Elastic Scattering of 5.25-Mev Protons from Co, Ni, Cu, and Zn†

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The angular distributions of 5.25-Mev protons scattered by natural targets of elements with $Z=27$ through $Z=30$ have been measured. All angular distributions show a minimum in the differential cross section relative to Coulomb scattering at about 85°. The Ni and Zn relative cross sections then rise linearly whereas there is no systematic behavior for the Co and Cu. The Cu relative cross section has maxima at about 45° and 130° and the Co has a region of approximately constant relative cross section at large angles.

The angular distribution for Ni has been fitted by Saxon using a complex potential well; however, when one uses the same parameter for Co, Cu, and Zn, essentially no agreement is found.

I. INTRODUCTION

RELATIVELY little work on the elastic scattering of protons of energies less than about 15 Mev has been reported in the literature other than low-energy studies involving individual resonances in the compound state.

The experiments of Wilkins,[1] Rhoderick,[2] Baker,[3] Goldman,[4] the Zurich group,[5] and the Liverpool group[6] in the energy range from 4 to 10 Mev have shown marked deviations from the predictions of the Rutherford formula. The analyses of such experiments are primarily that of Heitler *et al.*[7] who consider a spin-independent, one-body model, including only *S*-wave effects, and essentially fit Rhoderick's data at around 4 Mev, and that of Shapiro[8] who used the continuum theory to obtain predictions which are not in agreement with the 6- to 7-Mev data of Goldman and the Zurich group.

Extensive studies of the elastic scattering of 17- and 22-Mev protons have been made by Dayton and Schrank,[9] and Cohen and Neidigh.[10] These results are characterized by a smooth, rather slow, variation of shape with atomic number and by maxima and minima which have been interpreted as diffraction effects.

Feshbach, Porter, and Weisskopf[11] have used the complex square well nuclear potential model in obtaining good agreement with the 1- to 4-Mev neutron total cross sections and elastic differential cross sections. These results are also characterized by a slow dependence of shape on the atomic number of the scatterer.

Several attempts have been made at fitting the 17- to 22-Mev proton scattering data with similar potentials. Prowse[12] and Le Levier and Saxon[13] have obtained quite good agreement to the experimental data using a square, complex potential. Prowse fitted 9.5-Mev proton scattering from oxygen and Le Levier and Saxon 18-Mev proton scattering from aluminum. Chase and Rohrlich[14] have carried out similar calculations for aluminum, copper and silver and were unable to find satisfactory agreement particularly for the heavier targets. The disagreements become more marked as the target Z increases. In an attempt to improve this situation, Woods and Saxon[15] have introduced a rounded-off complex well. Melkanoff, Nodvik, Saxon, and Woods[16] have recently succeeded in obtaining a remarkably good fit to the 17-Mev proton scattering data of Dayton and Schrank at Princeton using this potential.

The experiments to be described were undertaken to test this model for the scattering of lower energy protons. It was considered of interest to examine the scattering at about 5 Mev since this energy corresponds to a reduced wavelength, $\lambda\!\!\!^{-}$, about one-third of the nuclear radius and therefore the scattering might depend more upon the details of the nuclear potential than scattering of high-energy particles where Fraunhofer diffraction might predominate. It was also believed that the results obtained might be more directly comparable to the neutron scattering data[11] than the higher energy proton scattering. The measurements reported here were on medium-Z nuclei in order that the level density in the region of interest in the compound nuclei would be sufficiently high to avoid specific resonance effects. Measurements reported[17] on the neighboring element manganese show a level spacing of <4.5 kev at about 1.8 Mev. It is

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[1] T. R. Wilkins, Phys. Rev. **60**, 365 (1941).
[2] E. H. Rhoderick, Proc. Roy. Soc. (London) **A201**, 348 (1950).
[3] Baker, Dodd, and Simmons, Phys. Rev. **85**, 1051 (1952).
[4] L. M. Goldman, Phys. Rev. **89**, 349 (1953).
[5] J. Rotblat *et al.*, Phys. Rev. **95**, 1220 (1954).
[6] H. Schneider *et al.*, Helv. Phys. Acta **27**, 170 (1954).
[7] H. Heitler *et al.*, Proc. Roy. Soc. (London) **A190**, 180 (1947).
[8] M. M. Shapiro, doctoral dissertation, Massachusetts Institute of Technology, 1951 (unpublished).
[9] We are indebted to Dr. D. Saxon for a reprint of his calculations fitting Dayton and Schrank's data.
[10] B. L. Cohen and R. V. Neidigh, Phys. Rev. **93**, 282 (1954).
[11] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).
[12] D. J. Prowse, as quoted by D. S. Saxon, Brookhaven National Laboratory Report BNL-331, 1955 (unpublished).
[13] R. E. Le Levier and D. S. Saxon, Phys. Rev. **87**, 40 (1952).
[14] D. M. Chase and F. Rohrlich, Phys. Rev. **94**, 81 (1954).
[15] R. Woods and D. S. Saxon, Phys. Rev. **95**, 577 (1954).
[16] M. A. Melkanoff *et al.*, University of California, Los Angeles, Technical Report 7/12/55, (unpublished).
[17] J. J. G. McCue and W. M. Preston, Phys. Rev. **84**, 1150 (1951).

therefore believed that the level density at 5.25 Mev is much less than our effective beam spread. (See Sec. II.)

The copper distribution was examined first to compare the results with those noted above at somewhat different energies. When it was found that the nickel distribution was considerably different from that from copper, the scattering from the adjacent elements zinc and cobalt was also examined in an attempt to uncover the reasons for the observed differences.

II. EXPERIMENTAL ARRANGEMENT AND PROCEDURE

The analyzed proton beam from the recently rebuilt 27-in. Rochester cyclotron has been used for this work.[18] Figure 1 is a plan view of the experimental facilities available for use with this machine. The beam-focusing system has been described in detail elsewhere.[19] Briefly, the emergent beam from the cyclotron is focused by a quadrupole lens pair to an object slit for the wedge analyzer. This analyzer also focuses the beam horizontally and vertically to an image point some twelve feet away. A collimating slit system produces a beam with an angular divergence of less than 0.5°, an energy spread of approximately 20 kev and an area on target of 3×9 mm.

The 36-in. diameter scattering chamber has independently rotatable arms, one carrying a high-pressure ionization chamber and the other a NaI(Tl) scintillation spectrometer. The beam incident upon the target was monitored independently by the scintillation counter and by a Faraday cage and electronic integrator system. The scattering data have been normalized at each angle to a given number of particles incident on the target, as determined by the monitors. Data were used only where the monitors were in agreement, thus eliminating error due to changes in the target or malfunctioning of either unit.

The angular distributions were obtained with the ionization chamber using an angular acceptance width of about 2.5° and a solid angle of approximately 10^{-4} of a sphere. We used a 3-mil unsupported aluminum window when locating inelastically scattered groups and when determining the beam energy, because of the availability of accurate range energy curves, but used a 1-mil titanium or rhodium window for the actual runs. The much greater tensile strength of the latter two metals permits the use of a thin window at high gas pressures without the marked bowing and consequent poorer energy resolution resulting with the aluminum. The chamber had an effective length of about 10 cm, had a Frisch grid, and was operated at approximately 8 atmos of 99.9% pure commercial tank argon. The

[18] We take this opportunity to thank the several graduate students and our engineering staff for the effort devoted to this rebuilding.

[19] D. A. Bromley and J. A. Bruner, U. S. Atomic Energy Commission Report NYO-3823, 1954 (unpublished).

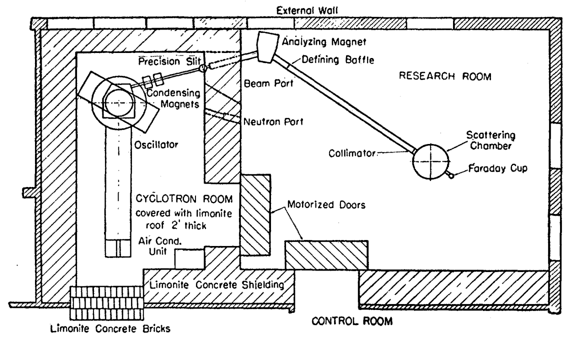

FIG. 1. Plan view of the Rochester 27-in. cyclotron installation.

collector electrode was at ground and the chamber shell at a negative high voltage. Previously, a battery supply for the high chamber voltage was used. It has been found that the Atomic Model 312 Power Supply is adequately stable.

The ion chamber had a modified Los Alamos Model 500[20] preamplifier mounted immediately beneath it in a sealed box on the scattering chamber arm. A cathode follower driver stage transmitted the output to a modified model 500 amplifier in the cyclotron control room whose output was analyzed by a 30-channel pulse-height analyzer.[21] The preamplifier and amplifier filaments were powered from shielded storage battery supplies. In each case the model 500 unit was modified to increase the rise time to several microseconds. The instrumental resolution of this system varied between 2 and 5% during the course of the experiments. The factors determining this resolution were the particular tank of 99.9% argon and the window used. The over-all resolution was determined primarily by the target thickness since, at some angles, the scattered protons had an appreciable ($\sim$1 mg/cm^2) path length to traverse.

The chamber rotation covered all laboratory angles less than 165° but the high background due to slit scattering at small angles restricted the useful range to angles greater than approximately 13°. The angular position could be set by hand to about 0.2°. The angular zero was determined by examining the scattering over the same angular range on either side of the beam axis. This, of course, did not completely determine the zero scattering angle since the plane of rotation of the detector aperture did not necessarily contain the beam line. By using a cathetometer it was established, in fact, that a tilt existed corresponding to an angular error of as much as 1°. However, the product of the differential cross section measured for the scattering of 5.25-Mev protons on gold and the appropriate $\sin^4(\theta/2)$ was constant to within $\pm 5\%$. The ratio of observed to Rutherford differential cross section was

[20] W. C. Elmore and M. Sand, *Electronics* (McGraw-Hill Book Company, Inc., New York, 1949).

[21] Fulbright, McCarthy, and McCutcheon, Phys. Rev. **87**, 184 (1952).

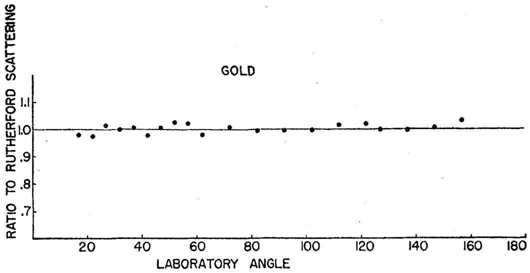

FIG. 2. Scattering of 5.25-Mev protons from gold. Since the absolute cross section was not measured, the ratio plotted here has been normalized to unity.

therefore normalized to unity as shown in Fig. 2, since the absolute cross section was not measured. We interpret this agreement to mean that to the accuracy of these experiments any errors due to tilt of the detector plane were small enough to be ignored.

The target angle was also adjustable to an accuracy of $\pm 3°$. Whenever it was necessary to change this angle, however, an empirical conversion factor was always obtained by measuring the relative intensities in each target position.

Since the scattering in these experiments was primarily Rutherford, for a fixed beam current the counting rate varied by several orders of magnitudes over the angular range studied. The integrator had several ranges of sensitivity, consequently we varied the ranges and beam current for convenience in data taking and as in the target position normalizations, the range factors for the integrator were obtained empirically by measuring the relative intensity in each range setting at a fixed angle.

The output of the DuMont 6291 photomultiplier of the scintillation spectrometer was transmitted directly, by another driver stage, to a model 500 amplifier with delay line pulse shaping, and to an Atomic Model 101-M discriminator and scaler. A second Atomic Model 312 power supply provided the photomultiplier positive high voltage.

The counting bias level for the scintillation spectrometer monitor was set at from $\frac{1}{2}$ to $\frac{2}{3}$ of the pulse height corresponding to the elastically scattered protons from the target. Since the contamination due to light elements was low, and since the inelastic cross sections were relatively low, particularly at small angles, the monitor, set as described, was insensitive to small gain changes.

In the analysis of the data all counts appearing in any one of the several channels of the analyzer containing the elastic peak were summed and normalized to a given number of incident protons on the target.

For the experiments on copper and nickel, the beam energy was determined by calibrating a proton resonance probe in the wedge analyzer field in terms of an indirect beam energy measurement based on a study of several nuclear reactions of known Q-value. In particular, we examined the scattering of protons from the ground, and first three excited states of Al^{27}.[22] The relative pulse height in the ion chamber was taken as a measure of the relative energy of the particles after traversal of a known absorber thickness. The beam energy was determined from the ground state and third excited of Al^{27} and checked against the unresolved combination of the first and second excited states. We independently verified this measurement with the first excited state of the nickel isotopes (see next section).

For the experiments on zinc and cobalt the resonance probe was recalibrated by means of the resonant elastic scattering from the 3.47-Mev level in F^{17} formed in the scattering from an O^{16} target.[23] It is believed that both of these measurements are accurate to within 0.20 Mev and that the energy used in all four experiments was the same within 0.15 Mev.

The nickel and copper targets were foils 5×10^{-5} inch thick obtained commercially.[24] The zinc target, of approximately 1-mg/cm^2 thickness, was prepared by vacuum evaporation of the metal on a thin Zapon film. Self supporting metallic cobalt targets were obtained by plating out cobalt from an aqueous solution of cobaltous acetate on a nickel foil. About 0.5 ampere per square inch of foil surface was used and it was found that when the cobalt deposit reached a thickness of about 1 mg/cm^2 it separated from the nickel and could then be mounted appropriately. The use of the acetate solution was indicated to avoid heavy contaminants such as chlorine or sulfur in the target. Spectrographic analysis of these cobalt target foils showed less than 0.1% copper, zinc, iron, and nickel contamination.

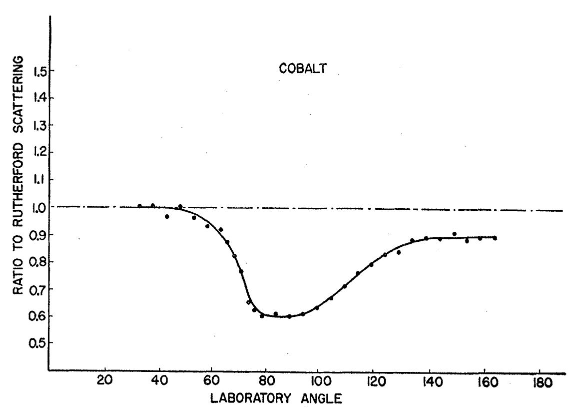

FIG. 3. Scattering of 5.25-Mev protons from cobalt. The ratio has been normalized to unity at forward angles.

[22] P. M. Endt and J. M. Kluyver, Revs. Modern Phys. **26**, 95 (1954).
[23] E. Baumgartner and H. W. Fulbright (private communication).
[24] International Chromium Corporation, Waterbury, Connecticut.

III. RESULTS

The angular distributions of 5.25-Mev protons elastically scattered from Ni ($Z=28$) and Cu ($Z=29$) have been published previously.[25] New results are shown for Co ($Z=27$) and Zn ($Z=30$) in Figs. 3 and 4. Figure 5 is a composite showing the distributions for all four elements. We emphasize that we have not measured absolute cross sections but rather have normalized to unity the product of the observed differential cross section and $\sin^4(\theta/2)$ (where θ is the angle of scattering), in the small-angle region where this product was constant to within experimental errors.

The error at each experimental point in the angular distribution is determined largely by an error in angle setting of up to about 0.5° in positioning the detector at small angles, and the statistical error in the number of counts observed at back angles. As can be seen from a simple calculation for the error in the product of the observed differential cross section and the $\sin^4(\theta/2)$, an angular error of 0.5° will produce at least a 10% error in the product at an angle of 20°. Because of the rapid variation of $\sin^4(\theta/2)$ with θ, the error due to the finite detector aperture is appreciable at angles of less than 20° but in our region of observation it is entirely negligible. The absolute error at any given angle is in fact estimated to be less than 10%. On the basis of reproducibility of the angular distribution at different times, however, such features as the ratio of the backward maximum to minimum in the vicinity of 90° are thought to be accurate to within 5%.

In these experiments the only interest in the inelastically scattered groups lay in the determination of their separation from, and intensity relative to, the elastically scattered groups. No precise measurements were made to locate the energy levels concerned; nor did we attempt to measure the angular distributions of any of these groups because of their low intensities. Our resolution was such that we could have detected states of excitation of 500 kev or higher if they had a relative intensity of greater than 10% of the elastically scattered group. Coulomb excitation studies by Temmer and Heydenburg[26] have shown no levels in Co, Ni, or Cu excited by 3-Mev α particles of less than 500-kev excitation. The 4% abundant isotope Zn^{67} does have two levels at 93 and 182 kev excited in this manner, but the corresponding groups could not be resolved in this experiment. Figure 6 is a histogram from the 30-channel pulse-height analyzer corresponding to the scattering by nickel of 5.25-Mev protons through a laboratory angle of 130°. The location of the groups corresponding to the first excited states in Ni^{58} and Ni^{60} as found by the Rice group[27] is indicated. It can be seen that the relative excitation cross section for these groups is less than 12% of the elastic cross section, in agreement with the Rice results.[27] The copper results show a proton group at an energy corresponding to the 0.67-Mev[27] state of Cu^{63}. Other states[27] probably contribute to the relatively high background. A spectrum of the protons scattered from Cu is shown in Fig. 7.

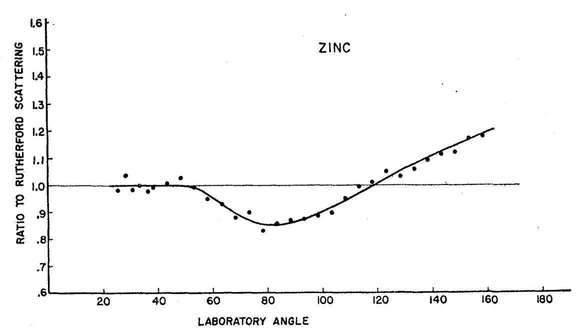

FIG. 4. Scattering of 5.25-Mev protons from zinc. The ratio has been normalized to unity at forward angles.

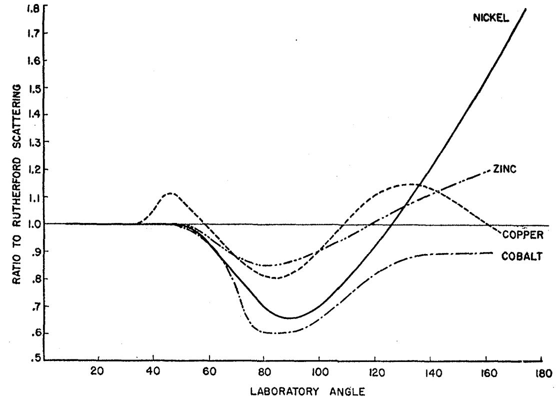

FIG. 5. A composite presentation of the best-fit curves drawn through the experimental points obtained in the scattering of 5.25-Mev protons from the indicated targets.

Both the Co and the Zn targets contained oxygen and carbon arising from cobalt acetate in the Co target and from the Zapon backing in the Zn targets. The presence of these elements in the targets is shown in the Zn histogram in Fig. 8. That these groups corresponded to light elements was shown by their energy shift, with detector angle, relative to the intense heavy element elastic proton group. In the Zn spectrum a small group can be seen in channel 8 corresponding to an excited state at about 1.5 Mev in one of the Zn isotopes. This small group appears in all histograms corresponding to back-angle scattering. No groups other than the elastically scattered protons from cobalt and the carbon and oxygen present in the cobalt target could be definitely observed in the scattering from the cobalt target.

These measurements show that with the possible exception of the zinc data, no inelastic contaminants

[25] D. A. Bromley and N. S. Wall, Phys. Rev. **99**, 1029 (1955).

[26] G. M. Temmer and N. P. Heydenburg, Phys. Rev. **93**, 351 (1954).

[27] J. P. Schiffer *et al.*, Phys. Rev. **99**, 655(A) (1955).

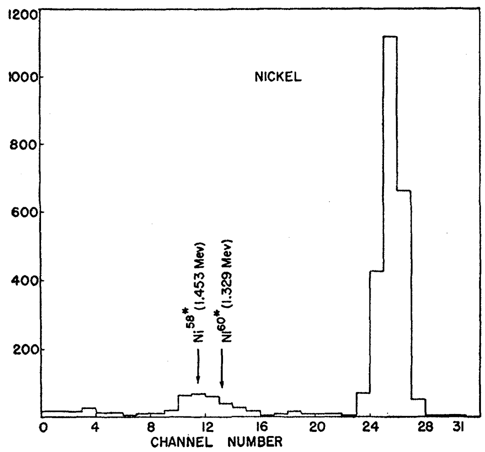

FIG. 6. Pulse-height spectrum for the elastic and inelastic scattering of 5.25-Mev protons from nickel at a laboratory angle at 130° as obtained with the 30-channel analyzer.

are present in what we have taken as the elastically scattered groups. With an isotopic abundance of 4%, and because the inelastic cross sections in this experiment seem small relative to the elastic cross sections, we believe that the inelastic "contamination" in the Zn case is negligible.

IV. DISCUSSION

As shown in Fig. 5, the angular distributions which we have measured are characterized by two main features. The first of these is that all the distributions have a minimum in relative cross section around 85° and the second is that although there are relatively large deviations from Coulomb scattering at large angles, there is little evidence for any systematic behavior.

As might be expected, if these minima resulted from destructive interference of Coulomb and nuclear scattering, the depth decreases with increasing atomic number. The relative depths however do not show a Z^{-2} dependence, but this is perhaps not surprising when the possibility of a change of relative phase of the nuclear and Coulomb amplitudes is included.

It is of interest to note that the distributions from nickel and zinc, both having even-even abundant isotopes which presumably have zero spin, are quite similar in behavior if not in the magnitude of the deviations from Coulomb scattering. At large angles the relative cross sections for these elements increases approximately linearly with angle whereas copper, with both isotopes having a spin of $\frac{3}{2}$,[28] has a maximum around 130° and cobalt, with a single isotope of spin 7/2,[28] has a flat region.

The fact that the 85° minimum appears in all curves strongly suggests that it results from a "gross structure" effect. That we are looking at a property of the nuclei in this region that is not characteristic of a few given resonances in the compound nuclei is borne out by the fact that although the binding energies of the last proton, added to the various abundant isotopes vary by over 2 Mev[29] the angular distributions in the angular region about 90° are quite similar. It should be mentioned that because of the dependence on $\sin^{2n}\theta$, "spin flip" (incoherent) scattering would contribute most strongly in this region and not at all at 180°. A complete phase-shift analysis of the experimental curves would be required to separate the direct and spin flip contributions and it has not been felt that the accuracy of the experiments was sufficiently high to justify the considerable labor which would be required for this analysis.

Further evidence that we are not looking at effects due to individual resonances comes from the fact that the measurements for copper with protons of 5.25, 6.0, 6.5, and 7.0 Mev have all yielded quite similar distributions.[30] It also should be mentioned again that the measurements on Mn^{55} indicate a level spacing, at an excitation energy to the binding energy of the last proton plus 1.8 Mev, of less than 4.5 kev. Although the beam incident on the targets had a 20 to 30 kev spread, the finite target thickness resulted in an effective beam spread of at least 100 kev. The Rice group,[27] in examining the deexcitation of the first excited state of Ni^{58} by γ-ray emission, has observed variations in the yield from a thin target with changes in the incident proton energy in the 4–5 Mev range. They attribute these resonances entirely to p-wave or higher J-value resonances on the basis of the fact that only one of some 37 resonances studied had an isotropic

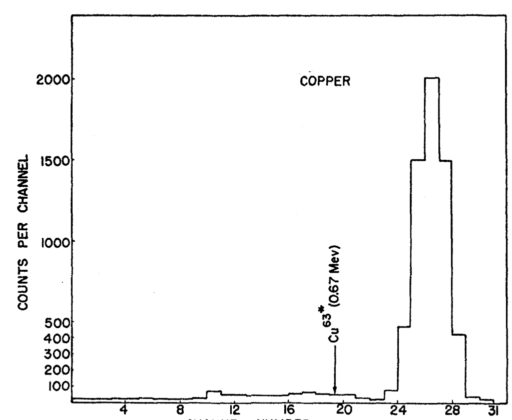

FIG. 7. Pulse-height spectrum for the elastic and inelastic scattering of 5.25-Mev protons from copper at a laboratory angle of 130°.

[28] P. F. A. Klinkenberg, Revs. Modern Phys. 24, 63 (1952).

[29] N. Feather, Phil. Mag. Suppl. 2, 141 (1953).

[30] Preliminary measurements on Ni and Cu have shown no marked changes in the angular distributions as the energy is changed by about 0.25 Mev. This also tends to confirm the supposition that the results are not dependent upon the exact region reached in the compound nucleus.

γ-ray distribution. For p-wave or greater resonances they observe 25 levels per Mev of width <3 kev each. Therefore in our experiments we are certainly looking at at least several resonances, and on the basis of the above arguments we are probably averaging over a large number of resonances.

If the assumption is correct, that the common 85° minimum, together with the subsidiary arguments advanced above, are evidence for a "gross structure" behavior then the results obtained should be amenable to analysis on the basis of a complex potential model. Furthermore, the marked differences which are observed at large angles (as well as the 45° maximum in the copper distribution) must also be due to some mechanism which does not depend on the effects of individual resonances. A possible mechanism will be suggested later in this section. It should be pointed out that only in the case of cobalt have we used a monoisotopic target. It is of course conceivable that the results obtained for Ni, Cu, and Zn are due to this use of natural targets rather than separated isotopes. This would require, however, that isotopes differing by pairs of equivalent neutrons should give appreciably different angular distributions.

Two attempts have been made to fit the data obtained in these experiments using current nuclear models. Following Chase and Rohrlich,[14] we have calculated the angular distribution to the expected from scattering 5.25-Mev protons by a nuclear potential of the form $V=-V_0(l+i\zeta)$ with $V_0=20$ Mev and $\zeta=0.05$ with partial waves up to and including $l=4$. In addition, Melkanoff and Saxon[31] have made preliminary calculations using a potential of the form

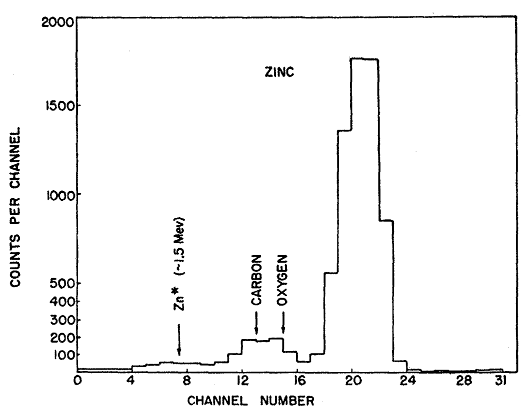

FIG. 8. Pulse-height spectrum for the elastic and inelastic scattering of 5.25-Mev protons from zinc at a laboratory angle of 130°. The energies of the elastically scattered protons from the carbon and oxygen in the Zapon lacquer target backing are as shown.

[31] We are greatly indebted to Dr. Saxon for communicating these results to us before publication and for his permission to quote them here. Much of our discussion of these results stems from Dr. Saxon's comments.

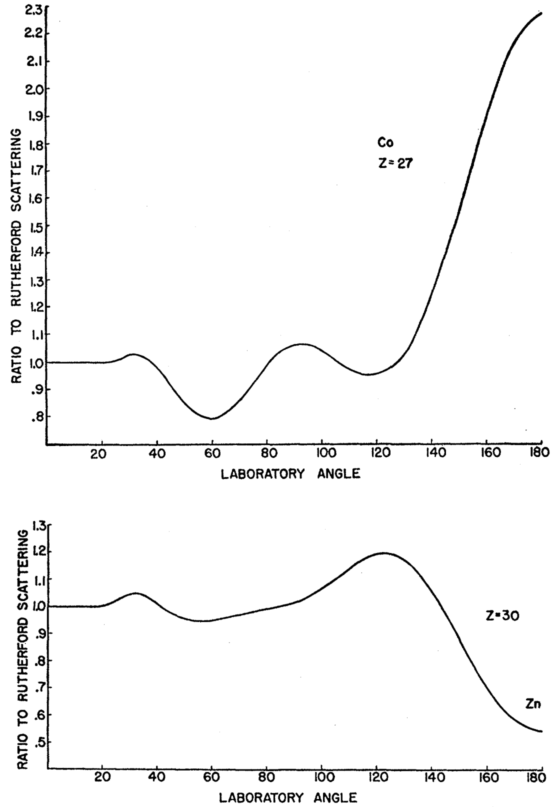

FIG. 9. The differential cross sections relative to Coulomb, as calculated using a square-well optical model potential for Co and Zn.

$$V=V_c+V_N:$$

$$-V_N=(V_0+iW_0)\Big/\left[1+\exp\left(\frac{r-R_0}{a}\right)\right]$$

$$=\frac{V_0+iW_0}{2}\left[1-\tanh\left(\frac{r-R_0}{2a}\right)\right],$$

$$V_c=\frac{Ze^2}{2R_0}\left[3-\left(\frac{r}{R_0}\right)^2\right],\quad r\leq R_0$$

$$=\frac{Ze^2}{r},\quad r\geq R_0.$$

Both of these calculations have been made in a spin-independent manner. Figure 9 shows the results of the square well calculation for the elements Co and Zn using a radius of $R=1.38A^{1/3}\times10^{-13}$ cm. Figure 10 shows the fit of the Melkanoff-Saxon analysis to the experimental data for Ni. The parameters which were used were $V_0=52.5$ Mev, $W_0=0.9$ Mev, $R_0/A^{1/3}=1.33\times10^{-13}$ cm and $a=0.375\times10^{-13}$ cm. Figure 11 shows the predicted results using these same parameters for Co, Cu, and Zn. As can be seen by comparing Figs. 9 and 11 to Fig. 5, there is essentially no fit between the angular distributions expected on the basis of these two models and experiment except in the case of Ni.

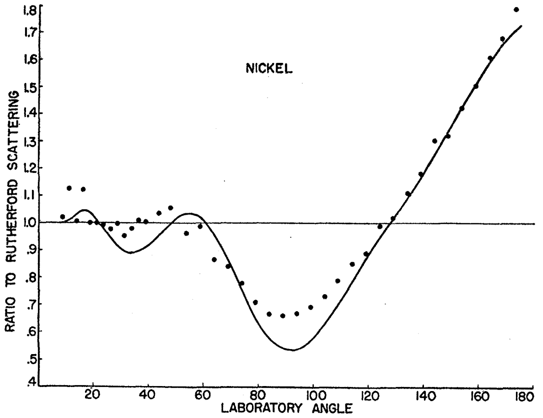

FIG. 10. A comparison of the experimental points for the nickel scattering data with the best fit curve calculated by Saxon and Melkanoff.

Since their formalism was spin-independent, Melkanoff and Saxon varied the parameter of their potential in order to fit the Ni data and then calculated the angular distribution for the adjacent elements with the same parameters. They report that the fit to the Ni was extremely sensitive to the actual values of the parameter they used. Because of this sensitivity, they feel that it may be possible to fit the Co, Cu, and Zn data with parameters not very different from those used for Ni. However, no results are available as yet. If such a fit to the experimental data is found, it will be of interest to see if the parameters reflect any variation of nuclear size or shape as a result of the closing of the $Z=28$ proton shell.

For S-wave protons, if one uses the Melkanoff-Saxon potential, the potential barrier has a maximum value of 5.3 Mev and occurs at about 7×10^{-13} cm for $Z=28$. Because of the fact that we are using 5.25-Mev protons, Saxon has suggested that the large differences in behavior in the calculated and measured angular distributions may be in large part due to the extreme sensitivity of this type of experiment to the exact details of the rounding of the potential well and therefore the nuclear surface. It seems, however, that the square-well calculations invalidate this hypothesis, since even these calculations show large qualitative differences in the angular distributions expected from nearby nuclei.[32] Therefore, relative to the present experiments it seems difficult to account for the different behavior of the various angular distributions by a barrier effect of any sort when as noted before, the copper angular distribution is essentially the same from 5.25 Mev to 7.0 Mev. In addition, if one computes the barrier penetrabilities for the various partial waves one finds only a factor of two difference between the $l=0$ and $l=2$ partial waves, and furthermore both of these are relatively high ($\sim$0.3 for $l=0$ and $\sim$0.15 for $l=2$).

The fact that both of the above calculations show marked variations in the angular distributions tends to weaken the hypothesis which we made earlier[25] that the difference in the angular distributions is due to some target spin effect. Conversely, the fact that the Zn and Ni distributions are qualitatively similar is suggestive of a spin dependence. If our hypothesis is in fact true, then agreement with the experimental results might be obtained if a detailed calculation were carried out using the channel-spin formalism or it may actually require a modification of the nuclear potential by some spin-dependent interaction. The usual single-particle spin-orbit interaction would not suffice for this since it does not take the nuclear spin into effect. To the best of our knowledge, no other spin-dependent interactions have been considered for the description of scattering phenomenon.

Note added in proof.—Recent calculations for $Z=28$ using a square well of depth $42(1+0.03i)$ Mev show no better fit to the experimental than the more shallow well given in the preceding. The deeper potential well does, however, result in a calculated curve which shows smaller deviations from Rutherford scattering than does the shallow well. To help determine the validity of any partial wave calculation with orbital angular momenta $l=0$ through $4\hbar$ a least-squares fit for the experimental data in Fig. 5, of the form

$$\frac{d\sigma}{d\omega}\bigg/\left(\frac{d\sigma}{d\omega}\right)_{\text{Coul}}=\sum_{l=0}^{4}a_lP_l(\cos\theta)$$

was made by Dr. E. B. Paul at the University of Toronto Computation Center. These data show that though the predominant term for all of the elements is the $l=0$, all terms up to $l=4$ are involved and there

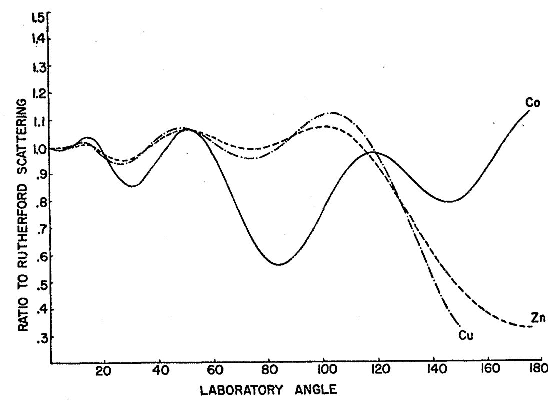

FIG. 11. The differential cross section relative to Coulomb for Co, Cu, and Zn calculated on the basis of Saxon's potential using the same parameters as in the case of Ni.

[32] With respect to the square-well calculation, it should be remarked that the large backward maxima in the $Z=27$ calculation results from a near cancellation in the expression for the $l=3$ partial wave phase shift. Therefore the change in character of the angular distribution may be characteristic of the property of a square-well potential to introduce apparently large maxima and minima. Further calculations to investigate this behavior are to be undertaken.

is no marked tendency for the contributions of the higher l values to decrease.

V. ACKNOWLEDGMENTS

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Calculation of Electric Dipole Coulomb Excitation and Dipole Bremsstrahlung*

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Numerical results are obtained for electric dipole Coulomb excitation. Tables of the dimensionless excitation function a_0 and the directional correlation parameter a_2 are presented over a very wide range of their arguments. It is shown that the dipole bremsstrahlung cross section can be written in terms of the a_μ, so that the tabulation also serves for the calculation of the bremsstrahlung differential cross section.

I. INTRODUCTION

The formalism and mathematical techniques necessary to an adequate quantum mechanical calculation for the general multipole in Coulomb excitation have been presented earlier.[1] These techniques have already been applied to the case of electric quadrupole excitation.[2] In the present paper numerical results for electric dipole excitation are given. Although at present electric dipole excitation is not as important experimentally as electric quadrupole excitation,[3] a few low-lying (1−) states have been reported[4] and there is reason to believe that low-lying states of odd parity may exist in the fissionable elements.

Results are given for the dimensionless excitation function a_0 and the correlation parameter a_2, for values of the arguments in the range $0.001\leq\eta\leq 40$, $1\leq\rho\leq 1.8$, where $\eta\equiv Z_1Z_2e^2/\hbar v_{\text{initial}}$ and $\rho\equiv k_{\text{initial}}/k_{\text{final}}$. This range includes all energies of experimental interest and covers energy losses of up to 70%. Numerical values for the dimensionless excitation function a_0 and the particle parameter a_2 are presented in Tables I and II, respectively. These values are plotted in two different ways in order to exhibit the general behavior of these functions. The qualitative features of the present calculation are very similar to those discussed in II.

The limitations of this work have been discussed in I. In particular, center-of-mass corrections and retardation effects have been neglected in the calculation presented below. For the excitation of medium and heavy nuclei, the error due to the assumption that the position of the target nucleus defines the center of mass should be small. Moreover, since it is expected that the present results will be applied to the excitation of very heavy nuclei, such corrections should be especially small. The neglect of retardation is a more serious source of error for electric dipole than for electric quadrupole excitation. In the electric dipole case the retardation expansion enters in lower order in $(k_{\text{rad}}r_{tp})^2$ than for quadrupole excitation. In addition the effect of the higher angular momenta is more pronounced in the dipole case. Nevertheless, the effect of retardation is expected to be less than one percent over most of the experimental region. Calculations to treat the exact dipole operator are now in progress and a quantitative discussion of retardation will appear later.

The relation between the calculations for electric dipole excitation and dipole bremsstrahlung will be discussed in Sec. III. It will be seen that the functions $a_{0,2}$ are likewise required in the calculation of dipole bremsstrahlung.

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[1] Biedenharn, McHale, and Thaler, Phys. Rev. **100**, 376 (1955). This reference is hereinafter designated in the text as I.

[2] Goldstein, McHale, Thaler, and Biedenharn, Phys. Rev. **100**, 436 (1955); and Biedenharn, Goldstein, McHale, and Thaler, Phys. Rev. **101**, 662 (1956). The latter reference is designated in the text as II.

[3] The absence of low-lying odd-parity states is accounted for by the fact that for nuclei whose intrinsic shapes are symmetric with respect to reflection, no states of parity opposite to that of the ground state can appear in the rotational spectrum. This question is discussed in detail by A. Bohr and his collaborators. For a survey of this work and a complete set of references, see, e.g., A. Bohr, *Rotational States of Atomic Nuclei* (Einar Munksgaard, Copenhagen, 1954). However, the theoretical interpretation of the anisotropies in the fragment distribution for photofission and fission produced by fast particles requires the abandonment of the assumption of reflective symmetry, so that, for $A\gtrsim 200$, low-lying (1−) states (capable of excitation by the Coulomb excitation process) should appear. For a discussion of this question, see A. Bohr, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (to be published), Paper U.N. 911.

[4] Stephens, Asaro, and Perlman, Phys. Rev. **96**, 1568 (1954), and **100**, 1543 (1955).