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Mesonic *vs* Nonmesonic Hyperfragment Decay*

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An attempt has been made to see whether reabsorption of pions, which would be produced inside a hyperfragment if the bound hyperon decayed as if it were free, might account for the striking decrease, with increasing hyperfragment size, of the number of hyperfragment decays in which a pion is actually observed. Optical-model calculations, as well as experimental yields for the analogous process of photopion production inside nuclei, indicate that much too few pions would be reabsorbed. Another model of hyperfragment decay, in which nucleons may influence the bound hyperon to decay without real pion emission, could be made to fit the data.

I. INTRODUCTION

SINCE Danysz and Pniewski[1] first reported the decay of a hyperfragment, as a nuclear fragment containing a bound Λ^0 hyperon is called, many such events have been observed.[2,3] Hyperfragment decays may be classified into two groups: (1) *mesonic decays*, in which a π meson is observed in the disintegration star, and (2) *nonmesonic decays*, with only nucleons and light nuclei emitted.

The probability P_π of mesonic decay as a function of the number of particles in the hyperfragment has been calculated from a simple theory. For this purpose, the hyperfragment is assumed to consist of a Λ^0 bound to a core nucleus with a definite nucleon distribution unaltered by the motion of the Λ^0. Because the Λ^0 binding energy is small for the lightest hyperfragments,[3] the Λ^0 may exist with large probability where nucleon density is negligible, that is, the Λ^0 may often lie "outside" the core nucleus. The eventual decay of the hyperfragment results from decay of the Λ^0, which might then occur when the Λ^0 is inside or outside the nucleus. Without specifying an exact decay mechanism, the following decay processes have been considered:

1. Free decay.—The Λ^0 is presumed always to decay as if it were free, by the mode:

$$\Lambda^0 \rightarrow p+\pi^- +37 \text{ Mev.} \tag{1}$$

(An alternative mode, with the emission of an unobservable neutron and π^0, would not be included in the experimentally determined P_π.) Reabsorption of some of the real pions by nucleons accounts for all observed nonmesonic decays.

2. Stimulated decay.—When in a region of strong interaction with a nucleon, the Λ^0 may decay *without* emitting a real pion; the Λ^0 has a mean life τ_n with respect to decay by this process. In addition, the Λ^0 may decay with pion emission by mode (1), reabsorption of real pions then accounting for a few nonmesonic decays, also. The mean life for decay by the latter process is τ_π when the Λ^0 is away from nucleons and τ_π' when it interacts strongly with nucleons. Three extreme cases are considered:

(a) $\tau_\pi' \ll \tau_\pi$.
(b) $\tau_\pi' = \tau_\pi$.
(c) $\tau_\pi' \gg \tau_\pi$, so that, for practical purposes, the Λ^0 only decays by emitting a pion when it lies *outside* a region of strong interaction with nucleons.

Models employing each of these decay schemes have been used to calculate P_π, and the results were compared with experiment. Data to date,[3] consisting of about one hundred events, indicate a sharp decrease of P_π with increasing hyperfragment size. All decays were mesonic for the lightest observed hyperfragment $_\Lambda H^3$ (deuteron plus Λ^0), about one-half were mesonic in hyperhelium, and only two mesonic decays were observed in heavier hyperfragments, $_\Lambda Be^9$ and $_\Lambda C^{11}$. Statistics are such that it should be sufficient that P_π fall from $\sim$90% at $A=3$ to 10% around $A=9$, where A is the number of particles in the hyperfragment, including the hyperon.

As will be shown in Sec. III, the free-decay model fails, because too few pions are reabsorbed to account for the number of observed nonmesonic decays. The stimulated decay model (2c), with $\tau_\pi' \gg \tau_\pi$, best reproduces the observed rapid fall of P_π with increasing A.[4]

II. ROLE OF THE Λ^0 DISTRIBUTION

The successful model employing decay process (2c) is distinguished from the other models by the fact that it limits decay of the Λ^0 by real pion emission to regions of the hyperfragment where there are few nucleons. It will be shown in the next section that the influence of

* Supported by a grant from the National Science Foundation.
[1] M. Danysz and J. Pniewski, Phil. Mag. **44**, 348 (1953).
[2] Fry, Schneps, and Swami, Phys. Rev. **99**, 1561 (1955).
[3] Fry, Schneps, and Swami, Phys. Rev. **101**, 1526 (1956).

[4] *Note added in proof.*—The result $\tau_\pi' \gg \tau_\pi$ may be partially justified by introducing the Pauli principle. One obtains $\tau_\pi'/\tau_\pi \approx 5$ for $A=5$ to $A=9$ from the following calculation suggested by Professor W. F. Fry. (See also Ruderman and Karplus, Phys. Rev., to be published.) Assume that when the Λ^0 decays by process (1) inside the nucleus, the emission of a proton with momentum less than k given by the Fermi sphere for the nucleus is forbidden. Then, using the Λ^0 wave function (3) to obtain the Λ^0 momentum distribution, and assuming isotropic emission of the proton in the center-of-mass system, one can calculate the probability P that the Λ^0 contributes enough momentum to the emitted proton so that the total proton momentum exceeds k. P will, of course, also be the probability that the Λ^0 is *not* forbidden to decay by pion emission when inside the nucleus. Then, $\tau_\pi'/\tau_\pi = 1/P$, since decay by pion emission is never forbidden when the Λ^0 lies outside the nucleus.

the nucleons is apparently felt at great enough distances so that, within some sphere with origin at the center of the core nucleus, nucleons are dense enough to prevent decay by the free mode (1) altogether. In fact, it turns out that R_0, the radius of this sphere, must be chosen so that the nucleon density outside the sphere is less than half-maximum. R_0 may thus be called the radius of the core nucleus. In essence, then, the bound-decay model assumes that the Λ^0 emits a pion only if it decays "outside" the core nucleus. The model succeeds because, like P_π, the probability P_0 that the Λ^0 lies outside the nucleus decreases rapidly with increasing A.

P_0 can be calculated as follows. The Λ^0 is assumed to be bound in a potential well with depth proportional to the density of nucleons in the core nucleus. Assuming the smoothed-uniform nucleon density obtained from electron scattering,[5]

$$\rho(r)=\rho_0\frac{1}{e^{q(r-R)}+1}, \tag{2}$$

the potential can be further reasonably approximated by a square well with range $R=a_0(A-1)^{\frac{1}{3}}$ and depth independent of A. The well parameters, range parameter $a_0=1.19\times10^{-13}$ cm and depth $V=21.2$ Mev, were chosen to reproduce experimental values of the Λ^0 binding energy, B_Λ, which increases rapidly from only 0.2 Mev for the hypertriton to equal average nucleon binding energy around $A=7$ as the Λ^0 becomes more tightly bound in the larger hyperfragments.[3] Since the Pauli exclusion principle does not act between the Λ^0 and the nucleons,[6] the assumption is made, here and in the determination of experimental values of the Λ^0 binding energy,[3] that the Λ^0 is in the ground state. The Λ^0 wave function then has the familiar form

$$\psi=\begin{cases} r^{-1}K\sin k'r, & r\leq R \\ r^{-1}Ke^{-k(r-R)}\sin k'R, & r>R \end{cases};$$

$$K=\left[\frac{k}{2\pi(kR+1)}\right]^{\frac{1}{2}}, \tag{3}$$

where $k'=(2mV)^{\frac{1}{2}}$, $k=(2mB_\Lambda)^{\frac{1}{2}}$, and m, equal to 2180 electron masses, is the mass of the Λ^0.

Finally,

$$P_0=4\pi\int_{R_0}^{\infty}\psi^2r^2dr, \tag{4}$$

where integration over angles has already been performed. For the case when the nuclear radius R_0 is taken equal to the well range R, P_0 falls from 0.64 at $A=4$ to 0.24 at $A=11$. For $R_0>R$, P_0 falls still faster.

III. DETAILS OF MODELS

A. Free-Decay Model

The calculation of the effect of reabsorption of pions, which in this model is assumed to account for all observed nonmesonic decays, is divided into two parts: (1) reabsorption of real pions emitted when the Λ^0 decays inside the core nucleus, and (2) reabsorption of pions emitted outside the core nucleus which then strike the nucleus.

1.—The problem of reabsorption inside the nucleus is analogous to the problem of the escape of pions in photoproduction treated by Brueckner, Serber, and Watson.[6] Following their method when a mean free path λ for pions in nuclear matter is known, the core nucleus is now assumed to be a homogeneous sphere, the Λ^0 decaying with equal probability anywhere inside the sphere and with isotropic emission of the pion. Then, the probability that a pion will be emitted in volume element $d\tau$ and traverse path r to a nuclear surface element dS without being absorbed, and thus escape through dS, is given by

$$\frac{dS\cos\theta}{4\pi r^2}\frac{d\tau}{(4/3)\pi R_0^3}e^{-r/\lambda}, \tag{5}$$

where θ is the angle between r and the normal to dS. Integrating over the volume and surface of the nucleus gives the total probability, f, that a pion emitted inside the nucleus will escape. The result is that obtained for the identical treatment of the photomeson problem,[7]

$$f=3\left\{\frac{1}{x^3}e^{-x}(1+x)-\frac{1}{x^3}+\frac{1}{2x}\right\}, \tag{6}$$

where $x=2R_0/\lambda$. For a mean free path[8] $\lambda=13\times10^{-13}$ cm for pions of $\sim$37 Mev [reaction (1)], and for a nuclear radius R_0 equal to the potential range R given in Sec. II, f falls very slowly from 0.91 at $A=4$ to 0.87 at $A=11$. Irrespective of questions of validity of the model used here, the relatively large observed photopion cross sections seem to exclude the possibility of absorption of a large number of the pions emitted. Experimental photomeson yields[9] roughly extrapolated to the pion energy under consideration indicate only slightly greater absorption than is indicated by f calculated here.

The contribution to P_π due to decays inside the nucleus is given by the product of the probability that the Λ^0 lies inside the nucleus times the probability f of pion escape,

$$f(1-P_0), \tag{7}$$

[5] D. G. Ravenhall and D. R. Yennie, Phys. Rev. **96**, 239 (1954).

[6] This fact is demonstrated by the existence of $_\Lambda H^4$ and $_\Lambda He^5$ and by the fact that the binding energy of the Λ^0 in $_\Lambda Be^9$ is greater than that of the last neutron in normal Be^9, as reported in reference 3.

[7] Brueckner, Serber, and Watson, Phys. Rev. **84**, 258 (1951).

[8] Frank, Gammel, and Watson, Phys. Rev. **101**, 891 (1956).

[9] R. M. Littauer and D. Walker, Phys. Rev. **83**, 206(A) (1951); **86**, 838 (1952).

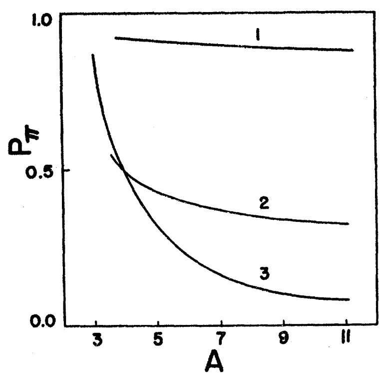

FIG. 1. The probability P_π of mesonic decay *versus* the number of particles A in the hyperfragment for (1) free decay model, (2) stimulated decay model, $\tau_\pi'=\tau_\pi$, and (3) stimulated decay model, $\tau_\pi'\gg\tau_\pi$. Experimental results indicate all mesonic decays for the hypertriton, about one-half mesonic decays in hyperhelium, and less than 10% mesonic decays in heavier hyperfragments,[3] in agreement with curve (3).

where P_0, the probability that the Λ^0 lies outside the nucleus, is given by (4).

2.—Reabsorption of pions emitted outside the nucleus may also be treated as a geometrical problem. With the assumptions that the Λ^0 distribution is given by the wave function ψ in (3), and isotropic emission of pions, then the probability that a pion will be emitted in volume element $d\tau$ outside the nucleus and then pass through a certain solid angle element $d\Omega$ is given by

$$(d\Omega/4\pi)\psi^2 d\tau. \tag{8}$$

Integrating over the solid angle subtended by the core nucleus at $d\tau$ and then integrating (numerically) over all space outside the nucleus gives the total probability, P, that a pion will be emitted outside the nucleus and then strike the nucleus. P has a value $\sim 0.2P_0$ for the range of interest, $A<12$.

The probability that pions will be reabsorbed depends on P and on the absorption cross section. Subtracting the probability of reabsorption from that of emission, one obtains a second contribution to P_π, namely, the probability that a pion will be emitted outside the nucleus and then escape without being reabsorbed,

$$P_0 - P(\sigma/\sigma_g). \tag{9}$$

Here, σ_g is the geometrical cross section of the core nucleus, and σ is the absorption cross section for 37-Mev pions in nuclear matter.

Values of σ, $\sim 0.25\sigma_g$ for $A<12$, were obtained from the relation[10] $\sigma=(f/\lambda)[(4/3)\pi R_0^3]$. For values f calculated above, σ agrees to within 3% with values obtained from $\sigma=\eta A\sigma_H$, where $\sigma_H=17$ mb is the experimental absorption cross section for 37-Mev π^- mesons in hydrogen,[11] and η, a factor accounting for nuclear binding effects, has a value 0.4 in agreement with experimental absorption cross sections of 25–45-Mev π^+ mesons in both aluminum[12] and lead.[13] (σ is about the same for positive and negative pions.) Also, the values σ are consistent with the results of Bernardini and his co-workers.[14]

Combining (7) and (9), one obtains for the probability that a π meson will be observed in a hyperfragment decay,

$$P_\pi = f(1-P_0) + [P_0 - P(\sigma/\sigma_g)]. \tag{10}$$

The result is curve (1) in Fig. 1. P_π decreases much too slowly as a function of A. In fact, even if the value for the mean free path λ were two or three times smaller than that used, and hence the absorption much greater, (10) still would not produce the observed fall-off of P_π with increasing A. It is to be concluded that the free-decay model, accounting for nonmesonic decays by reabsorption of pions, does not represent the process of hyperfragment decay.

B. Stimulated Decay Models

According to these models, the presence of nucleons may cause the Λ^0 to decay nonmesonically, that is, without emitting a real pion. Given a volume element $d\tau$ at a distance r from the center of the core nucleus, let $g(r)$ be the probability that a Λ^0 in $d\tau$ is influenced by nucleons and is thus able to decay nonmesonically. Then, the reasonable assumption is made that

$$g(r) = C\rho(r), \tag{11}$$

where $\rho(r)$ is the nucleon density at $d\tau$, and C is a constant of proportionality. The exception is made that $g(r)=1$ in any regions where $\rho(r)$ is so large that $C\rho(r)>1$. When the Λ^0 lies in such regions, there is always at least one nucleon close enough to the Λ^0 to influence its decay. Since the nucleon density (2) is a spherically symmetric, monotonically decreasing function of the radial distance r, the only region where $C\rho(r)>1$ must be a sphere with origin at $r=0$, the center of the core nucleus, and with radius, R_0, such that

$$C\rho(R_0)=1. \tag{12}$$

Then

$$g(r)=\begin{cases} 1, & r\le R_0 \\ C\rho(r), & r>R_0. \end{cases} \tag{11A}$$

Multiplying $g(r)$ by the probability $\psi^2 d\tau$ that the Λ^0 lies in volume element $d\tau$, where ψ is the Λ^0 wave function (3), and integrating over all space, one obtains the total probability, G, that the Λ^0 is influenced by nucleons and thus has the opportunity to decay nonmesonically.[15]

$$G = 4\pi\int_0^{R_0}\psi^2 r^2 dr + 4\pi\int_{R_0}^{\infty} C\rho(r)\psi^2 r^2 dr, \tag{13}$$

[10] N. C. Francis and K. M. Watson, Phys. Rev. **89**, 328 (1953).
[11] C. E. Angell and J. P. Perry, Phys. Rev. **90**, 724 (1953).
[12] J. F. Tracy, Phys. Rev. **91**, 960 (1953).
[13] K. J. Button, Phys. Rev. **88**, 956 (1952).
[14] Bernardini, Booth, and Lederman, Phys. Rev. **83**, 1075 (1951); G. Bernardini and F. Levy, Phys. Rev. **84**, 610 (1951).
[15] A result similar to (13), derived classically here, is obtained quantum-mechanically for the model reported by W. Cheston and H. Primakoff, Phys. Rev. **92**, 1537 (1953). See their Eq. (16a), analogous to (16) here.

where integration over angles has already been performed. It is to be understood that, if $C\rho(r)$ never exceeds unity, then $R_0=0$ and the first integral drops out. For convenience in integrating (13), the nucleon density function (2) is approximated by

$$\rho(r)=\begin{cases}\frac{1}{2}\rho_0(2-e^{-q(R-r)}), & r<R\\ \frac{1}{2}\rho_0 e^{-q(r-R)}, & r\geq R.\end{cases} \tag{14}$$

Values $qR=8$ and R equal to the potential range given in Sec. II give reasonable agreement with known nucleon distributions.[4]

Expression (13) does not apply to the hypertriton with its loosely bound deuteron core nucleus. G for this case can be estimated by interpreting C in (11) to be the volume of a sphere, centered on a nucleon, inside which the Λ^0 may be influenced to decay nonmesonically. Then nonmesonic decay occurs only in a fraction $2C/v$ of the deuteron volume $v=(4/3)\pi R_d^3$, where $R_d=5\times10^{-13}$ cm is the deuteron "radius." For the hypertriton,

$$G\simeq(2C/v)(1-P_0). \tag{15}$$

G is not very sensitive to P_0, the probability that the Λ^0 lies "outside" the deuteron. P_0 is $\sim$0.60, as calculated by the procedure of Sec. II with the assumption (crude in this case) that the Λ^0 is bound in a square-well potential with range $R=R_d$ and shallow well depth $V=2.6$ Mev chosen to produce the observed binding energy $B_\Lambda=0.2$ Mev.[3]

The probability per unit time, R_n, that the hyperfragment decays nonmesonically is given by the product of the probability, G, that the Λ^0 is influenced by nucleons and thus has the opportunity to decay by this mode, times the rate of decay, $1/\tau_n$, where τ_n is the mean life for nonmesonic decay of the Λ^0; that is,

$$R_n=G/\tau_n. \tag{16}$$

Since G is the probability that nucleons influence the decay of the Λ^0, then G is also the probability that the Λ^0 decays by pion emission with a mean life τ_π', and $(1-G)$ is the probability that it decays by pion emission with mean life τ_π. Then, the probability per unit time, R_π, that the hyperfragment decays with the emission of a pion is

$$R_\pi=G/\tau_\pi'+(1-G)/\tau_\pi. \tag{17}$$

The ratio, Y, of the number of nonmesonic decays to the number of mesonic decays is then the ratio of the two transition rates, (16) and (17),

$$Y=\frac{R_n}{R_\pi}=\frac{G/\tau_n}{G/\tau_\pi'+(1-G)/\tau_\pi}, \tag{18}$$

and the probability of mesonic decay is

$$P_\pi=1/(1+Y). \tag{19}$$

Small corrections to P_π, consistent with the results of Sec. III(A) must be made for the few real pions reabsorbed by nucleons.

1. Let $\tau_\pi'\ll\tau_\pi$. Then Y in (18), and hence P_π in (19), is a constant, τ_π'/τ_n. This result totally contradicts experimental results for P_π.
2. Let $\tau_\pi'=\tau_\pi$. Then Y in (18) becomes,

$$Y=\frac{G(1/\tau_n)}{G(1/\tau_\pi)+(1/\tau_\pi)-G(1/\tau_\pi)}=G\frac{\tau_\pi}{\tau_n}. \tag{20}$$

Y as given by (20), and hence P_π in (19), proves to be relatively insensitive to the adjustment of parameters R_0 and q. For values of nucleon distribution parameters q and R given above, the best result for P_π is curve (2) in Fig. 1, with $R_0=0$ and the product of constants $C\rho_0(\tau_\pi/\tau_n)=2.9$ to give $P_\pi=0.50$ for $A=4$. As for the free decay model, the rate of decrease with increasing A of P_π is much slower than the observed rate.

3. Let $\tau_\pi'\gg\tau_\pi$. Then Y in (18) becomes

$$Y=\frac{G/\tau_n}{(1-G)/\tau_\pi}=\left(\frac{G}{1-G}\right)\frac{\tau_\pi}{\tau_n}. \tag{21}$$

As was indicated in Sec. II, Y given by (21) is now more sensitive to R_0 as a parameter. Y changes at a sufficient rate for a value $R_0=1.2R$, where R, a function of $(A-1)^{\frac{1}{3}}$, is the nucleon distribution parameter in (14), which is taken equal to the range of the potential binding the Λ^0, as given in Sec. II. For this value R_0, $\rho(R_0)$ is less than half the maximum nucleon density. Also, take $qR=8$ in (14), and $\tau_\pi/\tau_n\simeq1$. Calculation of P_π by (19), using these parameters for Y in (21), gives as a satisfactory result curve (3) in Fig. 1.

IV. SUMMARY

It appears that reabsorption of real pions emitted in hyperfragment decay plays a minor role in producing observed nonmesonic decays. Thus a model assuming that the bound Λ^0 always decays as if free with real pion emission is untenable.

The number of mesonic decays seems to be predominantly a function of the probability, large for the lightest hyperfragments, that the bound Λ^0 lies outside the influence of nucleons. This suggests that the exact mechanism of hyperfragment decay should largely exclude any real pion emission if the Λ^0 decays while interacting strongly with nucleons.

V. ACKNOWLEDGMENTS

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