirical |
| 2.8±0.9 nm | 3.855±0.007 nm | +4.276 nm | +3.658 nm | +3.9896 nm |

the assumed assignment of nucleon configuration is probably correct and the parity of the $Co^{56}$ ground state is probably even, i.e., odd $(f_{7/2})\times$odd $(p_{3/2})=$even. Furthermore, we would expect the magnetic moment to be positive.

## ACKNOWLEDGMENTS

The cooperation of B. Rossi and the operators of the 60-in. cyclotron is gratefully acknowledged. Assistance on radiochemical procedures was kindly provided by P. Stevenson.

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