The stationary nature of Eq. (A13) follows analogously.

It is a consequence of describing the scattering in terms of the eigenstates of the tensor Hamiltonian that the variations δU_J, δW_J are asymptotically related. Thus asymptotically, U_J, W_J are given by

$$\begin{aligned} U_J^{(\alpha,\gamma)} &\sim A_J^{(\alpha,\gamma)}\{\sin[kr-(\pi/2)(J-1) \\ &\qquad +\delta u_J^{(\alpha,\gamma)}]/\cos\delta u_J^{(\alpha,\gamma)}\}, \\ W_J^{(\alpha,\gamma)} &\sim C_J^{(\alpha,\gamma)}\{\sin[kr-(\pi/2)(J+1) \\ &\qquad +\delta w_J^{(\alpha,\gamma)}]/\cos\delta w_J^{(\alpha,\gamma)}\}. \end{aligned} \tag{A18}$$

We must choose $\delta u_J^{(\alpha,\gamma)}=\delta w_J^{(\alpha,\gamma)}$ in order to uniquely fix the pairs of linearly independent radial functions $(u_J,w_J)^\alpha$, $(u_J,\omega_J)^\gamma$ so that the coupled wave mixture remains unchanged before and after the scattering. We may then define admixture parameters $\eta_J^{(\alpha,\gamma)}=-(C_J/A_J)^{(\alpha,\gamma)}$ giving the asymptotic mixing of the $L=J\pm1$ waves. Thus we have at large distances the relation

$$\delta W_J^{(\alpha,\gamma)}(r)=\eta_J^{(\alpha,\gamma)}\delta U_J^{(\alpha,\gamma)}(r), \quad r\rightarrow\infty. \tag{A19}$$

The reciprocal relationship between η_J^α, η_J^γ, which follows from the unitarity of the S-matrix and the reciprocity of scattering, may also be obtained from the variational principle of Eq. (A12) in a particularly simple fashion. We introduce the scale transformation

$$\begin{aligned} w_J^{(\alpha,\gamma)} &\rightarrow \eta_J^{(\alpha,\gamma)} w_J^{(\alpha,\gamma)}, \\ W_J^{(\alpha,\gamma)} &\rightarrow \eta_J^{(\alpha,\gamma)} W_J^{(\alpha,\gamma)}. \end{aligned} \tag{A20}$$

Equation (A12) may then be written as

$$[U_J^2\Gamma u_J+\eta_J^2W_J^2\Gamma w_J]_{r=\bar{r}_J}=a_J-2b_J\eta_J+c_J\eta_J^2. \tag{A21}$$

Variation with respect to η_J yields, by virtue of the stationary property,

$$\eta_J=b_J/[c_J-W_J^2(r_J)\Gamma w_J(r_J)]. \tag{A22}$$

Introducing Eq. (A22) into Eq. (A21), one obtains a quadratic equation in $W_J^2\Gamma w_J$, whose solutions are the eigensolutions of the scattering problem. The well-known relation

$$\eta_J^\alpha\eta_J^\gamma=-1 \tag{A23}$$

then follows immediately from Eq. (A22).

Finally, it may be noted that the D-wave admixture for the deuteron ground state may be written

$$\begin{aligned} \eta_D^2 &= \zeta^2/(1+\zeta^2) \\ &= -\left[\frac{U_1^2(\partial/\partial E)\Gamma v_1}{W_1^2(\partial/\partial E)\Gamma w_1}\right]_{r=\bar{r}}, \quad E=-\epsilon, \end{aligned} \tag{A24}$$

as follows from the stationary character of Eq. (A21). This then implies the more explicit expression (37) of the text.

PHYSICAL REVIEW VOLUME 102, NUMBER 3 MAY 1, 1956

Derivation of Low Scattering Formalism

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(Received January 23, 1956)

The Yang-Feldman formalism is applied to the derivation of the transition amplitude for boson-fermion scattering in a form which makes evident its connection with a method of derivation suggested by Wick.

LOW[1] has proposed a form of scattering theory which has proved extremely useful in the discussion of the consequences of causality for scattering.[2] Alternative derivations have been given by Goldberger,[3] Nambu,[4] Schweber,[5] and Wick,[6] the last of these only for the case of a fixed source. In this note we propose a concise derivation based upon the Yang-Feldman formalism[7] which makes evident the relationship to the method of Wick and shows that the latter is applicable to the relativistic case.

We consider a system described by a Hamiltonian

$$H=H_0+H_1, \tag{1}$$

where H_0 is a sum of Hamiltonians for noninteracting fields describing particles with their experimental masses and H_1 is their mutual interaction. We merely assume that H is expressed in terms of sets of canonical variables, which in the absence of interaction are the usual particle creation and annihilation operators,

$$H=H[a^\dagger(t),a(t);b^\dagger(t),b(t);\cdots]. \tag{2}$$

Here, the set $a^\dagger(t)$, $a(t)$ refer to the mesons, $b^\dagger(t)$, $b(t)$, to the nucleons, etc., and we have suppressed the dependence on spatial or three-momentum coordinates. For simplicity of notation we work with neutral mesons.

The field equations for the meson operators are

$$\begin{aligned} i\dot{a}(t) &= [a(t),H]=\delta H/\delta a^\dagger(t) \\ &= \omega a(t)+\delta H_1/\delta a^\dagger(t), \end{aligned} \tag{3}$$

[1] F. E. Low, Phys. Rev. **97**, 1392 (1955).
[2] See, for example, R. Oehme, Phys. Rev. **100**, 1503 (1955), and M. L. Goldberger, Phys. Rev. **99**, 979 (1955), from which the previous literature may be traced.
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[7] C. N. Yang and D. Feldman, Phys. Rev. **79**, 972 (1950); G. Källén, Arkiv Fysik **2**, 371 (1951).

where ω is the single meson energy operator. They can be integrated by the use of the Green's function, $G(t-t')$:

$$G(t-t')=-i\theta_+(t-t')e^{-i\omega(t-t')}, \tag{4}$$

$$\theta_+(t-t')=1,\ t>t'$$
$$=0,\ t<t', \tag{5}$$

which is the solution of the equations

$$i\partial/\partial t G(t-t')=\omega G(t-t')+\delta(t-t'), \tag{6}$$

$$G(t-t')=0,\ t<t'. \tag{7}$$

In terms of the incoming field $a_{-\infty}(t)$, we have, for example,

$$a(t)=a_{-\infty}(t)+\int_{-\infty}^{\infty} dt' G(t-t')j(t')$$
$$=a_{-\infty}(t)-i\int_{-\infty}^{t} dt' \exp[-i\omega(t-t')]j(t'), \tag{8}$$

with the definition

$$j(t)=\delta H_1/\delta a^\dagger(t). \tag{9}$$

Specializing to time $t=0$, we can easily derive from Eq. (8) the expressions

$$a(0)=a_{-\infty}(0)-\lim(\epsilon\to 0)$$
$$\times\textstyle\sum_n[\omega+H-E_n-i\epsilon]^{-1}j(0)\Psi_n\Psi_n^\dagger,$$
$$=a_{-\infty}(0)-\lim(\epsilon\to 0)$$
$$\times\textstyle\sum_n\Psi_n\Psi_n^+ j(0)[\omega+E_n-H-i\epsilon]^{-1}, \tag{10}$$

if we introduce a complete set of eigenstates, Ψ_n, of H and exhibit explicitly the time dependence of $j(t)$.

Of special interest also is the connection between the outgoing field, $a_{+\infty}$, and the incoming one, $a_{-\infty}$. From Eq. (8), for instance, we can derive the statements

$$a_{+\infty}(0)=a_{-\infty}(0)-i\int_{-\infty}^{\infty} dt' e^{i\omega t'}j(t')$$
$$=a_{-\infty}(0)-2\pi i\sum_n \delta(\omega+H-E_n)j(0)\Psi_n\Psi_n^\dagger$$
$$=a_{-\infty}(0)-2\pi i\sum_n \Psi_n\Psi_n^\dagger j(0)\delta(\omega+E_n-H). \tag{11}$$

Now the S-matrix element for the scattering of a meson with four-momentum q by a nucleon with four-momentum p, spin σ, resulting in the final state specified by q', p', σ', is

$$(q',p',\sigma'|S|q,p,\sigma)=(\Psi(p',\sigma'),a_{+\infty}(\mathbf{q}')a_{-\infty}^\dagger(\mathbf{q})\Psi(p,\sigma)),$$
$$=(\Psi(p',\sigma'),a_{+\infty}(\mathbf{q}')\Psi^{(-\infty)}(q,p,\sigma)), \tag{12}$$

where the Ψ's are eigenstates of H. By means of Eq. (11), the last form may be rewritten as follows:

$$(q',p',\sigma'|S|q,p,\sigma)=\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{q}-\mathbf{q}')\delta_{\sigma,\sigma'}$$
$$-2\pi i\delta(\omega'+E(\mathbf{p}')-E(\mathbf{p})-\omega)(q',p',\sigma'|T|q,p,\sigma), \tag{13}$$

which expresses the conservation of energy, and wherein T may be rendered in the alternative forms

$$(q',p',\sigma'|T|q,p,\sigma)=(\Psi(p',\sigma'),j(\mathbf{q}')\Psi^{(-\infty)}(q,p,\sigma))$$
$$=(\Psi^{(+\infty)}(q',p',\sigma'),j^\dagger(\mathbf{q})\Psi(p,\sigma)). \tag{14}$$

The transition from the first form of Eq. (14) to a version equivalent to that suggested by Low is now quite immediate. We need merely note that as an immediate consequence of Eq. (10) and the Hermitian adjoint equation we can claim that

$$\Psi^{(-\infty)}(q,p,\sigma)=a_{-\infty}^\dagger(\mathbf{q})\Psi(p,\sigma)$$
$$=a^\dagger(\mathbf{q})\Psi(p,\sigma)-\lim(\epsilon\to 0)$$
$$\times[H-\omega(\mathbf{q})-E(\mathbf{p})-i\epsilon]^{-1}j^\dagger(\mathbf{q})\Psi(p,\sigma), \tag{15}$$

and that

$$a(\mathbf{q})\Psi(p',\sigma')=-\lim(\epsilon\to 0)$$
$$\times[H-E(\mathbf{p}')+\omega(\mathbf{q})-i\epsilon]^{-1}j(\mathbf{q})\Psi(p',\sigma'). \tag{16}$$

If follows that

$$(q',p',\sigma'|T|q,p,\sigma)$$
$$=(\Psi(p'\sigma'),[j(\mathbf{q}'),a^\dagger(\mathbf{q})]\Psi(p,\sigma))$$
$$-\lim(\epsilon\to 0)(\Psi(p',\sigma'),\{j(\mathbf{q}')[H-\omega(\mathbf{q})$$
$$-E(\mathbf{p})-i\epsilon]^{-1}j^\dagger(\mathbf{q})+j^\dagger(\mathbf{q})[H+\omega(\mathbf{q})$$
$$-E(\mathbf{p}')+i\epsilon]^{-1}j(\mathbf{q}')\}\Psi(p,\sigma)). \tag{17}$$

It should be noted that, insofar as physically permissible values of $\omega(\mathbf{q})$ are concerned, the limiting prescription in Eq. (16) which applies to the last term of Eq. (17) is irrelevant and any other will do. For application of causality, on the other hand, it appears to be decisive.

From Eq. (17) one can pass immediately to the limit of a fixed-source theory. For the fully relativistic theory, the conservation of three-momentum remains to be extracted from the equation. Finally, the methods described above are easily extended to more complicated scattering and production processes.