its energy, $t_1 \approx 3\times10^7$ to 3×10^8 years, and it is necessary that $t_2 \gtrsim t_1$. For an electron, the rate of loss of energy

$$-dE/dt = 1.58\times10^{-15}H^2(E/mc^2)^2 - 4.8\times10^{-2}(v/c)^2(E/l) \text{ ergs/sec.}$$

The second term represents the energy gained by Fermi collision processes; l is a distance of the order of the separation of the turbulent clouds, and v is their velocity. It has been shown previously that for the halo, for energies near 1 Bev, $dE/dt \gtrsim 0$. Thus for the particles which originate in the disk but which diffuse into the halo, the flux can continuously build up, and perhaps gain considerable energy from the turbulent gas. Current estimates of v and l in the disk suggest that for the particles which remain in the disk the second term is small compared with the first and can be neglected. Thus, for example, if $E=5000$ Mev, the time taken to radiate an appreciable fraction of the energy is of the order of 10^7 to 10^8 years, so that $t_2 \approx t_1$.

These approximate arguments suggest that the radio emission from our galaxy, and perhaps from all normal galaxies, is a natural result of the presence of a cosmic-ray flux in the interstellar gas and magnetic field.

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Perturbation Treatment of the Nuclear Many-Body Problem

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It is generally believed that the results of treating the nuclear many-body problem by straightforward perturbation theory (based on replacing $\frac{1}{2}\sum\sum_{i\neq j}V_{ij}$ in the Hamiltonian by $\frac{1}{2}\sum_i U_i$ and treating the difference as a perturbation) indicate a poor convergence of the method.[1–3] A re-examination of Euler's original work[1] reveals the following situation.

Starting with an infinitely extended neutron-proton Fermi gas, imagine that nucleon-nucleon interactions are switched on. The resulting potential energy is, in first order, the expectation value (P.E.)$^{(1)}$ of $\frac{1}{2}\sum\sum_{i\neq j}V_{ij}$ for the unperturbed Slater determinant. In second order, a correction (P.E.)$^{(2)}$ appears, which expresses the result of correlations. The ratio (P.E.)$^{(2)}$/(P.E.)$^{(1)}$ (linear in the strength of the interaction) is a measure of the deviation from the uncorrelated state. Euler's formulas allow one to study this ratio as function of the density of the Fermi gas in the case when the interaction is a central, two-body, Gaussian potential of strength C and range a, acting in any one of the four two-particle states: triplet even, singlet even, triplet odd, singlet odd.

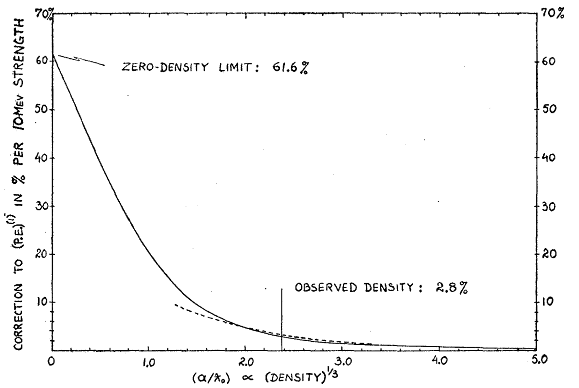

FIG. 1. Graph showing the damping out of correlations with increasing nuclear density. The ratio (P.E.)$^{(2)}$/(P.E.)$^{(1)}$ is plotted in percent per 10-Mev strength of a Gaussian interaction assumed to act in even states (full curve) or odd states (dotted curve). The abscissa, proportional to the cube root of the density, is the range of the Gaussian a, divided by λ_0 (the wavelength of the fastest particle in the Fermi gas, equal to $(8/9\pi)^{\frac{1}{3}}$ times the nuclear radius constant r_0).

At vanishingly small density (when the rare interactions that take place do so as essentially free two-body collisions), Euler's formulas give

$$\frac{(\text{P.E.})^{(2)}}{(\text{P.E.})^{(1)}} = \frac{1}{\sqrt{2}}\frac{C}{\hbar^2/Ma^2},$$

where M = nucleon mass. For a typical range $a=1.9\times10^{-13}$ cm (reference 2, p. 131), this gives (P.E.)$^{(2)}$/(P.E.)$^{(1)}$ = 61.6% per 10 Mev of strength. This agrees with the expectation that a Born approximation treatment of free nucleon-nucleon collisions is inadequate.

Figure 1 shows how the above ratio decreases with increasing nuclear density. At the observed density, corresponding to a nuclear radius constant[4] $r_0=1.216\times10^{-13}$ cm, it is 2.8% per 10 Mev of strength—a decrease by a factor of 22. For a typical triplet-even strength (about 43.7 Mev, reference 2, p. 131), the correlation correction is then 12.2%. For the weaker singlet-even interaction (about 27.2 Mev), it is 7.6%.

The rapid damping-out of correlations with increasing density is due partly to the increasing average collision velocities and partly to the exclusion principle,

which is, of course, taken into account in the perturbation treatment.

In his paper Euler does not discuss the correlations with reference to a well-defined criterion like the ratio $(\mathrm{P.E.})^{(2)}/(\mathrm{P.E.})^{(1)}$. His conclusion that the independent-particle starting point is inadequate would appear to have been influenced by the size of the ratio of the total correlation energy to the total energy per particle (the binding energy). However, for a system of particles where both attractions and repulsions are present, the final binding energy may be a rather small difference between larger positive and negative terms and the correlation energy may be considerable compared to this difference, although it still represents only small corrections to the first-order estimates of the separate contributions. This is precisely the case for nuclear models—including the one considered by Euler—in which, to ensure saturation, repulsions are introduced to cancel out exactly the largest contributions to the interaction energy, namely the first-order "direct integrals" which, in the case of an atom, for example, are responsible for the bulk of the electrostatic energy.

The magnitude of $(\mathrm{P.E.})^{(2)}/(\mathrm{P.E.})^{(1)}$, about 7.6—12.2%, suggests the conclusion that there is no evidence in the size of the correlation energy against the usefulness of the independent-particle starting point. Within the limitations of the perturbation method[5] (e.g., the effect of a repulsion of a range 0.5×10^{-13} cm cannot be estimated reliably if the strength exceeds about 500 Mev), this very straightforward treatment of the many-body problem should provide a *quantitative* basis for studying the relation between the characteristics of particle-particle forces and the resulting properties of a composite system. In such cases the approximate validity of the shell-model starting point is to be understood in the most naive sense, namely, that a system of independent particles in an over-all potential is a fairly good approximation to the exact solution of the many-body problem for particles obeying the exclusion principle.[6]

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