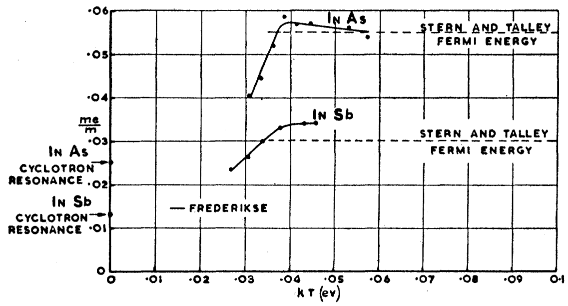

FIG. 1. Variation of effective electron mass with mean electron energy for indium arsenide and indium antimonide.

the band the mass ratio in both materials tends to a constant value, while for lower energies the mass ratio falls towards the cyclotron resonance values[1,2] found for very pure specimens at low temperatures. Frederikse[3] has reported an effective electron mass for indium antimonide between 160°K and 200°K from thermoelectric power measurements on pure specimens at low temperatures, and his values are indicated on the diagram. Hrostowski *et al.*[4] have found $m_e/m=0.015$ from electrical measurements on a pure indium antimonide specimen in a similar temperature range. On the other hand, Stern and Talley[5] have used an overlapping impurity band model to explain the shift of absorption edge with impurity concentration in degenerate specimens of InSb and InAs. Their theory would seem to represent an approximate fit with observation for values of Fermi energy greater than about 0.035 ev taking $m_e/m=0.055$ for InAs and $m_e/m=0.03$ for InSb.

We suggest that the mass found from cyclotron resonance may be best associated with the bottom of the impurity band alone since the bulk of the electrons will have energies below the bottom of the conduction band for very pure samples at low temperatures. For high electron concentrations, the majority of the electrons will have energies above the bottom of the conduction band and will be associated with an effective mass corresponding to the sum of the state densities in both impurity and conduction bands. Our measurements would appear to cover some of the range between these two regions. This explanation can only hold for the p-type sample of InSb quoted above if there is some degree of compensation in this sample.

This work is part of an investigation of the electrical properties of InAs and InSb, a full account of which will appear elsewhere.[6] Thanks are due to Dr. Avery (R.R.E., Malvern) for supply of InSb specimens.

[1] Dresselhauss, Kip, Kittel, and Wagoner, Phys. Rev. **98**, 556 (1955).
[2] C. Kittel, Phys. Rev. **98**, 1542 (T) (1955).
[3] H. P. R. Frederikse and E. V. Mielczarek, Phys. Rev. **99**, 1889 (1955).
[4] Hrostowski, Morin, Geballe, and Wheatley, Phys. Rev. **100**, 1672 (1955).
[5] F. Stern and R. M. Talley, Phys. Rev. **100**, 1638 (1955).
[6] Antell, Champness, Cohen, and Chasmar, Report on the Spring Meeting of the British Physical Society (to be published).

Exact Sum Rules in the Fixed-Source Meson Theory

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(Received April 20, 1956)

We want to report in this letter some exact relations obtained by making use of recent techniques connecting matrix elements of Heisenberg operators and scattering amplitudes[1] and to discuss their implications about the nature of the solution of the fixed-source meson theory. By starting from the relation[2]

$$\langle l\alpha;r|\operatorname{Im}T(\omega_p)|l'\alpha';r'\rangle = -\tfrac{1}{3}p^3v^2(p)f_0^2(\Psi_r,\,\tau_\alpha\sigma_l\delta(\omega_p-H)\tau_{\alpha'}\sigma_{l'}\Psi_{r'}) \quad (1)$$

connecting the element of the transition matrix T between pion-nucleon states (the subscript r denotes the four single-nucleon states, l and α the angular momentum and isotopic spin components of the meson) with the matrix elements of the source operators between physical single-nucleon states Ψ_r, and adopting the representation in which the total angular momentum J and the total isotopic T are diagonal, the following sum rules can be obtained:

$$4\alpha_3+4\alpha_2+\alpha_1=(1-r_2^2)f_0^2, \quad (2a)$$

$$\alpha_3-2\alpha_2+\alpha_1=(r_2-r_2^2)f_0^2, \quad (2b)$$

$$2\alpha_3-\alpha_2-\alpha_1=(r_2^2-r_1)f_0^2, \quad (2c)$$

where

$$\alpha_i=\frac{1}{3\pi}\int_1^\infty \frac{d\omega_p\operatorname{Im}g_i(p)}{p^3v^2(p)}. \quad (3)$$

In Eq. (3), $v(p)$ is the cutoff function; $g_i(p)$ is the eigenvalue of the transition matrix $-\pi T$ in the state $i[i=1(j=\tfrac{1}{2};\ T=\tfrac{1}{2});\ i=2(J=\tfrac{1}{2};\ T=\tfrac{3}{2})(J=\tfrac{3}{2};\ T=\tfrac{1}{2});\ i=3(T=\tfrac{3}{2};\ J=\tfrac{3}{2})]$. As is well known, for purely elastic scattering,

$$g_i=e^{i\delta_i}\sin\delta_i.$$

The constant r_2 is the ratio f/f_0 between renormalized and unrenormalized coupling constants. Both r_1 and r_2 can be defined by

$$\begin{aligned} r_1&=(\Psi_1,\sigma_z\Psi_1)=(\Psi_1,\tau_z\Psi_1),\\ r_2&=(\Psi_1,\sigma_z\tau_z\Psi_1), \end{aligned} \quad (4)$$

where Ψ_1 is the physical proton state with spin up.

In Eqs. (2), the expressions for g_i in the one-meson approximation given by Chew and Low for a theory with a square cutoff at ξ can be introduced, and g_1 and g_2 can be neglected with respect to g_3. Then one obtains

$$\frac{1}{6\pi}\int_1^\xi \frac{\sin^2\delta_3}{p^3}d\omega_p=f^2. \quad (5)$$

Numerical evaluation of the integral in Eq. (5) gives

$$f^2=0.013,$$

in strong disagreement with the experimental value $f^2=0.107\pm0.010$.[3]

This shows that agreement between theory and experiment in the low-energy region can only be obtained if high-energy contributions are dominant. This means that the usual viewpoint, according to which the cutoff factor of the fixed-source theory should make all the high-energy contributions negligible, is not correct.[4]

Two different situations may give rise to these high-energy contributions: (a) if the one-meson approximation is a good approximation to the exact solution, then only form factors $v(p)$ which go to zero sufficiently slowly as $p\rightarrow\infty$ are admissible; (b) if many-meson states contribute appreciably at high energies, there also a square cutoff can be accepted.

We derive now another sum rule according to which alternative (a) can be ruled out.

It can be shown that

$$(\Psi_r,a_\alpha^*(\mathbf{k})a_{\alpha'}(\mathbf{k})\Psi_r)$$
$$=f_0^2\frac{2\pi v^2(k)}{\omega_k}\sum_{ll'}k_lk_{l'}\left(\Psi_r,\sigma_l\tau_\alpha\frac{1}{(\omega_k+H)^2}\sigma_{l'}\tau_{\alpha'}\Psi_r\right), \quad (6)$$

where $a_\alpha(\mathbf{k})$ is a destruction operator of a meson with momentum $\mathbf{k}$ and isotopic spin component α.

Again using Eq. (1), one obtains for the total meson charge Q and the total number of mesons N in the cloud of a proton the expressions

$$Q=\frac{2e}{\pi}\int_0^\infty\frac{k^4dk}{\omega_k}v^2(k)\times\left[\frac{f^2}{\omega_k^2}+\gamma_1(\omega_k)+\gamma_2(\omega_k)-2\gamma_3(\omega_k)\right], \quad (7)$$

$$N=\frac{3}{\pi}\int_0^\infty\frac{k^4dk}{\omega_k^2}v^2(k)\times\left[\frac{f^2}{\omega_k}+\gamma_1(\omega_k)+4\gamma_2(\omega_k)+4\gamma_3(\omega_k)\right], \quad (8)$$

where

$$\gamma_i(\omega_k)=\frac{1}{3\pi}\int_1^\infty\frac{\mathrm{Im}g_i(\omega_p)}{v^2(p)p^3(\omega_k+\omega_p)^2}d\omega_p, \quad (9)$$

and e is the unit of charge.

The high-energy contributions to the integrals (9) are negligible because the convergence factor $(\omega_k+\omega_p)^2$ in the denominator insures that their value is determined with very good accuracy by the low-energy region. This has been proved in detail in a recent analysis of the Low equation in which comparison with experiment allows the determination of f^2 without any need to make use of the one-meson approximation.[4]

By imposing charge conservation, one gets

$$Q=\tfrac{1}{2}(1-r_1)e. \quad (10)$$

On the other hand, $r_1>-\frac{1}{3}$ for charge independence and angular momentum conservation.[5] This gives

$$Q\leqslant\tfrac{2}{3}e. \quad (11)$$

This condition, together with Eq. (7), gives a limitation on the cutoff function $v(k)$, where the quantities $\gamma_i(\omega_k)$ are evaluated by inserting in the integrals (9) the experimental values of $\mathrm{Im}g_i$. Since $v^2(p)\approx1$ at least up to $\omega_p\approx4$, one finds that, in order to fulfill condition (11), $v^2(p)$ cannot go to zero slowly enough for $\omega_p\rightarrow\infty$ to ensure that the one-meson approximation satisfies Eqs. (2). This is sufficient to rule out alternative (a).

With a square cutoff at ξ, Eqs. (7) and (11) give

$$\xi\leqslant5.5. \quad (12)$$

The average number of mesons in the cloud can be evaluated in this case also. Equation (8) gives

$$N\approx1.65. \quad (13)$$

This result suggests that two-meson states should be important. In this situation the only possible behavior of the exact solution is the one summarized by alternative (b).

[1] Lehmann, Symanzyk, and Zimmermann, Nuovo cimento **1**, 1 (1955); M. L. Goldberger, Phys. Rev. **97**, 508 (1955); F. E. Low, Phys. Rev. **97**, 1392 (1955); Y. Nambu, Phys. Rev. **98**, 803 (1955); M. Cini and S. Fubini, Nuovo cimento **2**, 860 (1955).

[2] See, e.g., M. Cini and S. Fubini, Nuovo cimento **3**, 764 (1956); S. Fubini, Nuovo cimento (to be published). In the present work, all energies and momenta are measured in terms of the meson mass μ.

[3] Cini, Fubini, and Stanghellini, Nuovo cimento (to be published).

[4] W. Thirring (private communication) has shown that the following inequality must hold rigorously: $\alpha_2>\alpha_3/4$. Since, in the energy range below the cutoff, $\delta_{31}\ll\delta_{33}$, the preceding inequality can be fulfilled only if strong contributions arise in α_2 and α_3 from the high-energy region.

[5] W. Thirring (private communication).

Errata

Photoelastic Observations of the Expansion of Alkali Halides on Irradiation, W. Primak, C. J. Delbecq, and P. H. Yuster [Phys. Rev. **98**, 1708 (1955)]. The photoelastic strain constant is related to the photoelastic stress constant through the modulus of rigidity, not Young's modulus; for from the last equation on p. 1712,

$$R=r\left(\frac{Y_y-c_{12}\Delta}{c_{11}-c_{12}}-\frac{X_x-c_{12}\Delta}{c_{11}-c_{12}}\right)$$
$$=(Y_y-X_x)r/(c_{11}-c_{12}).$$

For the determination of r, X_x is zero. The value of r for the data given is about 0.18, not 0.22. The agreement