

Selection Rules for $N\bar{N}$ Annihilation*

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Selection rules governing the annihilation of a nucleon-antinucleon pair into pions, photons, and heavy mesons are derived from the conservation of angular momentum, parity, charge parity, and isotopic spin. Special attention is given to the probability of annihilation purely into neutral pions.

WE consider annihilation of nucleonium, i.e., a nucleon-antinucleon pair, from a state of definite angular momentum, parity, isotopic spin, and charge parity (if neutral) into systems of pions, photons, and heavy mesons.

STATES OF NUCLEONIUM

A state of a weakly interacting nucleon-antinucleon pair can be described by the quantum numbers of total angular momentum j , spin s , orbital angular momentum l , and by a specification of the isotopic spin states, i.e., $p\bar{n}$, $n\bar{p}$, $p\bar{p}$, or $n\bar{n}$. From these we can express the parity $\mathcal{P} = -(-)^l$, the isotopic spin $i=1$ or 0 , the charge parity (in the case of neutral nucleonium, i.e., $i_z=0$) $C = (-)^{s+l}$, and the "CT" parity¹ $CT = (-)^{s+l+i}$. If the pair is interacting strongly, then s , l , and the "isotopic spin specification" are no longer good quantum numbers, but \mathcal{P} , i , and CT (and C if $i_z=0$) are good quantum numbers if we neglect electromagnetic interaction. For instance, if we call a nucleonium state 3P_2 we really mean the state having $j=2$, $i=1$, $\mathcal{P}=+1$, $CT=-1$ (and $C=+1$ if $i_z=0$).

DECAY INTO PIONS

The pion has CT parity -1 , so that an N -pion state has $CT = (-)^N$. The pion has an intrinsic spatial parity -1 , so that an N -pion state has $\mathcal{P} = (-)^N \mathcal{P}_{\text{orbital}}$. For two pions, j equals l because the pion is spinless, therefore $\mathcal{P} = (-)^j$; for three pions having $j=0$, $\mathcal{P}_{\text{orbital}}$ equals $+1$ and therefore $\mathcal{P} = -1$.

Thus a nucleonium state having $CT = \{+, -\}$ can decay into an $\{\text{even, odd}\}$ number of pions; with the qualifications that two pions are impossible if $(-)^j \mathcal{P} = -1$ (which is always the case for spin-singlet nucleonium), and three pions are impossible if the state is $(0+)$. Selection rules based on CT are not absolute

in the presence of electromagnetic interaction, but decays violating these selection rules will be slower than the corresponding radiative decays.¹

Table I lists the least number of pions into which each nucleonium state can decay; a state that decays into some (even or odd) number of pions can decay into any larger (even or odd) number of pions also. For states of higher odd or even l , the table would be precisely the same as given for P or D state respectively, except that the forbidding of three mesons from 3P_0 is not repeated, since the corresponding higher l states do not have $j=0$. If for a given l one weights the four spin states equally, and either weights the four isotopic spin states equally ("mixed" nucleonium, e.g., \bar{p} incident on D) or gives equal weight to the two $i_z=0$ states ("neutral" nucleonium, e.g., \bar{p} incident on H) one finds the ratios of the number of states with least pion numbers of 2, 3, 4, or 5 given in Table II.

If the annihilation takes place in a small volume, then low orbital angular momenta in the final state will be favored. The lowest- l configurations are listed in Table I for the least pion number states; the lowest- l configuration for a state of $2n$ additional pions simply has an additional factor of s^{2n} .

It is of some interest to estimate the proportion of decays leading to pure π^0 states, since these will yield the entire nucleonium rest mass as visible energy in a Čerenkov counter. Since $C=+1$ for a system of neutral pions, and (for a neutral system) we can write $CT = (-)^i C = (-)^N$, a system of N neutral pions has $i = \{0, 1\}$ if N is $\{\text{even, odd}\}$. If we neglect the space state of the N -pion final state, and thus assume that all isotopic spin states are equally populated, we can deduce the probability P_N that an N -pion state is a pure π^0 state, given that $i_z=0$ and $i = \{0, 1\}$ if N is $\{\text{even, odd}\}$.

N	2	3	4	5	6	7	8	9	10
P_N	0.3333	0.2000	0.0667	0.0286	0.0095	0.0037	0.0012	0.0005-	0.0001+

This table is derived in the appendix. Then, still neglecting the effects of space states, we can write the probability P_0 that neutral nucleonium, decaying into pions, decays into neutral pions only:

$$\begin{aligned} \text{even } l, > 2: & P_0 = \sum \frac{1}{8} P_N (\alpha_N^{(3)} + \alpha_N^{(4)}), \\ \text{odd } l, > 1: & P_0 = \sum \frac{1}{8} P_N (2\alpha_N^{(2)} + 3\alpha_N^{(3)} + \alpha_N^{(4)}), \\ \text{average } l: & P_0 = \sum \frac{1}{8} P_N (\alpha_N^{(2)} + 2\alpha_N^{(3)} + \alpha_N^{(4)}), \end{aligned}$$

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

¹ See A. Pais and R. Jost, Phys. Rev. **87**, 871 (1952). C is the eigenvalue of charge conjugation, and T is the eigenvalue of a rotation of 180° in isotopic spin space around an axis transverse

to the 3 or "charge" axis. Only systems with $i_z=0$ have a T parity, and then $T = (-)^i$. CT is the product of C and T , and is useful in that, in the usual representation of the τ matrices [where, under C : $\tau_{1,3} \rightarrow \tau_{1,3}$, $\tau_2 \rightarrow -\tau_2$], if the T -rotation is taken to be

TABLE I. Characteristics of the allowed final states from nucleonium annihilation.

Initial nucleonium state	K mesons and pions							
	C CT		Pions ^a	K \bar{K} ^b	Scheme A: the θ is (0+)		Scheme B: the θ is (1-) ^d	
					like K mesons	unlike K mesons	$\theta\bar{\theta}$	$\frac{\tau\bar{\theta}}{\theta\bar{\tau}}$
3S_1	-	+	$2p$	like	p_0p	$\begin{pmatrix} sp \\ ps \end{pmatrix}$	p	p
1S_1	-	-	$3p^2$	like	p_1p	$\begin{pmatrix} s_1p \\ p_1s \end{pmatrix}$	p	p
3S_0	+	-	$3s^2$	unlike	s_0s		p	p
1S_0	+	+	$4p^3$	unlike	s_1s		p	p
$^3P_{0,1,2}$	+	-	$5sp^3; 3sp; 3pd$	like; unlike; like	$-; \begin{pmatrix} s_0p \\ p_1s \end{pmatrix}; \begin{pmatrix} p_1d \\ d_0p \end{pmatrix}$	$ss; p\bar{p}; \begin{pmatrix} sd \\ p\bar{p} \\ d\bar{s} \end{pmatrix}$	$s; d; s$	$-; s; d$
$^1P_{0,1,2}$	+	+	$2s; 4sp^2; 2d$	like; unlike; like	$-; s_1p; d_1p$	$s_1s; p_1p; \begin{pmatrix} s_1d \\ p_1p \\ d_1s \end{pmatrix}$	$s; d; s$	$-; s; d$
3P_1	-	+	$4sp^2$	unlike	$\begin{pmatrix} s_1p \\ p_0s \end{pmatrix}$	$p\bar{p}$	s	s
1P_1	-	-	$3sp$	unlike	p_1s	p_1p	s	s
$^3D_{1,2,3}$	-	+	$2p; 4\begin{pmatrix} p^3 \\ spd \end{pmatrix}; 2f$	like; unlike; like	$p_0p; \begin{pmatrix} s_0d \\ p_0p \\ d_0s \end{pmatrix}; d_1d$	$\begin{pmatrix} sp \\ ps \end{pmatrix}; \begin{pmatrix} p\bar{d} \\ d\bar{p} \end{pmatrix}; \begin{pmatrix} p\bar{d} \\ d\bar{p} \end{pmatrix}$	$p; p; p$	$p; p; f$
$^1D_{1,2,3}$	-	-	$3p^2; 3\begin{pmatrix} p^2 \\ sd \end{pmatrix}; 3d^2$	like; unlike; like	$p_1p; p_1p; f_1f$	$\begin{pmatrix} s_1p \\ p_1s \end{pmatrix}; \begin{pmatrix} p_1d \\ d_1p \end{pmatrix}; \begin{pmatrix} p_1d \\ d_1p \end{pmatrix}$	$p; p; p$	$p; p; f$
3D_2	+	-	$3\begin{pmatrix} p^2 \\ sd \end{pmatrix}$	unlike	p_1p	$\begin{pmatrix} p\bar{d} \\ d\bar{p} \end{pmatrix}$	p	p
1D_2	+	+	$4\begin{pmatrix} p^3 \\ spd \end{pmatrix}$	unlike	d_1d	$\begin{pmatrix} p_1d \\ d_1p \end{pmatrix}$	p	p

^a The prefixed number is the least possible number of pions; the lowest- l configuration is given for this least number state.

^b "Like" signifies that the pair $K\bar{K}$ must be either $\theta\bar{\theta}$ or $\tau\bar{\tau}$; "unlike" that the pair is $\theta\bar{\tau}$ or $\tau\bar{\theta}$.

^c The lowest- l configuration is given, the first letter being the orbital state of the $K\bar{K}$ pair, with its isotopic spin as a subscript; the second letter is the orbital state of the pion around the K -meson pair. Where the isotopic spin of the $K\bar{K}$ pair can be either 0 or 1, it is not stated.

^d The lowest l orbital state is given.

where the $\alpha_N^{(m)}$ are the branching ratios for decay into N pions of a state that decays into at least m pions ($N-m$ even). The estimation of the α_N by the Lepore-Neuman statistical theory² indicates them to be rapidly decreasing functions of N ; i.e., $\alpha_{m+2}^{(m)} \ll \alpha_m^{(m)}$, so that $\alpha_N^{(m)} \approx \delta_{Nm}$. Further, since the phase space for a final state of two heavy mesons (see next section) is nearly the same as for two pions, the at-least-four-pion decay states would not decay into pions on this theory. Taking the estimations² $\alpha_2^{(2)} = 3/7$, $\alpha_3^{(3)} = \frac{1}{2}$, and all others zero, we find 0.033 for the probability that neutral nucleonium decays (directly) into π^0 's only.

around the 2-axis all three pions have odd CT parity. If the T -rotation is taken to be around some other axis in the 1-2 plane, the charged pions will have a different CT parity, which is much less convenient [Pais and Jost did in fact take the 1-axis, which makes $CT=+1$ for the charged pions]. Note added in proof.—T. D. Lee and C. N. Yang [Nuovo cimento 3, 749 (1956)] have also used the CT operator (G in their notation) to derive the selection rules for annihilation of nucleonium into pions.

² Lepore, Neuman, and Stuart, Phys. Rev. 94, 788 (1954). See the forthcoming paper by J. V. Lepore and D. H. Holland.

DECAY INTO HEAVY MESONS

We assume Gell-Mann's scheme, assigning $i=\frac{1}{2}$ to both the $\theta(\rightarrow 2\pi)$ and the $\tau(\rightarrow 3\pi)$, which we call collectively K mesons. The spin and parity of the two are known to the following extent. The θ must have $\mathcal{P}=(-)^i$ in order to decay into two pions, and further, j must be even for θ^0 to decay into two neutral pions.³ The τ seems to be $(0-)$ on the basis of Dalitz's analysis of the observed correlation in the final state. We shall accept the τ as $(0-)$ and consider the possibilities that the θ is either $(0+)$ or $(1-)$, calling the first assumption $[\theta \text{ is } (0+)]$ scheme A, the second $[\theta \text{ is } (1-)]$ scheme B.

³ J. Osher (unpublished) has evidence from an experiment similar to that of G. B. Collins [Proceedings of the Fifth Annual Rochester Conference on High-Energy Physics (Interscience Press, New York, 1955), p. 139] that the $\theta^0 \rightarrow \pi^0 + \pi^0$ decay does occur, with lifetime and abundance of the same order as for $\theta^0 \rightarrow \pi^+ + \pi^-$. Furthermore, from the theoretical point of view, if the θ has odd spin its two-pion final state would necessarily have $i=1$, so that the different lifetimes of θ^+ and θ^0 would be inexplicable under charge independence.

TABLE II. Relative numbers of nucleonium states decaying into each pion multiplicity.

$N_{\min} \backslash l$	"Mixed" nucleonium				"Neutral" nucleonium			
	0	1	even >0	odd >1	0	1	even or odd >1	
2	4.5	1	3	1	3	2	2	
3	1.5	4.5	3	5	3	$3\frac{3}{2}$	4	
4	0.5	2	2	2	1	2	2	
5	1.5	0.5	0	0	1	$\frac{1}{2}$	0	

In either case, a state of two K mesons has $i=0$ or 1. In Scheme A , a pair $\theta\bar{\theta}$ or $\tau\bar{\tau}$ (which we call *like* pairs) has $\mathcal{P}=(-)^l=(-)^i$, and $CT=(-)^{l+i}$ [and $C=(-)^l$ if $i_z=0$]. The pairs $\theta\bar{\tau}$ or $\tau\bar{\theta}$ (*unlike* pairs) are presumably interconvertible, as there is no selection-rule bar between them, and thus we can form linear combinations having $\mathcal{P}=-(-)^l$, and $CT=\pm 1$. In Scheme B , a pair $\theta\bar{\theta}$ has spin states $s_\theta=0, 1$, or 2, so that $CT=(-)^{l+s_\theta+i}=(-)^{s_\theta+i\mathcal{P}}$ can be ± 1 for a given i, j and \mathcal{P} . The unlike pairs in Scheme B have $\mathcal{P}=(-)^l$. Thus for a given $j \neq 0$, \mathcal{P} can be either ± 1 (CT likewise); but if $j=0$, then $l=s=1$, therefore $\mathcal{P}=-1$.

Thus in Scheme A , a state having $\mathcal{P}=\{+, -\}(-)^i$ can decay into a {like, unlike} pair.⁴ In Scheme B , any state can decay into a pair $\theta\bar{\theta}$; any state except $(0+)$ can decay into an unlike pair $\theta\bar{\tau}$ or $\tau\bar{\theta}$. A pair $\tau\bar{\tau}$ is the same in both schemes.

Any state can decay into a pair of K mesons plus one or more pions. The lowest- l configuration, along with the isotopic spin of the $K\bar{K}$ pair, is given in Table I.

The proportion of neutral nucleonium decaying into heavy mesons that go ultimately into neutral pions can be easily estimated. In Scheme A , the θ^0 would decay into charged or neutral pions in a ratio between 2:1 and 1:2, depending on the ratio of $i=0$ to $i=2$ in the final state; if charge independence is invoked, the hundred-times-shorter lifetime of θ^0 compared to θ^+ implies that θ^0 decays almost exclusively through $i=0$.⁵ Therefore the probability that a neutral $\theta\bar{\theta}$ pair will decay into four neutral pions is 1/18. We now have to estimate the proportion of like $K\bar{K}$ that are $\theta\bar{\theta}$ pairs. If at high energies " θ " particles and " τ " particles [i.e., $(0+)$ and $(0-)$ K mesons] are produced in equal numbers, the observed ratio of θ^+ , τ^+ , and $K_{\mu 2}^+$ decays, roughly 10:1:20, implies that a " θ^+ " has a chance of $\approx \frac{2}{3}$ of decaying into two pions. But then the hundred-times-shorter lifetime of the θ^0 insures that a " θ^0 "

⁴ Note added in proof.—On the scheme of Lee and Yang [Phys. Rev. **102**, 290 (1956)] and Gell-Mann (unpublished), the operator C_P , which interchanges τ and θ , is conserved in strong interactions. Now for a neutral unlike $K\bar{K}$ pair symmetry under complete exchange implies $1=(-)^i C C_P$, and thus $CT=(-)^{i+i} C_P$. Since to decay into an unlike pair, a nucleonium state must have $j=l$, it has $CT=(-)^{i+i+s}$ (and of course $C_P=+1$). Comparing this with the CT of the $K\bar{K}$ pair, we see that s must be zero. Thus conservation of C_P in scheme A would forbid decay of $j=l$ triplet nucleonium into a $K\bar{K}$ pair.

⁵ See S. Okubo and R. Marshak, Phys. Rev. **100**, 1809(A) (1956).

nearly always decays into two pions.⁶ Thus in Scheme A , a neutral $K\bar{K}$ pair equally likely to be like or unlike has a chance of 1/72 of decaying into neutral pions. Thus, on the Lepore-Neuman statistical theory (see the end of preceding section), the probability for neutral nucleonium to decay indirectly through a K -meson pair into π^0 's only is 0.007. In Scheme B , pure π^0 's could be produced only from the process $\tau^0 \rightarrow 3\pi^0$, which is negligible if τ^0 's are as rare as τ^\pm 's.

RADIATIVE DECAY

For completeness we shall sketch here the possible radiative decays, even though decays involving photons will be slower than purely mesonic decays by factors of the order of 100 for each photon emitted.

Since interaction with the electromagnetic field need not conserve isotopic spin or CT parity, charged nucleonium can decay into any system of mesons and photons, conserving, of course, charge and strangeness; in addition it must respect the "no 0-0 transition" rule, that a photon plus a $j=0$ system cannot have $j=0$. For neutral nucleonium, however, charge parity C is a good quantum number. A photon has $C=-1$; thus a final state consisting of mesons plus N photons has $C=(-)^N C_{\text{meson state}}$.

Thus a neutral nucleonium state having $C=-1$, i.e., spin-space states triplet-even or singlet-odd, can decay into $\pi^0+\gamma$. Any neutral state can decay into $\pi+\pi+\gamma$, the pions being charged if $C=1$, and being charged and neutral in the ratio 2:1 if $C=-1$ and $i=0$.

The decay of neutral nucleonium into a pure photon state is equivalent to positronium annihilation⁷: an N -photon state must have $(-)^N=C$, and $(1\pm)$ or $(\text{odd}-)$ states cannot decay into two photons.

APPENDIX

Given a system of N pions with total isotopic spin i_N and $i_z=0$, we wish to find the probability that all N pions are π^0 , assuming that all states of the system with isotopic spin $i_N, i_z=0$ are equally probable. The system may be decomposed into a system of one pion of isotopic spin one plus a system consisting of the remaining $N-1$. In order that we have $i_N=i$, i_{N-1} must be $i, i+1$, or $i-1$. Each state of the $(N-1)$ -pion system with $i_{N-1}=i, i+1$, or $i-1$ may give rise to a state of the N -pion system with $i_N=i$. Corresponding

TABLE III. Values of f_i^N .

$i \backslash N$	1	2	3	4	5	6	7	8	9	10
0		1/3		1/5		1/7		1/9		1/11
1	1		3/5		3/7		1/3		3/11	
2		2/3		4/7		10/21		40/99		...
3			2/5		4/9		14/33	
4				8/35		24/77	
5					8/63	

⁶ That is, unless the decay into fermions ($\mu^+ + \mu^-$ or $\nu + \bar{\nu}$) is also speeded up a hundredfold in the $i=0$ state relative to $i=1$.

⁷ C. N. Yang, Phys. Rev. **77**, 242 (1950).

to each such N -pion state is a definite probability that all N pions are π^0 . The quantities $P^N(i)$ are the averages of these probabilities, taken over all states.

Thus $P^N(i)$ is a sum of terms, each of which is a product of three factors a, b, c which are identified as follows:

Factor a : the fraction of the states with i_N formed from states with i_{N-1} , i.e., $n^{N-1}(i_{N-1})/n^N(i_N)$, where $n^N(i)$ is the number of states of the N -pion system with $i_N=i, i_z=0$.

Factor b : the probability that $(i_{N-1})_z=(i_1)_z=0$ for a state of N pions formed from a system of $N-1$ pions with i_{N-1} and one pion with i_1 . This quantity is symbolized by $|\langle i_N 0 | 00 \rangle_{1, i_{N-1}}|^2$. It can be shown that

$$|\langle i 0 | 00 \rangle_{1, i+j}|^2 = \frac{i(1-\delta_{j0}) + \frac{1}{2}(1+j-\delta_{j0})}{2i+1+2j},$$

where $j=0, \pm 1$.

Factor c : the probability that if $(i_{N-1})_z=0$, the $(N-1)$ -pion system is all π^0 . This is just $P^{N-1}(i_{N-1})$.

Multiplying a, b , and c , one obtains

$$\begin{aligned} P^N(i) &= \frac{n^{N-1}(i+1)}{n^N(i)} \frac{i+1}{2i+3} P^{N-1}(i+1) + \frac{n^{N-1}(i)}{n^N(i)} \\ &\quad \cdot 0 \cdot P^{N-1}(i) + \frac{n^{N-1}(i-1)}{n^N(i)} \frac{i}{2i-1} P^{N-1}(i-1) \\ &= \frac{1}{n^N(i)} \left[\frac{i+1}{2i+3} n^{N-1}(i+1) P^{N-1}(i+1) \right. \\ &\quad \left. + \frac{i}{2i-1} n^{N-1}(i-1) P^{N-1}(i-1) \right]. \end{aligned}$$

TABLE IV. Values of $P^N(i)$.

$i \setminus N$	2	3	4	5	6	7	8	9	10
0	1/3	1/5	1/15	1/35	1/105	1/273	1/819	1/2211	1/6633

Or, defining

$$f_i^N = n^N(i) P^N(i),$$

one has

$$f_i^N = \left[\frac{i+1}{2i+3} f_{i+1}^{N-1} + \frac{i}{2i-1} f_{i-1}^{N-1} \right].$$

The f_i^N must satisfy the conditions that $f_0^1=1, f_i^N=0$ for $N < 1$ or $i < 0$. Since f_i^N is the probability that a state of N π^0 's has isotopic spin i , it follows that $\sum_{i=0}^N f_i^N = 1$.

The recursion relations for the f 's may be solved with the help of a generating function.⁸ The solution is

$$f_i^N = \frac{N!(2i+1)}{(N-i)!(N+i+1)(N+i-1) \cdots (N+1-i)} \quad (\text{for } i \neq 0),$$

$$f_0^N = 1/(N+1) \quad (\text{for } i=0).$$

The n_i^N are found from the equations

$$n^N(i) = n^{N-1}(i+1) + n^{N-1}(i) + n^{N-1}(i-1) \quad (\text{for } i \neq 0).$$

and

$$n^N(0) = n^{N-1}(1) \quad (\text{for } i=0),$$

with $n^N(N)=1$, and $n^N(i)=0$ (if $N < i$).

Values of f_i^N and $P^N(i)$ are given in Tables III and IV.

⁸ This was pointed out by R. J. Riddell, Jr., who carried out the solution.