Pair Production in the Field of Orbital Electrons by a Total-Absorption Method at 319 Mev*

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The attenuation of (319±4)-Mev photons in liquid hydrocarbons has been measured and has been used to infer the cross section for pair production in the field of the orbital electrons (triplet production).

Targets of benzene (C_6H_6) and cyclohexane (C_6H_{12}) were used in good geometry. A pair spectrometer with three fast counter channels was used to measure the intensity of the transmitted photon beam. The synchrotron was monitored with a modified pair spectrometer that was sensitive only to the bremsstrahlung spectrum above 300 Mev.

The total absorption cross sections in hydrogen and carbon at 319 Mev have been found to be 18.0±1.8 mb per hydrogen atom, and 321.1±2.8 mb per carbon atom, respectively.

Subtraction of the theoretical nuclear pair and electron Compton cross sections and the experimental photomeson cross sections in hydrogen from the total hydrogen absorption cross section gives 6.8±1.8 mb for the experimental triplet cross section, as compared with the theoretical value of 7.68 mb. Similarly, when one subtracts the cross sections for competing processes in carbon, one gets for the triplet cross section at 319 Mev the value 46.6±6.4 mb, compared with the theoretical value of 44.4 mb. The large uncertainty in the carbon triplet cross section is due to uncertainties in the cross sections for the competing processes.

I. INTRODUCTION

THE possibility of the formation of an electron pair in the field of an atomic orbital electron by a gamma ray was first recognized by Perrin.[1] Landau and Rumer[2] suggested for the reciprocal case of bremsstrahlung by an electron in the field of orbital electrons that one should replace Z^2 by $Z(Z+1)$ in order to account for the total radiation cross section per atom. The first quantum mechanical calculation of the process was that by Wheeler and Lamb.[3] They used the first Born approximation, which assumes that $Z/137 \ll 1$. Screening was included and was found to be less effective on the electrons than on the nucleus. Thus, the so-called "triplet production" (pair production by photons in the field of an orbital electron) is greater than ϕ_{pair}/Z. This is discussed in more detail below (Sec. III). Several nonscreened theories have been given, of which the most complete is that of Borsellino.[4] He finds that with increasing photon energy, pair production in the field of an electron very slowly approaches that in the field of a nucleus of unit charge. At 319 Mev, the former is 5% smaller than the latter.

The first indisputable observation of triplet production was that reported in 1944 by Ogle and Kruger,[5] who used the 2.67-Mev gamma ray from Na^{24}. This energy is just above the threshold ($4mc^2$) for triplet production, and—as has been shown by Watson[6] and more rigorously by Votruba[7]—the three electrons tend to share the available energy about equally for a photon energy that is less than 5 Mev. For photon energies above this, the positron and one electron get most of the energy, with the third electron getting $\frac{1}{2}mc^2$ in the extreme relativistic limit. Thus, to observe the effect directly, one should use gamma rays just above the threshold. Ogle and Kruger observed the pairs and triplets formed in the air in their cloud chamber. They detected 56 pairs and 2 triplets with excellent momentum and energy balance. More recent cloud-chamber work was that by Phillips and Kruger[8] with the 6-Mev gamma from photons on fluorine, by Gaerttner and Yeater[9] with the average energy of 50 Mev from the General Electric betatron, and by Emigh[10] at the Illinois 300-Mev betatron. Besides these direct observations, DeWire[11] at 280 Mev, Lawson[12] at 88 Mev, and Berman[13] at 19.5 Mev have all found it necessary to introduce the theoretical absorption by triplet production to account for the total absorption experimentally observed. Only Berman, however, studied low-Z elements where triplet production is comparable to pair production, and his energy was too low to give rise to a pair cross section comparable to Compton cross section.

The subject of this paper is the study of triplet production by a total-absorption method at 319 Mev. This specific technique was employed because the direct effect is difficult to observe, as the third electron gets very little energy.

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II. COMPETING ABSORPTION PROCESSES AT 319 MEV

The absorption processes that compete with triplet production at 319 Mev are discussed below. The theoretical treatment of triplet production is given in Sec. III.

A. Pair Production in the Nuclear Coulomb Field

For 319-Mev gamma rays, the largest contribution to the total absorption cross section in all elements except hydrogen is that due to pair production in the nuclear Coulomb field. The theory for this process, including screening, was given by Bethe and Heitler[14] in 1934. They used the Dirac negative energy states and the first Born approximation, which requires that $Z/137\ll 1$, this being the requirement that the plane wave representation used for the pair members remain undistorted by the nuclear Coulomb field. For high-Z elements the condition is not satisfied. This was first seen experimentally by Adams[15] at 20 Mev. This discrepancy with theory has since been confirmed by several authors, among them Lawson[12] at 88 Mev; DeWire, Ashkin, and Beach[16] at 280 Mev; and Emigh,[10] who gets at 300 Mev substantially the same correction as that found by Lawson. The differential cross section in the limit of high energies, neglecting screening, has been calculated without recourse to the Born approximation by Maximon and Bethe[17] and has recently been integrated over positron energy.[18] It is shown in reference 18 that the correction is equally applicable to cases with complete, incomplete, or no screening. Just as the experiments had indicated, the correction term has a Z^2 dependence. It is to be noted that this correction is completely negligible for carbon, amounting to less than 0.1%, and for elements of lower Z. As is shown below, triplet production is most readily detectable for low-Z elements. Thus we are justified in calculating nuclear pair production from the Bethe-Heitler theory, in view of its experimental confirmation for low Z.

In calculating the screening of the nucleus by the orbital electrons, a Fermi-Thomas distribution of the electrons is assumed. This theory is of a statistical nature and becomes more applicable as Z increases. Fortunately, Wheeler and Lamb[3] have made an exact calculation for hydrogen.

A convenient form for the differential cross section, $\phi_p(E_+)dE_+$, for the creation of a pair whose positron has an energy in the range E_+ to $E_+ + dE_+$, whose negative electron has an energy in the range E_- to $E_- - dE_-$, and for the incident quantum whose energy is k_0, where $k_0 = E_+ + E_-$, is given by the relation

$$\phi_p(E_+)dE_+ = \frac{\bar{\phi}dE_+}{k_0^3} \times\{[E_+^2+E_-^2][\phi_1(\gamma)-(4/3)\ln Z] + \tfrac{2}{3}E_+E_-[\phi_2(\gamma)-(4/3)\ln Z]\},$$

where $\phi_1(\gamma)$ and $\phi_2(\gamma)$ are given graphically in reference 3 as functions of

$$\gamma = 100mc^2k_0/(E_+E_-Z^{\frac{1}{3}}),$$

and where[19]

$$\bar{\phi} = \frac{Z^2}{137}\left(\frac{e^2}{mc^2}\right)^2 = Z^2\times 0.5793 \text{ mb}.$$

The authors have calculated the value of this function for $k_0 = 319$ Mev for hydrogen and carbon as a function of the positron total energy, using a maximum interval between points of 10 Mev. The total pair cross sections were obtained by graphical integration of these curves (see Table I). From calculations such as this, a plot of the pair cross section *vs* photon energy may be obtained.

B. Electron Compton Effect

The electron Compton effect is the subject of the previous paper (hereafter referred to as AKM,[20] in which it was concluded that the Klein-Nishina formula was correct to within 16% at 319 Mev. The theoretical value at 319 Mev is

$$\phi_{\text{Compton}} = 3.04 \text{ mb/electron}.$$

C. Photomeson Production in Hydrogen and Carbon

Energy and angular distributions for π^+-meson production in hydrogen have been measured at various laboratories. The recent Berkeley total cross section at 275 Mev[21] is in excellent agreement with that from California Institute of Technology. Cross sections published by the latter institution show the total π^+-production cross section in hydrogen at 319 Mev to be 0.22 ± 0.03 mb.[22]

For π^0 production in hydrogen at 319 Mev, Oakley and Walker[23] show the total cross section to be 0.23 ± 0.03 mb.

Thus, the total photomeson cross section in hydrogen at 319 Mev is 0.45 mb, accurate to approximately 50%.

The ratio of π^+ production from carbon and from

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[14] H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146**, 83 (1934).
[15] G. D. Adams, Phys. Rev. **74**, 1707 (1948).
[16] DeWire, Ashkin, and Beach, Phys. Rev. **83**, 505 (1951).
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[19] 1 mb $=10^{-27}$ cm^2.
[20] Anderson, Kenney, and McDonald, preceding paper [Phys. Rev. **102**, 1626 (1956)].
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[22] Walker, Teasdale, Peterson, and Vette, Phys. Rev. **99**, 210 (1955); Tollestrup, Keck, and Worlock, Phys. Rev. **99**, 229 (1955).
[23] D. C. Oakley and R. L. Walker, Phys. Rev. **97**, 1283 (1955).

hydrogen per equivalent quantum is given as 2.16 by Jakobson, Shultz, and White.[24] Using this result and the π^+ cross section in hydrogen at 319 Mev, we find the total π^+ cross section in carbon is 0.47 mb.

Littauer and Walker[25] show the ratio of π^- to π^+ production in carbon to be 1.06 for 65-Mev mesons at 135° and for 310-Mev bremsstrahlung. Their work, combined with the recent work of Motz, Crowe, and Friedman,[26] shows no strong energy or angular dependence. Using the π^+ total cross section in hydrogen at 319 Mev, one finds the π^- cross section in carbon to be 0.5 mb.

In the paper by Panofsky, Steinberger, and Steller[27] the ratio of the cross sections per equivalent quanta for π^0 production in carbon and in hydrogen is 7.7. From the hydrogen cross section one gets 1.77 mb per 319-Mev photon.

Thus the total photomeson cross section in carbon at 319 Mev is 2.7 mb, accurate to within a factor of 2.

D. Other Processes

The photoproduction in carbon of stars with three or more prongs has been observed by Miller.[28] In the interval of 242 to 322 Mev, the average value for the total cross section is given as 1.5 mb, where an additional 10% was added to account for stars in which only neutrons were formed. This cross section should be good to within 50%.

The photoproton yield from carbon is obtained from the angular distribution found by Feld, Godbole, Odian, Scherb, Stein, and Wattenberg,[29] and the absolute differential cross section at 60° for 180-Mev photons by Weil and McDaniel.[30] With a $k_0^{-\frac{3}{2}}$ dependence assumed, the cross section is found to be 0.6 mb per 319-Mev photon, accurate to within approximately a factor of 3.

No other processes contribute significantly to the interaction of 319-Mev photons in hydrogen or carbon. The photoelectric effect, so important at lower photon energies, gives cross sections of the order of 10^{-36} cm² in hydrogen and 10^{-32} cm² in carbon. The proton Compton cross section is of the order of 10^{-31} cm².

III. PAIR PRODUCTION IN THE COULOMB FIELD OF THE ORBITAL ELECTRONS (TRIPLET PRODUCTION)—THEORETICAL

Triplet production is an absorption process for 319-Mev gamma rays which is second in magnitude only to pair production. The name is derived from the fact that the quantum forms a pair of electrons in the field of an orbital electron, and the electron recoils and is removed from the atom, giving a triplet of electrons. An elementary calculation shows that the threshold for the process is $4mc^2$, whereas it is $2mc^2$ in ordinary nuclear pair production.

The article by Wheeler and Lamb[3] is primarily devoted to pair production by electrons in the field of orbital electrons, but by the Weizsacker-Williams method they obtain the result for triplet production. The calculation closely follows that of Bethe and Heitler for ordinary pair production. They employ the first-order Born approximation, which assumes a plane-wave representation for the pair members. They also assume that the recoil energy of the third electron is negligible, an assumption which has remained in more recent theories and has been shown to be valid by Watson[6] and Votruba.[7] For screening, the Fermi-Thomas electron distribution is again assumed, except in the case of hydrogen, where exact wave functions were used. The differential cross section for the production of a pair of electrons in the field of an orbital electron, in which the positron has an energy in the interval E_+ to E_++dE_+, and for the incident quantum of energy k_0, is given by

$$\phi_t(E_+)dE_+=\frac{\bar{\phi}}{Z}\frac{dE_+}{k_0^3}\{[E_+^2+E_-^2]\times[\psi_1(\epsilon)-(8/3)\ln Z]+\tfrac{2}{3}E_+E_-[\psi_2(\epsilon)-(8/3)\ln Z]\},$$

where $\psi_1(\epsilon)$ and $\psi_2(\epsilon)$ are given graphically in reference 3 as functions of $\epsilon=100mc^2k_0/(E_+E_-Z^{\frac{2}{3}})$. The results of graphical integration of this function for $k_0=319$ Mev for hydrogen and carbon are given in Table I. It is to be noted that screening is less effective on the orbital electrons than it is on the nucleus. In the limit of very high energies and complete screening we find[31]

$$Z\phi_{\text{Wheeler-Lamb}}/\phi_{\text{Bethe-Heitler}}=1.4 \text{ in hydrogen.}$$

In this derivation, Wheeler and Lamb have assumed that the probability of producing a pair in the Coulomb field of a free electron is the same as in the field of a proton. Borsellino[4] has shown that this assumption is not correct, but he has neglected screening. In the limit of high energies, Borsellino's result for the total triplet cross section per atom is

$$\phi_{t,\text{ Borsellino}}=\frac{\bar{\phi}}{Z}\left\{\frac{28}{9}\ln 2\alpha-\frac{218}{27}-\frac{1}{\alpha}\times[(4/3)(\ln 2\alpha)^3-3(\ln 2\alpha)^2+6.84\ln 2\alpha+21.51]\right\},$$

where $\alpha=k_0/mc^2$. As one goes to very high energies this result has as its asymptotic limit the Bethe-Heitler nonscreened pair-production cross section divided by the

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[31] H. A. Bethe and A. Ashkin in *Experimental Nuclear Physics*, edited by E. Segrè (John Wiley and Sons, Inc., New York, 1953), Vol. I, Part II, p. 263.

atomic number. That is

$$\phi_{t,\text{ Borsellino}}\to\frac{\bar{\phi}}{Z}\left[\frac{28}{9}\ln 2\alpha-\frac{218}{27}\right]\quad\text{for}\quad\alpha\gg 1.$$

Borsellino's result approaches this function very slowly, differing from it by 5% at 319 Mev.

At the intermediate energy of 319 Mev it is necessary to consider both screening and the difference between electronic and nuclear effects. Bethe and Ashkin[31] point out that it is a good approximation to take Borsellino's result for the difference between the cross sections in the field of an electron and in the field of a nucleus of charge 1, and to subtract this from the Wheeler and Lamb cross section. That is,

$$\phi_{\text{triplet}}=\phi_{\text{Wheeler-Lamb}}-Z[\phi_{\text{pair }(Z=1),\text{ unscreened Bethe-Heitler}}-\phi_{\text{triplet }(Z=1),\text{ Borsellino}}].$$

The justification for this method of calculation of the triplet cross section is that Borsellino's difference between the production probabilities for electronic and nuclear fields is due mainly to large momentum transfers, while the screening effect arises from small momentum transfers. The corrected theoretical triplet cross sections for hydrogen and carbon are given in Table I.

IV. METHOD AND APPARATUS

A. General

The most direct method of obtaining the triplet cross section would be to detect the three electrons for a given photon energy. It has been mentioned previously that the third electron gets very little of the energy at 319 Mev. Thus the direct observation of the effect, either electronically or by cloud chambers, would be exceedingly difficult. Superimposed upon this difficulty is the problem of working with a bremsstrahlung spectrum.

A simpler approach is to measure the total absorption cross section for a given photon energy in elements for which the triplet cross section is an appreciable fraction of that total absorption cross section. If from theory and experiment one can make reasonable estimates of the cross sections for the competing processes at this energy, a value of the triplet cross section can be obtained.

B. Total Absorption Cross Sections

The total absorption cross sections at 319 Mev for hydrogen and carbon were determined in a good-geometry attenuation experiment by use of the pair-spectrometer coincidence-counting equipment and monitor discussed in AKM (see Fig. 2 of that paper).

TABLE I. Theoretical pair and triplet cross sections at 319 Mev.

	Hydrogen mb/atom	Carbon mb/atom
Pair (Bethe and Heitler)	7.72	251.4
Triplet (Wheeler and Lamb)	8.05	46.7
Triplet (Corrected)	7.68	44.4

C. Choice Targets

It is desirable to have the triplet cross section as large as possible with respect to the nuclear pair cross section. Since the former has a Z dependence and the latter a Z^2 dependence, one chooses as small a Z as is practical. Hydrogen is the obvious choice, since at 319 Mev the two effects are about equal in magnitude, each contributing about 40% of the total absorption cross section, with the Compton effect giving 18% and photomeson production the remainder. Unfortunately, hydrogen in gas or liquid form is difficult to work with, since the attenuation would be very small with existing targets, and the hydrogen density cannot be precisely determined.

One could use other elements in liquid or solid form, but benzene (C_6H_6) and cyclohexane (C_6H_{12}) have some definite advantages which ultimately led to their being selected. They are readily available in high-purity samples, and they are free of the explosion hazard and general handling problems inherent in hydrogen targets. Their hydrogen density, comparable to that of liquid hydrogen, can be determined to within a few parts in 10^4. Unfortunately, their use requires a subtraction type of experiment to determine any hydrogen cross section. However, one simultaneously obtains the total absorption cross section in carbon to a much higher precision than would be possible with a graphite target, owing to the density uncertainties in the latter.

From the total absorption cross section in carbon, one can also determine the triplet cross section. It is true that carbon lacks the simplicity of hydrogen, in which the number of possible competing processes is limited, but the competing reactions in carbon (other than nuclear pair, triplet, and Compton effect) still amount to less than 2% of the total. At the same time, the triplet effect contributes about 12%. Thus one has the possibility of measuring the triplet cross section to even higher accuracy than is possible with hydrogen. Owing to screening, which is a function of the atomic number, this will not be simply six times the cross section of hydrogen.

D. Target Assembly

The two hydrocarbons were contained in identical aluminum cylinders, 29 inches long and 2.5 inches in diameter, with 5-mil aluminum end windows. The absorber length was chosen so as to give approximately $1/e$ attenuation of the gamma-ray beam at 319 Mev.

Table II. Triplet cross sections in hydrogen and in carbon at 319 Mev.

Interaction	Hydrogen cross section (mb/atom)	Error	Carbon cross section (mb/atom)	Error
Nuclear pair (theoretical)	7.72		251.4	
Electron Compton (theoretical)	3.04		18.3	
Photomeson (experimental)	0.45	50%	2.7	factor of 2
Photostar (experimental)	···		1.5	50%
Nuclear photoeffect (experimental)	···		0.6	factor of 3
Total (less triplet)	11.21±0.23		274.5±5.7	
Experimental total-absorption cross section	18.0 ±1.8		321.1±2.8	
Experimental triplet cross section	6.8 ±1.8		46.6±6.4	
Theoretical triplet cross section	7.68		44.4	

Since the method of the experiment was to measure the ratio of the number of 319-Mev quanta that penetrate the absorber to the number reaching that point with no absorber, it was necessary to have a third target. This dummy target was identical with the other two except that it was evacuated to less than 15 microns. The dummy target provides a true measure of the target-out condition, and eliminates the necessity of subtracting the absorption due to the Al windows, and of correcting for the displaced air column.

V. RESULTS AND CONCLUSIONS

A. Data

A total of 1 500 000 coincidence counts were taken in the course of three independent runs over a period of one year. The results of all runs were consistent within statistics.

The principal data consist of observations of photon beam intensities transmitted by each of the three targets, and the results of interest are the two independent ratios of their intensities. Effects of slow non-periodic variations in the experimental conditions were canceled to first order by observing the three intensities in cyclic order with a cycle period of one hour.

Because the absorbers attenuated the beam by a factor of $1/e$ at 319 Mev, the flux through the pair spectrometer in the desired energy interval would vary over a factor of almost 3 on switching from absorber to dummy target. In order to assure that the spectrometer was always operating at the same efficiency, the synchrotron beam intensity was varied so that the flux through the spectrometer was maintained at a constant value.

B. Cross Sections

The total absorption cross sections at 319 Mev in hydrogen and carbon were found to be 18.0±1.8 mb per hydrogen atom and 321.1±2.8 mb per carbon atom. The errors listed are root-mean-square errors on counting statistics and the best estimate of systematic errors. The temperature of the targets was monitored and the data were corrected for density variation and change in target length. Over all, these latter corrections amounted to less than 0.1%. The accidentals were subtracted from the total counts; and they amounted to less than 0.5%.

The calculations of the experimental triplet cross sections in hydrogen and carbon at 319 Mev are summarized in Table II, and the theoretical cross sections are given for comparison. It is to be noted that, with the exception of nuclear pair production, pair production in the field of the orbital electrons, and Compton effect, all other absorption processes in carbon contribute but 1.5% of the total absorption cross section. This amounts to 10% of the total triplet cross section in carbon, and when the uncertainties in some of these other cross sections are reduced the data will then be available for the calculation of a better value for the triplet cross section in carbon.

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