question arises whether there is any means of discriminating between (0−) and (2−). The fact that no polarization was observed is weak, but inconclusive, evidence against nonzero spin. There is no hope of discriminating against (2−) by means of the likelihood ratio because the shape of f_{2-} can be made to resemble f_{0-} very closely for certain values of the arbitrary parameter A/B (see Table I). In Fig. 1 f_{2-} is plotted for the choice $A/B=1$. The small differences between f_{2-} and f_{0-} in this plot could be made to overlap by letting the interaction have a small energy dependence. Thus in principle it is impossible to rule out (2−) by a Dalitz-type analysis. The distribution function f_{2-} also has an arbitrary plus or minus sign which is related to the relative phases of the two spherical waves ($l=0$, $L=2$) and ($l=2$, $L=0$). We assume this relative phase is limited to either 0 or 180 degrees by time reversal arguments.[17] Since the term following this plus or minus sign is so small compared to the other terms, either choice of sign can give $P_{2-}/P_{0-}\sim 1$.

The (3+) distribution function also contains A/B and a plus or minus sign as arbitrary parameters. Because of the common factor ϵ in f_{3+}, it is impossible for P_{3+}/P_{0-} to be ~ 1 for any choice of the arbitrary parameter here. This and higher spin states can be analyzed by finding the maximum-likelihood solution[11] for the arbitrary parameters and using those values to get the relative probability. In the case of f_{3+} we went through this procedure and found $A/B=1.4$ for the maximum-likelihood solution, with the minus sign giving a much better fit. Using just the Columbia data, this gave the relative probability $P_{3+}/P_{0-}=10^{-2}$. The values of $A/B=1.0$ and 1.6 reduced this relative probability a factor 10. We conclude that (3+) is not ruled out for the τ meson, but that it is unlikely.

In summary, we feel that the only reasonable possibilities left for the τ meson are (0−) and (2−) with (3+) and (4−) as weak possibilities. (1+) is strongly ruled out, as are all possibilities which permit two-pion decay. The effects of the centrifugal barrier and conservation of strangeness should rule out all higher spins—at least for spins above ~ 5. The data are statistically quite consistent with f_{0-} and can be made consistent with f_{2-} and f_{4-} when the arbitrary parameters in these distributions are so adjusted. The lack of any indication of pion-pion interaction effects can be used to set an upper limit on the s-wave scattering length for the pion-pion interaction. This upper limit turns out to be $a_0<R$ where R is the range of the pion-pion interaction.

[17] This is strictly true in the absence of pion-pion forces, but is probably a good approximation in our case. We wish to thank T. D. Lee and V. L. Telegdi for bringing this simplification to our attention.

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Symmetries in Isotopic Spin Space and the Charge Operator

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A general relation is shown to exist between the charge and the operator inducing a symmetry with respect to the 1,2 plane in isotopic spin space. This relation is unique, i.e., it is the same for all types of fields (baryons and mesons).

THE connection between the charge Q and the third component I_3 of isotopic spin is well known. However, the appearance of an additive constant in the relation $Q(I_3)$ and, above all, the fact that this constant must be chosen differently for each type of field have long been a kind of puzzle for some physicists.

The experimental finding of the hyperons and heavy mesons and the discovery of the fact that their main properties are well accounted for by the Gell-Mann model[1] made this question even more acute but at the same time offered some hints to a possible answer. Under the assumption that the strong interaction Hamiltonians are (a) of the Yukawa type, and (b) invariant not only under rotations but also under re-

[1] M. Gell-Mann, Phys. Rev. **92**, 833 (1953); M. Gell-Mann and A. Pais, *Proceedings of the Glasgow Conference on Nuclear and Meson Physics*, 1954 (Pergamon, London, 1955), p. 342.

flections in isotopic spin space, it could be shown[2,3] that all the possible Hamiltonians constructed with isoscalar and isospinor fields of the first and second kind and isopseudovector fields[4] commute with a certain operator U. Furthermore, this new constant U is precisely the one that appears in the equation[5]

$$Q=-I_3+\tfrac{1}{2}U, \tag{1}$$

connecting I_3 and the charge. The values of U postulated by Gell-Mann could thus be derived from the theory. Moreover, a limitation of the admissible kinds of fields was thereby suggested[3] for, if one enlarges the list of fields given above (if, e.g., one includes isopseudoscalar or isovector fields in the set), an operator U commuting with all Hamiltonians satisfying (a) and (b) no longer exists.

These considerations have recently been reformulated in a simpler way by Racah.[6] The advantage of Racah's approach is that one need not write down the Hamiltonians explicitly. He simply points out that the Cartan spinors ξ of the first and second kind are transformed to $i\xi$ and $-i\xi$, respectively, by an inversion through the origin, while scalars and pseudovectors remain of course unchanged. Starting from the thus defined "parity" $p(p=i,\ -i,\ 1,\ 1)$, this enables him to define U through

$$p=i^U. \tag{2}$$

It is now clear that in any strong interaction [satisfying (b)] the total U is either conserved or changed by no less than $4n$ (n being an integer). It can be seen immediately that with Yukawa Hamiltonians and the restricted list of fields given above, U is in fact always conserved: the results of both approaches are therefore identical as they should be.

Formally there is an analogy between the definition (2) of U, written as

$$p=e^{iU\pi/2}, \tag{2$'$}$$

and the following well-known operator identity: if $A(\alpha)$ is, for a given field type, the operator that induces a rotation α around the third axis in isotopic spin space, one has, quite generally,

$$A(\alpha)=e^{-iI_3\alpha}. \tag{3}$$

What we would like to point out in this note is that the charge operator Q can be expressed in a very similar way: in fact it is immediately seen by (1), (2′), and (3) that

$$pA(\pi)=A(\pi)p=e^{iQ\pi}. \tag{4}$$

Now $pA(\pi)$, the product of a rotation of 180° around the third axis by an inversion through the origin, is just a symmetry with respect to the (1,2) plane in isotopic spin space. If, therefore, for any given field type, B is the operator which induces such a symmetry, the charge operator for this same field type is given by the general relation:

$$e^{iQ\pi}=B. \tag{5}$$

As is the case for (2′) and (3), relation (5) is formally the same for every type of field, in contradistinction with the relation connecting Q with, e.g., rotation around the third axis. Equation (5) therefore indicates that, broadly speaking, the charge is naturally linked with the symmetry with respect to the (1,2) plane in much the same way as I_3 and U are naturally linked with the rotations around the third axis and the inversion through the origin, respectively. For isospinors of the first and second kind, and isopseudovectors,

$$B=\tau_3, -\tau_3, \quad \text{and} \quad \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix},$$

respectively, while

$$Q=\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} -1 & & \\ & 0 & \\ & & 1 \end{pmatrix}$$

for $\mathfrak{N}$, Ξ, and (π or Σ), respectively. The direct check of the validity of (5) in these cases is of course elementary.

It may finally be mentioned that relations of the form

$$\mathcal{O}=\exp(iC^{st}\mathcal{O}_1)$$

between operators $\mathcal{O}_1$ associated with physical quantities and operators $\mathcal{O}$ associated with geometrical transformations are not surprising, the former operators being Hermitian while simple geometrical transformations usually lead to unitary operators.

[2] B. d'Espagnat and J. Prentki, Phys. Rev. **99**, 328 (1955).

[3] B. d'Espagnat and J. Prentki, Compt. rend. **240**, 2486 (1955); CERN Report 55-11 (CERN Ed., Geneva, 1955); Nuclear Phys. **1**, 33 (1956).

[4] For a precise definition of spinors of the first and second kinds in 3-dimensional space, see E. Cartan, *Leçons sur la théorie des spineurs I* (Hermann and Cie, Paris, 1938) or reference 3.

[5] Contrary to what was done in references 2 and 3 we revert here to the older convention of attributing, e.g., $I_3=-\frac{1}{2}$ to the proton, $I_3=+\frac{1}{2}$ to the neutron.

[6] G. Racah, Nuclear Phys. **1**, 302 (1956). Our best thanks are due Professor Racah for communicating his manuscript to us before publication.