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Magnetic Moment of the 247-kev Excited State of Cd^{111} †*‡

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The magnetic moment of the 247-kev ($I=5/2$) excited state of Cd^{111} was determined from the effects of an external magnetic field on the directional correlation of the Cd^{111} gamma rays. Two methods of measurement were employed. The observation of the *attenuation* of the directional correlation yielded for the magnetic moment $|\mu_{5/2}|=(0.80\pm0.04)$ nm, taking into account the time-dependent quadrupole interaction in the liquid sources which were used for this measurement. The more accurate method of determining the *azimuthal shift* of the correlation pattern about the axis of the magnetic field gave a more precise value of the magnetic moment, $\mu_{5/2}=-(0.783\pm0.023)$ nm.

1. INTRODUCTION

CONSIDERABLE attention has been devoted recently to the study of magnetic moments of nuclei. The experimental material on the magnetic moments of nuclear ground states[1] has played an important role in the formulation of theories of nuclear structure based on the independent-particle model with strong spin-orbit interaction[2,3] and its extensions, including mixed configurations of nuclear states,[4] or on the collective aspect of nuclear motion.[5–7] This role is due partly to the fact that magnetic moments can be measured very accurately—in fact, besides nuclear masses, they are the most precisely measured nuclear quantities—and partly to their relatively simple relationship to the structure of nuclei. The latter statement, however, does not imply that we can predict exactly the value of the magnetic moment of a given nuclear state, because the actual magnetic moment vector $\boldsymbol{\mu}$ is not a constant of the motion as is the nuclear angular momentum vector $\mathbf{I}$. The significant numerical (or "effective") value $\boldsymbol{\mu}_I$ of the magnetic moment is given by the expectation value $\langle\boldsymbol{\mu}_z\rangle$ of the z-component of the magnetic moment operator $\boldsymbol{\mu}_{\text{op}}$ calculated for the state of the nuclear system for which the total angular momentum $\mathbf{I}\hbar$ has its maximum projection along the z-axis ($m=I$). One often refers to this quantity simply as the "magnetic moment" $\boldsymbol{\mu}_I$. The direction of this vector quantity coincides with the nuclear axis $\mathbf{I}$. If the wave functions $\Psi(I)$ of the nuclear states were known the expectation value $\langle\boldsymbol{\mu}_z\rangle$ could easily be calculated using the well-known methods of quantum mechanics. Whereas the parities, and to a certain extent the angular momenta I, of states can be predicted on the basis of a rather qualitative and partial knowledge of the wave function of nuclear states, the computation of $\langle\boldsymbol{\mu}_z\rangle$ requires the exact knowledge of Ψ including the amplitudes of normally rather insignificant admixture components.

Since we have two nuclear constituents—neutrons and protons—with different intrinsic magnetic moments and since the nuclear magnetic moments depend on the distribution of the intrinsic and the orbital angular momentum between neutrons and protons, the nuclear magnetic moment, in general, cannot be expressed in terms of a simple Landé g factor as is possible for an atomic state. Consequently, any calculation of $\boldsymbol{\mu}_I$ requires the assumption of a rather detailed nuclear model and the comparison with experimental magnetic moments provides a very sensitive test of the validity of such a model.[8,9]

One further complication should be kept in mind. It is the fact that when several nucleons interact with each other the meson field which is responsible for this interaction may be different from that of a free nucleon and thus intrinsic magnetic moments might be altered if a nucleon is brought into interactions with other nucleons in a bound nuclear state.[10] However, some evidence against this quenching theory of nuclear magnetic moments has been pointed out recently by de-Shalit.[11]

In view of these considerations it is desirable to have accurate information of magnetic moments of nuclei in their excited states. Such measurements are within the realm of present-day experimental techniques if the lifetime of the excited state under consideration is of such a magnitude that directional correlations involving this state as an intermediate step can be influenced

† A brief account of this work was given previously: R. M. Steffen and W. Zobel, Phys. Rev. 97, 1188 (1955).
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[3] Haxel, Hensen, and Suess, Z. Physik **128**, 295 (1950).
[4] A. de-Shalit and M. Goldhaber, Phys. Rev. **92**, 1211 (1953).
[5] D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953).
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[7] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).
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[10] F. Bloch, Phys. Rev. **83**, 839 (1951).
[11] A. de-Shalit, Phys. Rev. **90**, 83 (1953).

by external magnetic fields.[12–14] From a precise measurement of this effect, the magnitude and sign of the magnetic moments can be extracted.[15,16] Few magnetic moments of excited states have been measured with this method, i.e., Cd^{111},[17] Pb^{204},[18] Ta^{181},[19] Np^{237},[20] and none had been determined with an accuracy good enough to draw any quantitative conclusions at the time this work was started.

The magnetic moment $\mu_{5/2}$ of the 247-kev excited state of Cd^{111} ($I=5/2$) was the first magnetic moment of an excited state of a nucleus measured. In their pioneer work the Swiss group[17] obtained the value $\mu_{5/2}=(-0.70\pm0.12)$ nm. The error in this measurement was too large to decide whether the magnetic moment of the excited state is different from that of the ground state ($I_0=1/2$): $\mu_{1/2}=-(0.59492\pm0.00008)$ nm.[1] Furthermore, the measurements were made with sources where the static quadrupole interaction was considerable, a fact which was not known at the time and therefore no allowance was made for it. During the course of the work described in this paper the Swiss group repeated the measurements of the magnetic moment $\mu_{5/2}$ with greatly improved accuracy.[21] Their result $\mu_{5/2}=(-0.725\pm0.047)$ nm, however, was not corrected for a possible time-dependent quadrupole interaction in the liquid sources which were used for these measurements.

In the present investigation use was made of two different methods to determine the magnetic moment of the 247-kev Cd^{111} state:

(i) The magnetic moment was extracted from the measured attenuation of the directional correlation of the Cd^{111} gamma cascade displayed by liquid sources as a function of the externally applied magnetic field. Owing to the interaction of the nuclear quadrupole moment with the fluctuating electric gradients in the liquid, which is not entirely negligible in sources of this kind,[22] the value of $\mu_{5/2}$ obtained in this way must be corrected accordingly.

(ii) The second method, which gives a result unaffected by the presence of a quadrupole interaction, if it is not too strong, made use of the relationship between the magnetic moment $\mu_{5/2}$, the magnitude of the external magnetic field H and the azimuthal shift of the correlation pattern about H as the axis.

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[18] V. Krohn and S. Raboy, Phys. Rev. **97**, 1017 (1955).
[19] S. Raboy and V. Krohn, Phys. Rev. **95**, 1689 (1954).
[20] Krohn, Novey, and Raboy, Phys. Rev. **98**, 1187 (1955).
[21] Albers-Schönberg, Heer, Novey, and Scherrer, Helv. Phys. Acta **27**, 547 (1954).
[22] R. M. Steffen, Phys. Rev. **102**, 116 (1956), preceding paper. This paper is referred to as I and its notation employed here.

2. INFLUENCE OF A STATIC MAGNETIC FIELD ON DIRECTIONAL CORRELATIONS IN THE PRESENCE OF A TIME-DEPENDENT ELECTRIC QUADRUPOLE INTERACTION

The influence of a static magnetic field on a directional correlation can be understood by a simple semiclassical model. A static field $\mathbf{H}$ exerts a torque on the magnetic dipole $\boldsymbol{\mu}_I$ of the intermediate nuclear state of angular momentum $\mathbf{I}\hbar$. The resulting Larmor precession of the nuclear axis $\mathbf{I}$ about $\mathbf{H}$, with angular frequency

$$\omega_m=\mu_I H/I\hbar, \tag{1}$$

causes a reorientation of the nuclei in the intermediate state and consequently alters the directional correlation. The directional correlation, which, if unperturbed by extranuclear fields, is given by (compare I)

$$W^0(\Theta)=1+\sum_{k=1}^{k_{\max}} A_{2k}{}^0 P_{2k}(\cos\Theta), \tag{2}$$

will then not depend simply upon the angle Θ between the propagation directions $\mathbf{k}_1$ and $\mathbf{k}_2$ of the two cascade radiations but will be a function of the orientation of $\mathbf{k}_1$ and $\mathbf{k}_2$ with respect to the axis of the external magnetic field $\mathbf{H}$. Calculations of the influence of the external magnetic field on directional correlations, neglecting electron shell interactions ($I-J$ hyperfine coupling) have been made by Alder[23] and Lloyd.[24] If the polar angles θ_1, φ_1, and θ_2, φ_2 specify the directions of $\mathbf{k}_1$ and $\mathbf{k}_2$ in a coordinate system where $\mathbf{H}$ is in the z axis, then the directional correlation is given by

$$W(\theta_1\varphi_1,\theta_2\varphi_2)=\sum_{k=0}^{k_{\max}}\sum_{l=-k}^{l=+k}\frac{A_{2k}{}^0}{4k+1}e^{-i\omega_m 2lt}Y_{2k}{}^{2l}(\theta_1\varphi_1)Y_{2k}{}^{-2l}(\theta_2\varphi_2), \tag{3}$$

where the $Y_{2k}{}^{2l}(\theta,\varphi)$ are spherical harmonics and t is the time during which the intermediate nuclear state is exposed to the magnetic interaction. If, in particular, the external magnetic field $\mathbf{H}$ is perpendicular to the correlation plane (defined by the propagation directions $\mathbf{k}_1$ and $\mathbf{k}_2$ of the two nuclear radiations) the directional correlation function becomes especially simple and useful. For symmetry reasons the correlation becomes again a function of the angle $\Theta=\varphi_1-\varphi_2$ and the time t only. As compared to the unperturbed correlation (2), the perpendicular magnetic field causes an azimuthal shift, $\phi=\omega_m t$, of the correlation pattern about $\mathbf{H}$:

$$W(\Theta,\omega_m,t)=W^0(\Theta-\omega_m t)=1+\sum_{k=1}^{k_{\max}} A_{2k}{}^0 P_{2k}[\cos(\Theta-\omega_m t)], \tag{4}$$

if the second radiation is observed at the time t after the emission of the first radiation. If, in addition to the

[23] K. Alder, Helv. Phys. Acta **25**, 235 (1952).
[24] S. P. Lloyd, Phys. Rev. **82**, 277 (1951).

external magnetic field H, a time-dependent quadrupole interaction (e.g., in a liquid source) is present, the A_{2k}^0 must be multiplied with attenuation coefficients[25]

$$G_{2k}(t)=e^{-\lambda_{2k}t} \tag{5}$$

(compare I) and the directional correlation is then given by

$$W(\Theta,\omega_m,t)=1+\sum_{k=1}^{k_{\max}} A_{2k}^0 e^{-\lambda_{2k}t} \quad P_{2k}[\cos(\theta-\omega_m t)]. \tag{6}$$

In practice, a directional correlation is determined by means of a coincidence analyzer which accepts the second nuclear radiation within a finite time interval, τ_0, the resolving time, after the detection of the first radiation. If no artificial delay is introduced into the coincidence channel which detects the first radiation, the directional correlation observed is the weighted average of $W(\Theta,\omega_m,t)$ [Eq. (6)] over the time interval $t=0$ to $t=\tau_0$:

$$\begin{aligned}\bar{W}_0(\Theta,\omega_m)&=\int_0^{\tau_0} W(\Theta,\omega_m,t)e^{-\lambda t}dt \Big/ \int_0^{\tau_0} e^{-\lambda t}dt\\ &=1+\frac{\lambda}{e^{-\lambda\tau_0}-1}\sum_{k=1}^{k_{\max}} A_{2k}^0\int_0^{\tau_0} e^{-(\lambda+\lambda_{2k})t}\\ &\qquad\times P_{2k}[\cos(\Theta-\omega_m t)]dt.\end{aligned} \tag{7}$$

$\lambda=1/\tau_N$ is the decay constant of the intermediate nuclear state. It must be recognized that this expression is correct only under the condition that each of the two radiation detectors accepts but one of the radiations involved in the nuclear cascade. In case the two detectors cannot distinguish between the first and the second radiation one must average over positive *and* negative values of ω_m, i.e., all terms in Eq. (7) which change sign if ω_m is replaced by $-\omega_m$ must be omitted. The magnetic interaction no longer causes a rotation of the correlation pattern, but a mere attenuation:

$$\bar{W}_0(\Theta,\omega_m)_{\mathrm{sym}}=1+\sum_{k=1}^{k_{\max}} \bar{A}_{2k}(\omega_m)P_{2k}(\cos\Theta), \tag{8a}$$

where the $\bar{A}_{2k}(\omega_m)$ are complicated functions of ω_m, λ_{2k}, τ_0, and A_{2k}^0. The explicit expression of $\bar{A}_2(\omega_m)$ for the case where higher A_{2k}^0 terms can be neglected, becomes

$$\bar{A}_2(\omega_m)=\frac{A_2^0}{1+\left(\dfrac{2\omega_m\tau_N}{1+\lambda_2\tau_N}\right)^2}\times\frac{4}{\dfrac{4(1-e^{-\lambda\tau_0})(1+\lambda_2\tau_N)+A_2^0(1-e^{-(\lambda+\lambda_2)\tau_0})}{1+e^{-(\lambda+\lambda_2)\tau_0}\left(\dfrac{2\omega_m\tau_N}{1+\lambda_2\tau_N}\sin2\omega_m\tau_0-\cos2\omega_m\tau_0\right)}-A_2^0}. \tag{8b}$$

[25] A. Abragam and R. V. Pound, Phys. Rev. 92, 943 (1953).

It is evident that from a measurement of $\bar{A}_{2k}$ as a function of the magnitude of an external magnetic field H, the numerical relationship between H and ω_m can be established and by virtue of (1) the magnetic moment μ_I of the intermediate state can be extracted. This method requires, essentially, the measurement of the attenuation of the correlation anisotropy in the magnetic field, hence we call it the "*attenuation method*." For a precise determination of μ_I, however, either the time-dependent quadrupole interaction in the source, characterized by λ_{2k}, or the unperturbed correlation (the coefficients A_{2k}^0) must be known accurately from other measurements. One might surmise that for the case where more than one coefficient A_{2k}^0 is important in the expansion of the unperturbed directional correlation function the quadrupole interaction can be eliminated if the H dependence of several of the coefficients $\bar{A}_{2k}(\omega_m)$ is known. In general, however, this proves to be a highly inaccurate procedure.

It was pointed out by Abragam and Pound[25] that an accurate μ_I determination is possible, without detailed knowledge of a (weak) quadrupole interaction in the source, if the magnetic field is kept at a constant value H_0 and the coincidence rate at a fixed angle, Θ_0 say, which is proportional to $W(\Theta_0-\omega_m t)$, is measured as a function of t. An oscillating function of t is then observed. The determination of the period $2\pi/\omega_m=2\pi I\hbar/\mu_I H_0$ thereof, then allows one to determine μ_I directly. Similarly, $W(\Theta_0-\omega_m T)$ determined with a constant time delay T is represented by an oscillating function of H with a period of $2\pi I\hbar/\mu_I T$. Again, the measurement of the particular values H_l of the magnetic field which correspond to l complete periods of $W(\Theta_0-\omega_m T)$ (or known fractions thereof) allows one to compute directly the magnetic moment. From an experimental point of view the latter method is simpler than to vary t and was adopted for our measurements. Since the azimuthal shift of the correlation pattern about the magnetic field is observed with this technique, we call it the "*azimuthal-shift*" method. Taking into account the finite resolving time τ_0 of the coincidence analyzer which records the second radiation between $T-\tau_0$ and $T+\tau_0$ after the emission of the first, and if the delay in one channel is given by T, the correlation function becomes

$$\bar{W}_T(\Theta,\omega_m)=\int_{T-\tau_0}^{T+\tau_0} W(\Theta-\omega_m t)e^{-\lambda t}dt \Big/ \int_{T-\tau_0}^{T+\tau_0} e^{-\lambda t}dt. \tag{9}$$

If the time-dependent quadrupole interaction is small ($\lambda_{2k}\tau_N<1$), the period of the time-averaged function $\bar{W}_T(\Theta_0,\omega_m)$ as a function of ω_m (with constant T) is almost independent of the quadrupole interaction parameters λ_{2k}. If the $P_2(\cos\Theta)$ is the only important term in the expansion of the unperturbed correlation

function, $\bar{W}_T(\Theta,\omega_m)$ is given by the following expression:

$$\bar{W}_T(\Theta,\omega_m)=1+\frac{A_2^0}{4}\frac{1}{1+\lambda_2\tau_N}\frac{\sinh[(\lambda+\lambda_2)\tau_0]}{\sinh[\lambda\tau_0]}e^{-\lambda_2 T}$$
$$+\frac{3}{8}\frac{A_2^0}{1+\lambda_2\tau_N}\frac{1}{1+\left(\dfrac{2\omega_m\tau_N}{1+\lambda_2\tau_N}\right)^2}\frac{e^{-\lambda_2 T}}{\sinh[\lambda\tau_0]}$$
$$\times\Big\{e^{(\lambda+\lambda_2)\tau_0}\cos2[\Theta-\omega_m(T-\tau_0)]$$
$$-e^{-(\lambda+\lambda_2)\tau_0}\cos2[\Theta-\omega_m(T+\tau_0)]$$
$$+\left(\frac{2\omega_m\tau_N}{1+\lambda_2\tau_N}\right)e^{(\lambda+\lambda_2)\tau_0}\sin2[\Theta-\omega_m(T-\tau_0)]$$
$$-\left(\frac{2\omega_m\tau_N}{1+\lambda_2\tau_N}\right)e^{-(\lambda+\lambda_2)\tau_0}\sin2[\Theta-\omega_m(T+\tau_0)]\Big\}. \quad (10)$$

3. APPARATUS AND PREPARATION OF SOURCES

3.1 Geometry of the Detector Arrangement

Three scintillation detectors (NaI(Tl) crystals canned and mounted on DuMont 6292 Photomultiplier tubes) were used to observe the directional correlation of the Cd^{111} gamma rays in an external magnetic field. Two of the detectors, I and II, are in fixed positions, and the third detector III is movable. A schematic diagram of the detector arrangement is shown in Fig. 1.

3.2 Electronics

The electronics part of the equipment—the block diagram is shown in Fig. 2—is rather conventional. The resolving time of the two coincidence analyzers can be varied from 80 to 200 millimicroseconds. The use of three channels permitted two simultaneous measurements of the directional correlation and thus improved the accuracy of the experiment.

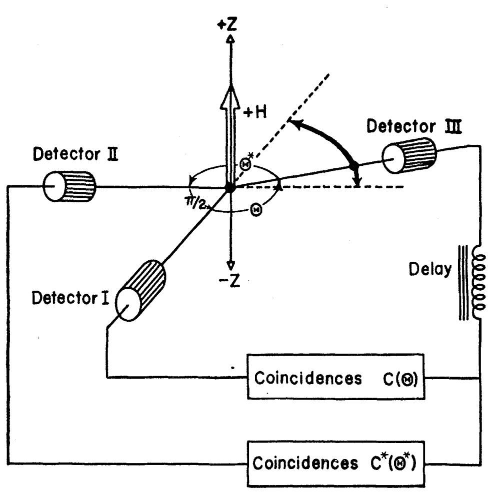

FIG. 1. Schematic diagram of the detector arrangement. (The attenuation measurements were made without delay in channel III.)

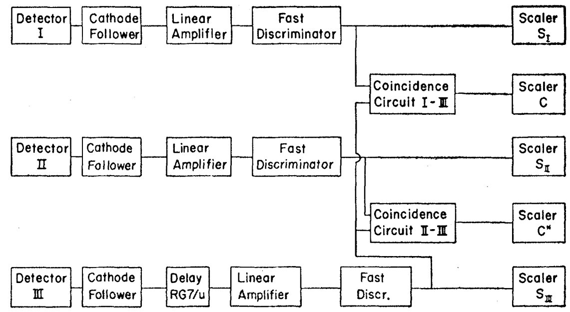

FIG. 2. Block diagram of the electronics.

For the attenuation measurements, where no delay was employed, the integral pulse height discriminators in all three channels were set just below the photopeak of the 172-kev gamma radiation of Cd^{111}. This gamma-energy discrimination insures freedom from effects which result from backscattered radiation and eliminates, to a large extent, effects due to large-angle Compton-scattering of the gamma radiation. The crystals were selected and adjustments made in such a manner that the ratio of the detection efficiencies for the 172-kev and the 247-kev gamma ray was the same in all the channels. In this way the symmetry of the detection arrangement was the same for both gamma rays—an important requirement for the attenuation method.

For the azimuthal shift measurements a delay of (375 ± 3) millimicroseconds was inserted into channel III. The delay was determined accurately by a high-frequency oscillator and an oscilloscope. To improve the genuine-to-chance coincidence rate ratio for these measurements, the discriminators in the channels without delay (channels I and II) were set just below the 247-kev photopeak.

3.3 The Magnet

A high-voltage low-current electromagnet provided the magnetic field at the radioactive source. The field over the extension of the source was uniform within 0.2% up to fields of 7750 oersteds. The current which energized the magnet was stabilized and kept the magnetic field constant within 0.5% over a fifty-hour period. A proton resonance fluxmeter was used to determine the magnitude of the magnetic field within 0.2%.

Elaborate shielding of the photomultipliers with Mu metal was necessary to eliminate any influence of the fringing field of the electromagnet on the detectors. Small-angle Compton scattering of the gamma rays at the pole pieces of the magnet, which are close to the radioactive source, might be expected to disturb the measured correlation. Using the gamma-energy discrimination employed for the attenuation measure-

ments as discussed above, the experimental A_2 ($H=0$) coefficient of the Cd^{111} directional correlation measured with the magnet in position differed from the value measured with the magnet removed by less than 0.003. For lower energy discrimination ($E_{\text{Disc.}} \simeq 50$ kev), however, the difference in the experimental values of A_2 may be as large as 0.02.

3.4 Preparation of the Radioactive In^{111} Sources

The radioactive sources used for the μ_I measurements were dilute aqueous solutions of $InCl_3$ and were prepared in the same way as described in I. Thus the information obtained there as to the presence of the time-dependent quadrupole interaction in sources of this kind should be immediately applicable to an interpretation of the results of the experiments described here.

4. ATTENUATION OF THE Cd^{111} DIRECTIONAL CORRELATION IN A MAGNETIC FIELD

4.1 Experimental Procedure and Corrections

The experimental determination of the directional correlation function $\bar{W}_0(\Theta,H)$ requires the measurement of the coincidence rate $C(\Theta,H)$ (corrected for chance coincidences, decay of the source, background, etc.) between two detectors (of finite size) as a function of the angle Θ subtended at the source by the axes of the two detectors. $C(\Theta,H)$ is proportional to $\bar{W}_0'(\Theta,H)$, the time-integrated directional correlation function, $\bar{W}_0(\Theta,H)$, smeared out over the finite solid angle subtended by the two detectors and, if the extension of the radioactive source cannot be neglected, over the finite volume of the latter.

Neglecting the source correction and assuming cylindrical symmetry of the detectors, Frankel[26] has shown that $\bar{W}_0'(\Theta)$ is of the same form as the theoretical directional correlation $\bar{W}_0(\Theta)$, but each term in the Legendre polynomial expansion (2) must be multiplied with a geometry-correction factor Q_{2k} ($Q_{2k}<1$)[27,28]:

$$\bar{W}_0'(\Theta)=1+\sum_{k=1}^{k_{\max}} Q_{2k}A_{2k}P_{2k}(\cos\Theta). \tag{11}$$

The Q_{2k} are given by[28]

$$Q_{2k}=q_{2k}/q_0, \tag{12a}$$

$$q_{2k}=\int_{\text{Detector 1}} \epsilon_1(\vartheta_1)P_{2k}(\cos\vartheta)\sin\vartheta d\vartheta \times\int_{\text{Detector 2}} \epsilon_2(\vartheta_2)P_{2k}(\cos\vartheta)\sin\vartheta d\vartheta, \tag{12b}$$

where $\epsilon_i(\vartheta_i)$ is the efficiency of detector i at the angle ϑ_i between the counter axis and the direction of propagation of the incident radiation.

Strictly speaking, this method of correction is valid only if the directional correlation depends only upon the angle Θ between the propagation directions of the two cascade radiations. In the case of an external magnetic field, however, where the field H is applied normal to the detector axes and where the radiation in the divergent beam striking the finite size detectors is, in general, not exactly perpendicular to the direction of H, the theoretical correlation function which must be smeared out over the extension of the detectors is of the general form (3) and $\bar{W}_0'(\Theta,H)$ must be calculated from

$$\bar{W}_0'(\Theta,H)=\frac{\int \bar{W}(\theta_1\varphi_1\theta_2\varphi_2)\epsilon_1(\theta_1\varphi_1)\epsilon_2(\theta_2\varphi_2)\sin\theta_1\sin\theta_2 d\theta_1 d\theta_2 d\varphi_1 d\varphi_2}{\int \epsilon_1(\theta_1\vartheta_1)\epsilon_2(\theta_2\varphi_2)\sin\theta_1\sin\theta_2 d\theta_1 d\theta_2 d\varphi_1 d\varphi_2}. \tag{13}$$

For symmetry reasons $\bar{W}_0'(\Theta,H)$ is again a function of Θ and H only, but its form is considerably more complicated than Eq. (11). However, numerical calculations showed that the error introduced by using Eq. (11) instead of Eq. (13) is negligibly small, at least for the geometry and the fields used in these experiments. Thus, the much simpler correction method of Frankel, Eq. (11), was used throughout. The detector efficiencies $\epsilon_i(\vartheta_i)$ were determined experimentally by means of well collimated gamma-ray beams of the same energy as used in the correlation experiments.[29] It must be kept in mind, however, that the simplified procedure described above gives a good approximation only if, instead of the actually measured magnetic field H, the average normal component to the correlation plane $\bar{H}$ is used for the evaluation of the data. $\bar{H}$ is slightly different from H due to the divergence of the gamma radiation to the finite size detectors. In addition a very small diamagnetic correction must be applied. Allowance made for both effects results in $\bar{H}=0.988H$.

A typical run for one value of H was of twenty hours length. From the corrected coincidence rate measured at five different angles Θ the correlation coefficient $\bar{A}_2(\bar{H})$ for a particular H was obtained from a least-square fit of the experimental points to a function of the form $\bar{W}_0(\Theta,\bar{H})=\text{const}(1+Q_2\bar{A}_2(\bar{H})P_2(\cos\Theta))$. A_4 is negligibly small for the Cd^{111} gamma-gamma cascade (compare I) and higher terms do not exist. Some repre-

[26] S. Frankel, Phys. Rev. **83**, 673 (1951).
[27] A. M. Feingold and S. Frankel, Phys. Rev. **97**, 1025 (1955).
[28] M. E. Rose, Phys. Rev. **91**, 610 (1953).
[29] J. S. Lawson and H. Frauenfelder, Phys. Rev. **91**, 649 (1953).

sentative experimental correlation curves are shown in Fig. 3.

4.2 Computation of $\mu_{5/2}$

The experimental values of $\bar{A}_2(\bar{H})$ obtained for magnetic field strengths up to 7700 oersted are shown in Fig. 4. The data are plotted as a function of the average magnetic field $\bar{H}$. In order to extract from these experimental points a value for $\mu_{5/2}$ a comparison with the theoretically predicted $\bar{A}_2(\omega_m)$ [Eq. (8b)] must be made.

The computation of this function $\bar{A}_2(\omega_m)$ requires knowledge of A_2^0, λ_2, and τ_N. The values of A_2^0, λ_2, and τ_N are taken from I and from Simms and Steffen,[30] respectively: $A_2^0=-0.180\pm0.002$, $\lambda_2=(5.6\pm1.6)\times10^6$ sec^{-1}, $\tau_N=(122.3_{-0.8}^{+1.8})\times10^{-9}$ sec. The resolving time τ_0 used in the attenuation measurements was $\tau_0=(174\pm2)$ millimicroseconds. Inspection of Eq. (8b) shows that, whereas the errors of A_2^0, λ_2, and τ_0 give small contributions to the error of the final result of $\mu_{5/2}$, the uncertainty in the lifetime τ_N seriously affects the value obtained for $\mu_{5/2}$. $\bar{A}_2(\omega_m)$ was computed and plotted *versus* ω_m and then the $\bar{H}$ scale of the experimental curve $\bar{A}_2(\bar{H})$ adjusted (by trial and error) to obtain the least-square fit to the calculated $\bar{A}_2(\omega_m)$. In this way the experimental constant of proportionality be-

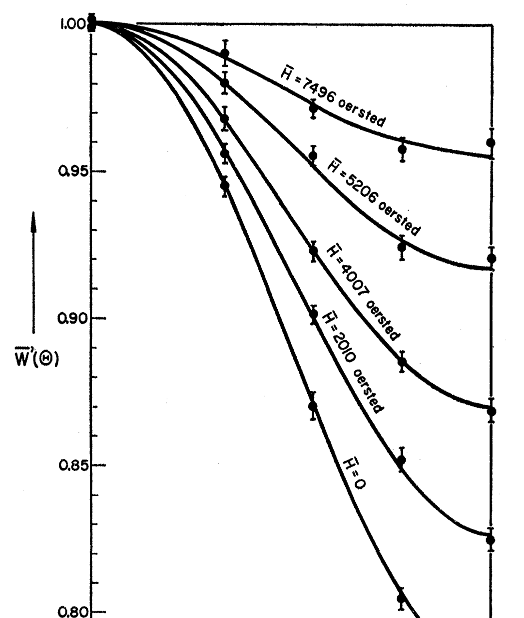

FIG. 3. The directional correlation $\bar{W}_0'(\Theta,\bar{H})$ of the Cd^{111} gamma cascade observed with perpendicular magnetic fields $\bar{H}$ of different magnitude. (The curves were normalized to $\bar{W}_0'(90°)=1$ *after* the least-square fit calculation.)

[30] P. C. Simms and R. M. Steffen, Bull. Am. Phys. Soc. Ser. II, **I**, 207 (1956), and to be published in *The Physical Review*.

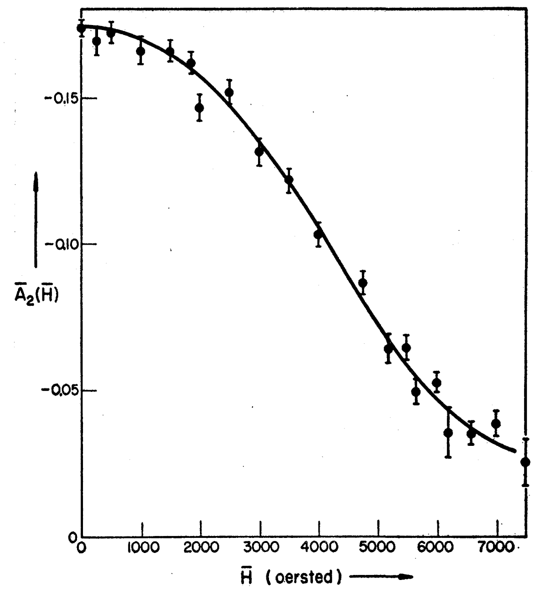

FIG. 4. The experimental correlation coefficient $\bar{A}_2(\bar{H})$ as function of the magnetic field $\bar{H}$. The curve corresponds to a magnetic moment of $|\mu_{5/2}|=0.80$ nm.

tween ω_m and $\bar{H}$ was determined as

$$|\omega_m/\bar{H}|=(1535\pm75)\ \text{sec}^{-1}\ \text{oersted}^{-1},$$

from which we obtain at once, by using Eq. (1), the magnitude of the magnetic moment of the 247-kev excited state of Cd^{111}

$$|\mu_{5/2}|=(0.80\pm0.04)\ \text{nm}\quad(\text{standard error}).$$

By far the largest contribution to the error is due to the uncertainty in τ_N and λ_2.

5. AZIMUTHAL SHIFT OF THE DELAYED Cd^{111} DIRECTIONAL CORRELATION IN A MAGNETIC FIELD

5.1 Method of Measurement

To determine the azimuthal shift as precisely as possible with the available experimental equipment the following technique was used. The three detectors were kept in fixed positions, such that the axes of the detectors I and III subtended an angle of $\Theta_0=3\pi/4$ at the source (Fig. 1). We denote by $C(3\pi/4,+\bar{H})$ the (corrected) coincidence counting rate observed by these two detectors for a magnetic field $\bar{H}$ in the $+z$ direction, and $C(3\pi/4,-\bar{H})$ is the same but for $\bar{H}$ in the $-z$ direction. Obviously, for $\bar{H}=0$ the ratio $R=C(3\pi/4,+\bar{H})/C(3\pi/4,-\bar{H})$ is unity. The same is true for all values of $\bar{H}=\bar{H}_l$, which cause the directional correlation pattern to rotate by an azimuthal angle of $\phi\cong l\pi/2$ (l=integer). The same argument holds for the ratio $R^*(\bar{H})=C^*(3\pi/4,-\bar{H})/C^*(3\pi/4,+\bar{H})$, where $C^*(3\pi/4,\pm\bar{H})$ are the coincidence counting rates of the detector pair II and III, whose axes also subtend an angle of $\Theta_0^*=3\pi/4$ (compare Fig. 1). In

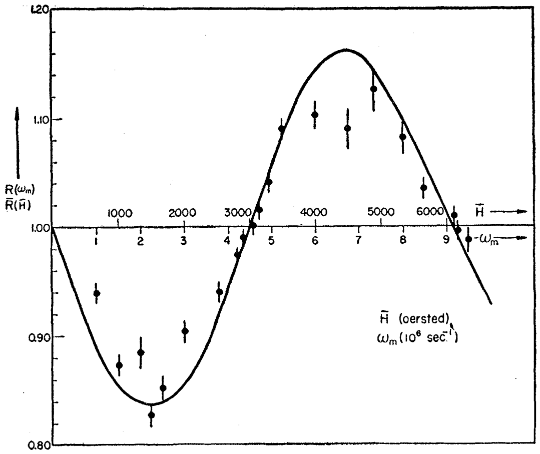

FIG. 5. The ratio R calculated as function of the Larmor frequency ω_m, and the experimentally measured ratio $\bar{R}(\bar{H})$ as function of the magnetic field $\bar{H}$.

fact, for perfect symmetry and for any field $\bar{H}$, $R^*=R$. From a comparison of the experimentally determined values $\bar{H}_l$ of the magnetic field for which $R(\bar{H}_l)=1$ or $R^*(\bar{H}_l)=1$, with the theoretical predictions, the magnetic moment can be computed. It is evident [compare Eq. (6)] that, for a delay T and infinitely short resolving time τ_0 of the equipment, R and R^* become unity for the azimuthal shift angles $\phi_l=\omega_m{}^lT=(\bar{H}_l\mu_I/I\hbar)T=l\pi/2$.

5.2 Experimental Procedure

The ratios $R(\bar{H})$ and $R^*(\bar{H})$ were determined as a function of the magnetic field strength $\bar{H}$ with a constant delay of $T=(375\pm3)$ millimicroseconds. The resolving time of the coincidence analyzers used for these measurements was $\tau_0=(98\pm2)$ millimicroseconds.

A typical run for one value of $\bar{H}$ took about two days. The magnetic field was reversed about every hour and care was taken to have the same magnitude of the magnetic field in the two opposite directions. The ratios R and R^* were then determined from the coincidence counting rates measured with magnetic field up and with magnetic field down.

5.3 Results

In Fig. 5 the experimental points for $\bar{R}(\bar{H})=(w^*R^*+wR)/(w^*+w)$, where w^* and w are the weights of the measured values of R^* and R, respectively, are plotted as a function of $\bar{H}=0.988H$. No correction for finite detector size was made in computing the experimental points for $\bar{R}(\bar{H})$.

From Eq. (10) $R(\omega_m)$ can be calculated as a function of the Larmor frequency ω_m. The explicit expression of this quantity $R(\omega_m)$ for $\Theta_0=3\pi/4$ is given by

$$R(\omega_m)=\frac{\bar{W}_T(\frac{3}{4}\pi,+\omega_m)}{\bar{W}_T(\frac{3}{4}\pi,-\omega_m)}=\frac{1+0.25\dfrac{A_2{}^0}{1+\lambda_2\tau_N}\dfrac{\sinh[(\lambda+\lambda_2)\tau_0]}{\sinh[\lambda\tau_0]}e^{-\lambda_2T}-0.375\dfrac{A_2{}^0}{1+\lambda_2\tau_N}\dfrac{e^{-\lambda_2T}}{\sinh[\lambda\tau_0]}r(\omega_m)}{1+0.25\dfrac{A_2{}^0}{1+\lambda_2\tau_N}\dfrac{\sinh[(\lambda+\lambda_2)\tau_0]}{\sinh[\lambda\tau_0]}e^{-\lambda_2T}+0.375\dfrac{A_2{}^0}{1+\lambda_2\tau_N}\dfrac{e^{-\lambda_2T}}{\sinh[\lambda\tau_0]}r(\omega_m)}, \tag{14a}$$

where

$$r(\omega_m)=\frac{1}{1+\left(\dfrac{2\omega_m\tau_N}{1+\lambda_2\tau_N}\right)^2}\Bigg[e^{(\lambda+\lambda_2)\tau_0}\sin[2\omega_m(T-\tau_0)]+\frac{2\omega_m\tau_N}{1+\lambda_2\tau_N}e^{(\lambda+\lambda_2)\tau_0}\cos[2\omega_m(T-\tau_0)]$$
$$-e^{-(\lambda+\lambda_2)\tau_0}\sin[2\omega_m(T+\tau_0)]-\frac{2\omega_m\tau_N}{1+\lambda_2\tau_N}e^{-(\lambda+\lambda_2)\tau_0}\cos[2\omega_m(T+\tau_0)]\Bigg]. \tag{14b}$$

The function $R(\omega_m)$, corrected for the finite size of the detectors, is shown as a solid curve in Fig. 5. The same values of $A_2{}^0$, λ_2, and τ_N as in the previous calculation of $\bar{A}_2(\omega_m)$ were used. The values of $\bar{H}_1$ and $\bar{H}_2$ for which $R(\bar{H})$ becomes unity are practically independent of the quadrupole interaction parameter λ_2. The adjustment of the $\bar{H}$ scale to the ω_m-scale, such that the experimental points $\bar{R}(\bar{H})$ provide a least-square fit to the function $R(\omega_m)$, establishes the relationship between $\bar{H}$ and ω_m from which the magnetic moment $\mu_{5/2}$ can be computed:

$$\omega_m/\bar{H}=-(1510\pm30)\ \text{sec}^{-1}\ \text{oersted}^{-1}.$$

In particular, the experimentally determined values of $\bar{H}_l$, where $\bar{R}(\bar{H}_l)=1$ are given in Table I.

From these data together with the direction of the azimuthal shift, we extract the magnitude and sign for the magnetic moment $\mu_{5/2}$:

$$\mu_{5/2}=-(0.783\pm0.023)\ \text{nm}\quad(\text{standard error}).$$

The agreement with the result of the attenuation

TABLE I. Experimental values of $\bar{H}_l$ for azimuthal shifts Φ_l of $\pi/2$ and π.

l	$\bar{H}_l$ oersted	$\omega_m{}^l$ sec^{-1}	Approximate value of corresponding azimuthal shift Φ_l of correlation pattern
1	3030 ± 60	-4.59×10^6	$\pi/2$
2	6210 ± 100	-9.26×10^6	π

method is satisfactory. It is felt, however, that the result obtained with the azimuthal shift method is more reliable, since it is almost independent of the presence of any weak quadrupole interaction in the source. On the other hand, the fact that the two methods give results consistent with each other is an indirect indication that the time-dependent quadrupole interaction in the dilute $InCl_3$ solution is correctly described by $\lambda_2\tau_N=0.07$ (compare I).

6. DISCUSSION

The magnetic moment of the 247-kev excited state of Cd^{111} is considerably larger than the magnetic moment of the ground state, $\mu_{1/2}=-(0.59492\pm0.00008)$. The single-particle shell model without configuration mixing predicts for the magnetic moments of odd-proton and odd-neutron states of angular momentum I (the Schmidt limits)[8]:

$$(\mu_I)_{\mathrm{sp}}=\frac{g_l+(g_s-g_l)}{2l+1},\quad I=l\pm\tfrac{1}{2},$$

where l is the orbital quantum number of the odd particle, and $g_s=5.585$ or -3.826, and $g_l=1$ or 0, are the spin and the orbital g factors for the proton and neutron, respectively. According to this much oversimplified model, the magnetic moment of the ground state as well as of the first excited state of Cd^{111} should be due to an odd neutron with its spin parallel to its orbital angular momentum. Both states would have the same moments, $(\mu_I)_{\mathrm{sp}}=-1.91$. By taking into account configuration mixing, calculated on the basis of simple perturbation theory and estimates of the two-body interaction strengths and integrals determined from the empirical data on pairing energies, Arima and Horie[31] computed the magnetic moments of the Cd^{111} states. Their result, $\mu_{1/2}=-0.49$ and $\mu_{5/2}=-0.70$ for the moments of the ground and first excited state of Cd^{111}, respectively, agree fairly well with the experimental values.

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[31] A. Arima and H. Horie, Progr. Theoret. Phys. (Japan) **12**, 623 (1955).