Interaction Contribution to Nuclear Isomerism*†

LARRY SPRUCH AND AUBEY ROTENBERG‡
Physics Department, New York University, Washington Square, New York, New York
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A number of observed magnetic multipole isomeric transitions are forbidden on a strict Mayer model of the nucleus, and even on a number of variations thereof, if the ordinary magnetic moment operator is used. Possible explanations include modifications of the nuclear model, of the magnetic moment operator, or of both. Modifications of the operator involve consideration of the currents due to the exchange of mesons between nucleons which give rise to interaction contributions to the magnetic moment operator. An exploratory investigation has been carried out to see to what extent these interaction effects by themselves can account for these so-called forbidden transitions. Using certain simplicity arguments the operators were chosen from the set which can be written down, phenomenologically, with the radial functions arbitrary. The dipole calculations were performed on both the Fermi model and the shell model. On the Fermi model, the transition matrix element involves the unknown radial function only in an integral which is identical for all transitions. By an appropriate but arbitrary choice of the numerical value of this integral, the data can be accounted for, though somewhat crudely; the difference between transitions in odd-proton and odd-neutron nuclei is not explained. There are very few data on "forbidden" octupole transitions; the data that do exist can be more or less understood in terms of the interaction moments.

I. INTRODUCTION

ALTHOUGH the Mayer[1] model of the nucleus successfully accounts for many observed properties of nuclei, modifications of this model are necessary in order to account for some data not in accord with the Mayer predictions. In particular, the magnetic moments of nuclei are in qualitative agreement with these predictions (the Schmidt lines[2]) but do not agree quantitatively with them. There are two principal ways in which these deviations from the Schmidt lines might be explained. One approach is to assume that the Mayer scheme determines the parity and angular momentum and perhaps the largest part of the true wave function at least for the ground state but that it must be modified to include admixtures[3,4] from a few of the outer particles. For the excited states the above situation may hold and in addition collective vibrations of the core[5] may have to be included in the nuclear wave functions.[6] The other is to retain the Mayer picture but to generalize the magnetic moment operator to include, besides the contribution associated with the orbital and intrinsic spin angular momenta, the interaction effects arising from the exchange of mesons between nucleons. These interaction effects can be treated from a fundamental meson theoretic viewpoint[7,8] or, as first pointed out by Sachs,[9] from a phenomenological viewpoint.[10–12] The underlying theory of these effects is that internuclear forces are produced by a field bearing charged mesons which give rise to a charge-exchange potential. Once the possibility of these exchange forces is admitted, any meson theory must give rise to an interaction current in order to satisfy the requirements of charge conservation. It has been pointed out by Sachs and Ross[13] that there are some observed isomeric transitions in odd-A nuclei which would not be expected to occur on the Mayer model of the nucleus. These are magnetic dipole ($M1$) transitions, involving a change of two units of orbital angular momentum (but of course only one unit of total angular momentum) and are therefore forbidden by the ordinary magnetic moment operator which can connect states differing at most by one unit of orbital angular momentum. The forbidden transitions, too, can be explained either by admitting admixtures to the wave functions and/or by interaction effects. Ross[12] has shown that the types of admixtures which could explain the forbidden transitions require what seem to be unreasonably large departures from the Mayer model. The current trend is not to take the Mayer model too seriously, and both admixtures and interaction effects may be needed to explain both the

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‡ Now at the A. E. C. Computing Facility, New York University, New York, New York.

[1] M. G. Mayer, Phys. Rev. **78**, 16 (1950); Haxel, Jensen, and Suess, Z. Physik **128**, 295 (1950).
[2] T. Schmidt, Z. Physik **106**, 358 (1937).
[3] M. Mizushima and M. Umezawa, Phys. Rev. **85**, 37 (1952). B. H. Flowers, Phil. Mag. **43**, 1330 (1952). A. de Shalit, Phys. Rev. **90**, 83 (1953); **91**, 1479 (1953). A. de Shalit and M. Goldhaber, Phys. Rev. **92**, 1211 (1953). R. J. Blin-Stoyle and M. A. Perks, Proc. Phys. Soc. (London) **A67**, 885 (1954). A. Russek, Phys. Rev. **99**, 834 (1955). R. K. Osborne and E. D. Klema, Phys. Rev. **100**, 822 (1955).
[4] A. B. Volkov, Phys. Rev. **94**, 1664 (1954).
[5] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953); K. Siegbahn, *Beta and Gamma Ray Spectroscopy* (Interscience Publishers, Inc., New York, 1955), Chap. XVII.
[6] Mention should be made also of recent work on the theory of nuclear models. See, for example, Brueckner, Eden, and Francis, Phys. Rev. **99**, 76 (1955), and references therein.
[7] S. T. Ma and C. F. Yu, Phys. Rev. **62**, 118 (1942); C. Møller and L. Rosenfeld, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **20**, No. 12 (1943); W. Pauli and S. Kusaka, Phys. Rev. **63**, 400 (1943); F. Villars and A. Thellung, Helv. Phys. Acta **21**, 355 (1948).
[8] F. Villars, Helv. Phys. Acta **20**, 476 (1947).
[9] R. G. Sachs, Phys. Rev. **74**, 433 (1948).
[10] R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951); N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951).
[11] A. Russek and L. Spruch, Phys. Rev. **87**, 1111 (1952).
[12] M. Ross, Phys. Rev. **88**, 935 (1952).
[13] R. G. Sachs and M. Ross, Phys. Rev. **84**, 379 (1951).

deviations from the Schmidt lines and the forbidden isomeric transitions. However, rather than study both effects, we want to see to what extent the data can be explained by isolating interaction effects.[14] It should be expressly noted that in spite of our use of the Mayer model, our results are not completely restricted to the Mayer model; we need not rule all admixtures out of consideration, since for many of the possible admixtures the selection rules are violated for the ordinary magnetic moment operator. The only effect of these admixtures would be to require a renormalization of the wave functions which would change the calculated transition probabilities only by a few percent for reasonable admixtures. Once the possibility of interaction effects is admitted, then they will be expected to contribute to all isomeric transitions. The forbidden transitions, however, offer a possibility of isolating the interaction effects, since the ordinary magnetic moment operator cannot contribute in these cases. Ross[12] investigated the possibility of explaining the forbidden dipole transitions by means of interaction effects, but at that time there was very little experimental data available; only one lifetime was known with any accuracy. Since then nearly a score of lifetime measurements have been made of forbidden magnetic dipole transitions. In addition, there is some evidence for the existence of forbidden magnetic octupole transitions. Magnetic quadrupole and magnetic 2^4-pole forbidden transitions are not expected on the Mayer model for the same reason, pointed out by Mayer[1] in her original paper, that $E1$ isomeric transitions are not to be expected, *viz.*, the levels connecting the states between which transitions would give rise to the radiation in question lie in different Mayer shells. 2^5 and higher forbidden multipole transitions can take place only in the region of the naturally radioactive and transuranic elements and because of the small probability for the occurrence of such transitions, they are not likely to be observed in competition with other modes of decay.

In view of the fact that so much more data are now available, it seems then that a reinvestigation of the forbidden transitions might throw light on the role played by interaction effects inside nuclei.

II. THE OPERATORS

Because there is no firm foundation for the various meson calculations, the operators are obtained in a phenomenological manner; that is, the empirical evidence is expressed in terms of the simplest operators involving the nuclear coordinates. This is the approach which has been used with some success in explaining the deviations of the magnetic moments from the Schmidt lines,[11,15] and in investigations of the neutron-deuteron cross section, the neutron-proton cross section,[16] and the photodisintegration of the deuteron.[17] Also, interaction effects have been considered[18] in calculations of the hyperfine structure splitting in He^3.

The problem of writing down the interaction operator phenomenologically has been considered by several authors and in greatest detail by Berger and Foldy[19] who wrote down, subject to certain conditions of invariance and symmetry, the most general two-body velocity-independent dipole operator. In principle, the operators for the higher multipoles could be obtained in the same way. However, this would be a lengthy procedure which does not seem justified since in any event the radial functions and the strength of the individual terms are undetermined on a phenomenological approach. An idea suggested by Stern and Schwinger and used by Stern[20] and by Villars and Weisskopf[21] seems to be a simple and reasonable method of choosing a gauge-invariant operator which gives rise, for all multipoles, to a spin-dependent interaction. (Only spin-dependent interactions can account for the forbidden transitions in which we are interested.) It gives rise in the dipole case to just three of the operators written down by Berger and Foldy, and these are the ones used by Villars[8] in explaining the H^3-He^3 anomaly and also by Russek and Spruch[11] in dealing with the deviations of the magnetic moments from the Schmidt lines.

Ross[12] has shown that if interaction effects alone are to account for both light- and heavy-body data, the operator must change rather markedly in passing from light to heavy nuclei. Interaction effects may account for heavy-body data if the interaction is due both to two-body and to many-body interactions. It is possible to retain the *form* of a two-body interaction if it is assumed that the two-body and many-body interactions can be combined to give an effective two-body operator which will not change very markedly over the range of medium and heavy nuclei. This corresponds to the assumption that the effect of the neighboring nucleons on an interacting pair has a smoothed-out average influence which need not, however, be small.[12]

In their derivation of the interaction operators, Villars and Weisskopf[21] express the two-body charge-interaction potential, V_{12}, taken from the experimental data, in a form which often appears in meson-theoretic calculations, namely,

$$\begin{aligned} V_{12}(\mathbf{r}_{12},\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) &= \tau_1\cdot\tau_2 V(\mathbf{r}_{12},\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2) \\ &= (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)[(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla}_1)(\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla}_2)u_1(r_{12})+u_2(r_{12})], \quad \text{(1a)} \end{aligned}$$

[14] Volkov (reference 4) has taken the other point of view and has studied the effects of admixtures in connection with the magnetic moments and forbidden isomeric transitions.

[15] H. Miyazawa, Progr. Theoret. Phys. Japan **6**, 801 (1951).

[16] N. Austern, Phys. Rev. **83**, 672 (1951); **85**, 147 (1952); **92**, 670 (1953).

[17] J. M. Berger, Phys. Rev. **94**, 1698 (1954).

[18] A. M. Sessler and H. M. Foley, Phys. Rev. **98**, 6 (1955).

[19] J. M. Berger and L. L. Foldy, Technical Report No. 18 of the Nuclear Physics Laboratory, Case Institute of Technology (unpublished); see also L. L. Foldy, Phys. Rev. **92**, 178 (1953); R. K. Osborne and L. L. Foldy, Phys. Rev. **74**, 433 (1948).

[20] A. Stern, thesis, Harvard University, 1951 (unpublished).

[21] F. Villars and V. F. Weisskopf (unpublished).

where $\boldsymbol{\tau}_i$ is the isobaric spin operator of the ith nucleon. The functions u_1 and u_2 are determined by this equation. Then they construct what is probably the simplest charge-exchange spin dependent gauge-invariant potential, namely,

$$W_{12}=2\boldsymbol{\tau}_1^+\boldsymbol{\tau}_2^-\left\{\left(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla}_1-\frac{ie}{\hbar c}\boldsymbol{\sigma}_1\cdot\mathbf{A}(\mathbf{r}_1)\right)\right.$$
$$\left.\times\left(\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla}_2+\frac{ie}{\hbar c}\boldsymbol{\sigma}_2\cdot\mathbf{A}(\mathbf{r}_2)\right)u_1+u_2\right\}$$
$$\times\exp\left\{-\frac{ie}{\hbar c}\int_{\mathbf{r}_1}^{\mathbf{r}_2}\mathbf{A}\cdot d\mathbf{r}\right\}+\text{Herm. conj.},\quad(1\text{b})$$

where $\mathbf{A}$ is the vector potential and $\tau^{\pm}=\frac{1}{2}(\tau^x\pm i\tau^y)$.

For weak fields, a good approximation to Eq. (1b) can be obtained by expanding in powers of $\mathbf{A}$ and retaining only those terms linear in $\mathbf{A}$. This corresponds to the assumption that the field is sufficiently weak so that it does not alter the motions of the mesons nor of the nucleons appreciably. For magnetic moment calculations the vector potential associated with the external field must be used and this gives, upon eliminating the isobaric spin, the operators

$$\begin{aligned}\mathbf{M}_1&=\mathbf{M}_{\text{long.}}=(ie/2\hbar c)\textstyle\sum_1\sum_2\mathbf{r}_1\times\mathbf{r}_2V(\mathbf{r}_{12},\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)P_{12},\\ \mathbf{M}_2&=(e/2\hbar c)\textstyle\sum_1\sum_2 r_{12}{}^2f_2(r_{12})(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)P_{12},\\ \mathbf{M}_3&=(e/2\hbar c)\textstyle\sum_1\sum_2 f_3(r_{12})[\mathbf{r}_{12}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)]\mathbf{r}_{12}P_{12},\end{aligned}\quad(2)$$

where $\mathbf{r}_i$ and $\boldsymbol{\sigma}_i$ are the position and spin operators for the ith particle, V is defined by Eq. (1a), f_2 and $f_3\equiv f$ can be expressed in terms of the u_n which follow from Eq. (1a), and P_{12} is the space exchange operator. Only one of these operators, $\mathbf{M}_1$ has a firm theoretical foundation,[9] and an investigation by Spruch[22] has shown that it alone cannot account for the deviations of the magnetic moments of nuclei from the Schmidt lines. It will be noticed that of these three operators only the last one, $\mathbf{M}_3$, has the possibility of carrying off two units of orbital angular momentum. (It should be noted that the operator $\mathbf{r}_1\times\mathbf{r}_2$ transforms as an $L=1$ operator.) For calculations of magnetic transition probabilities in which the emitted photon carries off L units of orbital angular momentum with projection M, we use the vector magnetic potential of order L, M[23] generated by the interaction currents themselves, and normalized to one quantum per unit energy state. The operators have the same form as those in Eq. (2) but of course involve the wave number of the emitted radiation because of the normalization chosen. Of the operators obtained in this way, the only one which contributes to the forbidden isomeric transitions can be written, again after eliminating the isobaric spin,

$$H^{1,0}=2A_1(ie/\hbar c)(3/8\pi)^{\frac{1}{2}}f(r_{12})[(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{r}_{12}]z_{12}P_{12},\quad(3\text{a})$$

where H^{LM} serves as the perturbation which gives rise to a photon with quantum numbers L and M; z_{12} is the projection of $\mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2$ on an arbitrarily chosen z-axis and $f(r_{12})$ is an undetermined function of the interparticle distance which is expected, however, to have a range comparable to the range of nuclear forces. In the Villars-Weisskopf formalism V_{12} is considered known, one then determines $u_1(r_{12})$, and $f(r_{12})$ follows from the relation

$$f(r_{12})=[\partial/\partial r_{12}(u_1)]/r_{12}.$$

From our viewpoint, V_{12} is the effective potential in the presence of the other nucleons and as such is not known. We have therefore attempted to perform the calculations, wherever possible, in such a way that the specific form of $f(r_{12})$ need not be known. It is understood that this operator must be summed over all pairs of particles and that it acts on wave functions which have been antisymmetrized separately in neutrons and protons. A_1 arises from the introduction of the vector potential; the general form is given by

$$A_L=i(2\hbar c)^{\frac{1}{2}}[\pi L(L+1)]^{-\frac{1}{2}}k^{L+1}/(2L+1)!!,$$

where k is the wave number of the emitted radiation and

$$(2L+1)!!\equiv(2L+1)(2L-1)\cdots3\cdot1.$$

Similarly for the octupole, keeping only those terms which can carry off four units of orbital angular momentum, we find

$$\begin{aligned}H^{3,0}=\frac{15}{2}\left(\frac{7}{4\pi}\cdot\frac{1}{12}\right)^{\frac{1}{2}}\frac{ie}{\hbar c}A_3f(r_{12})\{&(z_1z_2+z_{12}{}^2)[z_{12}\boldsymbol{\sigma}_{12}\cdot\mathbf{r}_{12}\\ &+i(\boldsymbol{\sigma}_2\cdot\mathbf{r}_{12})(\mathbf{r}_{12}\times\boldsymbol{\sigma}_1)^z+i(\boldsymbol{\sigma}_1\cdot\mathbf{r}_{12})(\mathbf{r}_{12}\times\boldsymbol{\sigma}_2)^z]\\ &-z_2{}^2z_{12}[\boldsymbol{\sigma}_{12}\cdot\mathbf{r}_2+i(\boldsymbol{\sigma}_2\cdot\mathbf{r}_{12})(\boldsymbol{\sigma}_1\times\mathbf{r}_{12})^z\\ &+i(\boldsymbol{\sigma}_1\cdot\mathbf{r}_{12})(\boldsymbol{\sigma}_2\times\mathbf{r}_{12})^z]\\ &+z_1{}^2z_{12}[\boldsymbol{\sigma}_{12}\cdot r_2-i(\boldsymbol{\sigma}_1\cdot\mathbf{r}_{12})(\boldsymbol{\sigma}_2\times\mathbf{r}_{12})^z\\ &-i(\boldsymbol{\sigma}_2\cdot\mathbf{r}_{12})(\boldsymbol{\sigma}_1\times\mathbf{r}_{12})^z]\}P_{12}.\end{aligned}\quad(3\text{b})$$

As already observed, use of the suggestion of Villars and Weisskopf in deriving the operators gives a much more restricted set than that which arises on a strict phenomenological basis. The latter gives thirteen dipole operators,[19] only six of which give contributions by virtue of the selection rules to forbidden transitions. If the further requirement of charge independence plus no nucleon recoil is imposed, only three operators contribute. One of these is considered here [Eq. (3a)], and one gives zero on the Fermi model and is therefore expected to give a small contribution on a shell model. Thus the essential difference between the approach taken here and a strict phenomenological one is the

[22] L. Spruch, Phys. Rev. **80**, 372 (1950).
[23] See, for example, G. Goertzel and N. Tralli, Phys. Rev. **83**, 399 (1951).

neglect of the operator designated $\mathbf{M}_{\mathrm{III}'}$ by Berger and Foldy.[19] For higher multipoles there are many more operators and our approach involves the neglect of more than just one of the operators arising on a phenomenological basis. In a previous calculation[11] only operators arising on lowest order meson theory were used. This is inconsistent with the present viewpoint that many-body interactions are significant. However, the assumption made above that the two-body and many-body interactions can be combined to give an effective two-body interaction means that the two calculations are formally the same.

III. TRANSITION PROBABILITIES

The transition probability for emission of radiation of multipole order L is found by using the well-known formula,

$$\omega_L=(2\pi/\hbar)(2j_i+1)^{-1} \times\sum_M \sum_{m_i} \sum_{m_f} |H'_{m_i m_f}|^2 \rho(E)dE, \quad (4a)$$

where, because of the normalization chosen for the vector potential,[23]

$$|H'_{m_i m_f}|^2\rho(E)dE =(2\pi/k\hbar c)|(\Psi_{m_f}(f), \sum_{1,2} H^{LM}\Psi_{m_i}(i))|^2, \quad (4b)$$

where $\Psi_{m_f}(f)$ and $\Psi_{m_i}(i)$ are the nuclear wave functions in the final and initial states antisymmetrized separately in neutrons and protons; m_f and m_i are the projections of the total angular momenta j_f and j_i of the outer particle in the two states. If the summations over m_f and m_i are done first, the result must be independent of M by the principle of spectroscopic stability.[24] Thus the summations over M can be done first by choosing a particular value of M, say $M=0$, and multiplying by the number of M states, $2L+1$. Now only those terms for which $m_i=m_f\equiv m$ will be allowed, so the triple sum is reduced to a single one and we have

$$\omega_L=\frac{2\pi}{\hbar}\frac{2\pi}{k\hbar c}\frac{2L+1}{2j_i+1}\sum_m |(\Psi_m(f), \sum_{1,2} H^{L0}\Psi_m(i))|^2. \quad (5)$$

We assume that an odd-A nucleus consists of a core with spin, orbital angular momentum, and total angular momentum quantum numbers equal to zero, plus one odd nucleon in the state with radial, orbital angular momentum, and total angular momentum quantum numbers as given by the Mayer model. It can be shown then that only one summation over core particles is needed, since only interactions between an outer proton and core neutrons or an outer neutron and core protons give nonzero contributions. The matrix element in Eq. (5) then reduces to

$$(\Psi_m(f), \sum_{1,2} H^{L0}\Psi_m(i)) =\sum_2(\Phi(2)\psi_{n_f l_f j_f m}(1), H^{L0}\psi_{n_i l_i j_i m}(1)\Phi(2)), \quad (6)$$

[24] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, Cambridge, 1951).

where 1 is the outer nucleon and 2 is a core nucleon which is a neutron if 1 is a proton and vice versa. Φ and ψ_{nljm} are respectively the core particle and outer particle wave functions. n_i, l_i, and n_f, l_f are the radial and orbital angular momentum quantum numbers in the initial and final states, respectively. Now in H^{L0} we shall omit those terms involving $\boldsymbol{\sigma}_2$ since they give zero.[11]

To proceed further requires that specific assumptions be made with regard to the wave functions. While the radial parts of the wave functions are not known with any accuracy, it is expected that calculations on either a Fermi gas model or shell model will give fairly reliable results. The former treats the core as if all the particles were alike but has the advantages that it is comparatively easy to handle mathematically and that it involves only one integral which need not be evaluated since it is the same in all cases. The latter, while it is expected to give better results since it takes into account the assumed shell structure of the core, involves a great deal of calculation and turns out to be not too reliable for cases involving more than a very few nodes in the outer particle wave functions, since they are very sensitive to the parameters characterizing the range of the nuclear force and the size of the nucleus. Of course, for these cases the Fermi model results are unreliable at least to the same extent. On both models the outer nucleon wave function is assumed to be separable into radial, angular, and spin parts, i.e.,

$$\psi_{nljm}(\mathbf{r}_1,\boldsymbol{\sigma}_1)=\sum_\mu \langle l\,\tfrac{1}{2}\, m-\mu\, \mu | l\,\tfrac{1}{2}\, j\, m\rangle \phi_{nlm}(\mathbf{r}_1)\chi(\mu), \quad (7)$$

where $\chi(\mu)$ is the spin wave function, being equal to α, or spin up, for $\mu=+\frac{1}{2}$ and β for $\mu=-\frac{1}{2}$. $\langle l\,\frac{1}{2}\, m-\mu\, \mu | l\,\frac{1}{2}\, j\, m\rangle$ is the usual Clebsch-Gordan coefficient with phase defined by Condon and Shortley.[24] Under these circumstances, the spin inner product over the outer particle can be carried out at once[25] and gives

$$[\chi(\mu_f),\boldsymbol{\sigma}_1\chi(\mu_i)] =(-)^{\mu_f-\frac{1}{2}}\sqrt{2}\langle \sigma_i\, \sigma_f\, \mu_i\, -\mu_f | \sigma_i\, \sigma_f\, 1\, \mu_i-\mu_f\rangle \mathbf{u}_{\mu_i-\mu_f}, \quad (8)$$

where $\sigma_i=\sigma_f=\frac{1}{2}$ and

$$\mathbf{u}_1=-(2)^{-\frac{1}{2}}(\mathbf{i}-i\mathbf{j}); \quad \mathbf{u}_{-1}=(2)^{-\frac{1}{2}}(\mathbf{i}+i\mathbf{j}); \quad \mathbf{u}_0=\mathbf{k}.$$

Villars and Weisskopf[21] showed that the calculations of the matrix elements for the static magnetic moments could be carried out quite nicely on the Fermi model.[26] On this model, the core is replaced by a Fermi gas filling the sphere $|\mathbf{p}|<P$ in momentum space, where P is the momentum of the last nucleon in the core and is determined by the density of nucleons. The wave function for a core nucleon is written, the spin having been eliminated, in the form

$$\Phi(\mathbf{r}_2)=(2\pi)^{-\frac{3}{2}}\exp\{i\mathbf{k}_2\cdot\mathbf{r}_2\}, \quad (9)$$

[25] M. E. Rose, *Multipole Fields* (John Wiley and Sons, Inc., New York, 1955), p. 22.
[26] H. A. Bethe and R. F. Bacher, Revs. Modern Phys. **8**, 82 (1936), Sec. 25.

where $\hbar\mathbf{k}_2$ is the momentum of the nucleon in question. The summation over the core particles is replaced by an integration over $\mathbf{k}_2$ from 0 to κ_2, where κ_2 is the wave number of the last even nucleon in the core. Noting that the operator involves a space exchange, we set $\mathbf{r}_1-\mathbf{r}_2=\mathbf{r}$ and expand the exponential in the usual way[27]:

$$\exp\{i\mathbf{k}\cdot\mathbf{r}\}=4\pi\textstyle\sum_L\sum_M(i)^L j_L(kr)\bar{Y}_{LM}(\mathbf{k}_0)Y_{LM}(\mathbf{r}_0), \quad (10)$$

and obtain

$$\int_0^{\kappa_2}\exp\{i\mathbf{k}_2\cdot\mathbf{r}\}d\mathbf{k}_2=4\pi\kappa_2^2r^{-1}j_1(\kappa_2r), \quad (11)$$

where $j_L(kr)$ denotes the spherical Bessel function and $\mathbf{k}_0$ and $\mathbf{r}_0$ are unit vectors along $\mathbf{k}$ and $\mathbf{r}$, respectively. We shall also use the notation $\mathbf{r}_{i0}$ for a unit vector in the direction $\mathbf{r}_i$.

The outer nucleon is assumed to occupy a level whose kinetic energy is $E=p_1^2/2m$, and its wave function is written

$$\phi_{nlm}(\mathbf{r}_1)=Y_{lm}(\mathbf{r}_{10})\int_0^\infty j_l(kr)g(k-\kappa_1)dk, \quad (12)$$

where Y_{lm} is the usual spherical harmonic, $\kappa_1=p_1/\hbar$ is the wave number of the outer particle, and $g^2(k-\kappa_1)=2\kappa_1^2\pi^{-\frac{1}{2}}\delta(k-\kappa_1)$, this function being used so that ϕ is normalized to unity. Now in the matrix element, the outer particle becomes a function of $\mathbf{r}_2$ owing to the space exchange operator but this can be expanded as a function of $\mathbf{r}_1$ and $\mathbf{r}$ by using the expansion[28]

$$j_{l_i}(kr_2)Y_{l_i,\,m-\mu_i}(\mathbf{r}_{20})=\sum_{\nu=0}^{\infty}\sum_{\mu=-\nu}^{\nu}\sum_{p}\left[\frac{(2l_i+1)(2\nu+1)4\pi}{2p+1}\right]^{\frac{1}{2}}$$
$$\times\langle l_i\,\nu\,0\,0|l_i\nu p0\rangle\langle l_i\,\nu\,m-\mu_i\,\mu|l_i\,\nu\,p\,m-\mu_i+\mu\rangle(i)^{\nu+p-l_i}$$
$$\times j_p(kr)j_\nu(kr)Y_{p,\,m-\mu_i+\mu}(\mathbf{r}_{10})Y_{\nu\mu}(\mathbf{r}_0). \quad (13)$$

On integrating over $\mathbf{r}_1$ and $\mathbf{r}$ spaces, the selection rules on the angular integrations give $p=l_f$, $\nu=2$, and $\mu=\mu_i-\mu_f$ and the angular integrations can be carried out at once.[29] It is found that the radial part of the $\mathbf{r}_1$ integration gives unity because of the way the wave function was originally chosen. The $\mathbf{r}$ integration gives

$$I_F\equiv\kappa_2^2\int f(r)j_1(\kappa_2r)j_2(\kappa_1r)r^3dr, \quad (14)$$

and we find for the value of the matrix element [Eq. (6)]

$$(\psi_m(f),\textstyle\sum_{1,2}H^{L0}\psi_m(i))=\dfrac{Bek^2}{15\pi^2}\left[\dfrac{2}{\hbar c}(2l_f+1)(2l_i+1)\right]^{\frac{1}{2}}$$
$$\times\langle 1\,1\,0\,\mu_i-\mu_f|1\,1\,2\,\mu_i-\mu_f\rangle(-)^{m-\mu_i}\langle l_fl_i00|l_fl_i20\rangle$$
$$\times\langle l_f\,l_i\,-m+\mu_f\,m-\mu_i|l_f\,l_i\,2\,\mu_f-\mu_i\rangle I_F, \quad (15a)$$

where

$$B=\textstyle\sum_{\mu_i}\sum_{\mu_f}(-)^{\mu_f-\frac{1}{2}}\langle l_f\,\sigma_f\,m-\mu_f\,\mu_f|l_f\sigma_fj_fm\rangle$$
$$\times\langle l_i\,\sigma_i\,m-\mu_i\,\mu_i|l_i\sigma_ij_im\rangle$$
$$\times\langle\sigma_i\,\sigma_f\,\mu_i\,-\mu_f|\sigma_i\,\sigma_f\,1\,\mu_i-\mu_f\rangle. \quad (15b)$$

In writing Eq. (15), we have set $\langle 1100|1120\rangle=(2/3)^{\frac{1}{2}}$.

Combining all the Clebsch-Gordan coefficients involving the projections μ_i and μ_f in Eq. (15) we can carry out the summations by established methods[30] obtaining

$$\textstyle\sum_{\mu_i}\sum_{\mu_f}(-)^{\mu_f-\frac{1}{2}+m-\mu_i}\langle l_f\,\sigma_f\,m-\mu_f\,\mu_f|l_f\sigma_fj_fm\rangle$$
$$\times\langle l_i\,\sigma_i\,m-\mu_i\,\mu_i|l_i\sigma_ij_im\rangle\langle\sigma_i\sigma_f\mu_i\mu_f|\sigma_i\,\sigma_f\,1\,\mu_i-\mu_f\rangle$$
$$\times\langle l_f\,l_i\,-m+\mu_f\,m-\mu_i|l_f\,l_i\,L+1\,\mu_f-\mu_i\rangle$$
$$\times\langle L\,1\,0\,\mu_i-\mu_f|L\,1\,L+1\,\mu_i-\mu_f\rangle$$
$$=(-)^{m+\frac{1}{2}}\left(\frac{2L+3}{2L+1}\right)^{\frac{1}{2}}\langle j_ij_fm\,-m|j_ij_fL0\rangle, \quad (16)$$

where $L=1$ for the dipole. Actually the relationships in reference 30 give the result in terms of Racah coefficients but analytic expressions for these are readily available[30] since in all cases at least one of the six parameters is equal to $\frac{1}{2}$. Now the sum over m [see Eq. (5)] is trivial and we get for the dipole transition probability

$$\omega_1=\frac{32}{45}\frac{e^2k^3}{\hbar^3c^2}\frac{(2l_i+1)(2l_f+1)}{\pi^2(2j_i+1)}|\langle l_f\,l_i\,0\,0|l_f\,l_i\,20\rangle|^2I_F^2. \quad (17)$$

The biggest disadvantage of the Fermi model calculations from the point of view of this investigation is that its usefulness is limited to the dipole case. The integrals diverge for all higher multipoles. This can be seen as follows. Classically, the magnetic moment is defined as the product of current and area of the current loop. For the interaction operator the current is that due to the exchange of mesons between nucleons. On the Fermi model, the core particles extend over all of space but have the density of real nucleons inside the nucleus. The outer particle is smeared over all of space. The short range of the nuclear force means that the area of the current loop is, on the Fermi model, effectively the same as that on the shell model so the fact that the core particles extend over all of space

[27] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), p. 1573.

[28] B. Friedman and J. Russek, Quart. Appl. Math. **12**, 14 (1954). Equation (13) can be derived directly and rather simply by manipulation of Eq. (10). The method used by these authors is necessarily more complicated since they were interested in more general expansions.

[29] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc.,, New York, 1952), p. 793.

[30] See, for example, Biedenharn, Blatt, and Rose, Revs. Modern Phys. **24**, 249 (1952).

does not alter their contribution from what it would be on the shell model. As for the outer particle, it is extended over a vast region but its density is correspondingly diminished and so the Fermi model gives a reasonable result.

For higher multipoles the situation is different, however. Consider the quadrupole. The classical definition of a magnetic quadrupole moment is the product of current, area of current loop, and displacement between two such loops. The picture for a quadrupole interaction moment would be of two current loops, i.e., a little current loop part of the time in one region of space and part of the time in another. Since any current loop has a finite probability of being anywhere inside the nucleus, the allowable displacements between loops is limited by the radius of the nucleus and *not* by the range of nuclear forces. If the nuclear radius is allowed to become infinite, then it is clear the quadrupole moment will also become infinite. For higher multipoles, which involve still higher powers of the nuclear radius, the integral will diverge even more rapidly.

In order to obtain an approximate value for the octupole transition probability, a shell model calculation is carried out. The angular wave functions are taken to be spherical harmonics and the radial functions to be harmonic oscillator wave functions with all the quantum numbers taken from the Mayer model. To facilitate the calculations, a Gaussian shape is chosen for the $f(r_{12})$, the arbitrary function of the interparticle distance. (We note again that on the Fermi model it was not necessary to specify this function, since all transitions depend upon it in the same way.) It was this model which was used[11] in calculating the interaction contributions to the magnetic moments. The integral was evaluated by expressing the integrand as a function of r_1^2, r_2^2 and the scalar product $\mathbf{r}_1\cdot\mathbf{r}_2$. However, the magnetic moment calculations were performed without taking into account the nodes in the radial wave functions. These calculations involve the diagonal matrix elements where wave functions on either side of the operator are the same. Since the interaction operator involves space exchange, the initial and final wave functions will differ in the integral in that they will have their spatial coordinates interchanged. If we write the matrix element in the form

$$\int \psi_a(1)\psi_b(2)f(r_{12})\psi_a(1)\psi_b(2)dV,$$

then because of the short range of the interaction force, contributions will come only from regions where the two nucleons 1 and 2 are close together and we can write the matrix element approximately as

$$\int |\psi_a|^2|\psi_b|^2 f(r_{12})dV,$$

where the arguments of the wave functions can be taken as either 1 or 2. Since the squares are positive definite, whether or not nodes are taken into account is not expected to alter the character of the result. As far as the core is concerned, there is the additional argument that most of the nucleonic wave functions have no nodes anyway.

For the off-diagonal elements where the initial and final states are quite different and the wave functions do not overlap, neglect of the nodes could crucially alter the nature of the results. This is true for the outer nucleon wave functions, but for the core, where the wave functions are the same initially and finally, nodes can be ignored. In the first place, as already observed, most of the core wave functions have no nodes in the radial wave functions. Secondly, as in the magnetic moment case there is a close overlap of these wave functions for a short-range force and the effect of nodes is expected to be unimportant. Finally, the fact that the density of particles inside the nucleus is relatively constant indicates that the calculations are not too sensitive to details of the core wave functions. The contributions from shells having $n>1$ are arbitrarily taken to be the same as for those having $n=1$ (no nodes).

A somewhat more direct method of calculation than that used previously[11] was employed, retaining the same model. A part of the Gaussian exponential was expanded as a series of spherical Bessel functions, and in this form the integrations were carried out using the formula[31]

$$\int_0^\infty j_l(r_1r_2)\ \exp\{-\alpha r_1^2\}r_1^{l+2}dr_1=\left(\frac{\pi}{2}\right)^{\frac{1}{2}}\frac{(r_2)^l}{(2\alpha)^{l+\frac{3}{2}}}\exp\left\{\frac{r_2^2}{4\alpha}\right\}.$$

For the details of combining the various angular momenta on the shell model, see the thesis by A.R. referred to at the beginning of this article.

The value of the strength of the interaction was chosen to agree on the average with the detailed results obtained below with the Fermi model. This required a strength of 100 Mev for a Gaussian well of range 1.6×10^{-13} cm, and this value was used for the octupole calculations. It is expected that both the shell and Fermi model results will vary roughly in the same manner with the change of parameters, so that a different choice of the range will not greatly affect the results. Evaluation of the integral I_F shows that the Gaussian well strength for the Fermi model is 45 Mev. Russek and Spruch,[11] using a larger range (2.0×10^{-13} cm), required a well strength of about 300 Mev for their operator $\mathbf{M}_3$. Although that is three times the value found in this work, the difference is not very significant for two reasons. Firstly, as observed above they did not consider nodes in the nuclear wave functions, and although it was pointed out that this procedure seems justifiable, the results may not be unaf-

[31] H. Bateman, *Higher Transcendental Equations* (McGraw-Hill Book Company, Inc., New York, 1953), Vol. II, p. 50.

TABLE I. Comparison of experimental and theoretical half-lives. The dipole calculations are based on the Fermi model, with the single free parameter adjusted to give an average value of unity for the ratio of theoretical and experimental values. The octupole calculations are based on the shell model.

Nucleus	Transition $l_i \to l_f$ dipoles		Energy kev	τ_γ (sec) Expt.	Theor.	τ_γ (Theor.)/τ_γ (Expt.)	Reference
${}_{30}Zn_{37}{}^{67}$	$2p_{3/2}$	$1f_{5/2}$	92	9.5×10^{-6}	3.4×10^{-9}	0.0004	33
${}_{33}As_{40}{}^{73}$	$1f_{5/2}$	$2p_{3/2}$	66	$<5\ \times10^{-9}$	1.9×10^{-8}	>0.4	34
${}_{52}Te_{71}{}^{123}$	$2d_{3/2}$	$3s_{1/2}$	159	3.3×10^{-10}	1.2×10^{-9}	3.6	35, 36
${}_{52}Te_{73}{}^{125}$	$2d_{3/2}$	$3s_{1/2}$	35.4	4.4×10^{-8}	1.1×10^{-7}	2.5	35
${}_{54}Xe_{77}{}^{131}$	$3s_{1/2}$	$2d_{3/2}$	80	2.1×10^{-9}	4.7×10^{-9}	2.2	35
${}_{56}Ba_{77}{}^{133}$	$2d_{3/2}$	$3s_{1/2}$	11.7	$\sim3.3\times10^{-7}$	3.0×10^{-6}	~9.1	37
${}_{55}Cs_{78}{}^{133}$	$2d_{5/2}$	$1g_{7/2}$	81	2.7×10^{-8}	5.3×10^{-9}	0.20	35
${}_{55}Cs_{80}{}^{135}$	$2d_{5/2}$	$1g_{7/2}$	248	4.4×10^{-10}	1.8×10^{-10}	0.41	35
${}_{57}La_{82}{}^{139}$	$2d_{5/2}$	$1g_{7/2}$	166	2.7×10^{-9}	6.1×10^{-10}	0.24	38
${}_{59}Pr_{82}{}^{141}$	$1g_{7/2}$	$2d_{5/2}$	145	3.9×10^{-9}	9.2×10^{-10}	0.24	39
${}_{61}Pm_{86}{}^{147}$	$1g_{7/2}$	$2d_{5/2}$	91	1.1×10^{-8}	5.0×10^{-9}	0.44	35
${}_{63}Eu_{90}{}^{153}$	$3s_{1/2}$	$2d_{3/2}$	69.1	$\{1.3\times10^{-9}$	7.3×10^{-9}	5.5	40, 41
				$\{6.0\times10^{-8}$	7.3×10^{-9}	0.12	42
${}_{73}Ta_{108}{}^{181}$	$2d_{5/2}$	$1g_{7/2}$	480	$3\ \times10^{-8}$	3.4×10^{-12}	0.0001	42–44
${}_{80}Hg_{119}{}^{199}$	$3p_{3/2}$	$2f_{5/2}$	50	$<1.6\times10^{-8}$	2.1×10^{-8}	>1.3	35
${}_{81}Tl_{122}{}^{203}$	$2d_{3/2}$	$3s_{1/2}$	280	$\sim1\ \times10^{-9}$	2.2×10^{-10}	~0.2	45
	Octupoles						
${}_{72}Hf_{107}{}^{179}$	$1h_{9/2}$	$3p_{3/2}$	160	8.9×10^{2}	7.0×10^{2}	0.81	46
${}_{73}Ta_{108}{}^{181}$	$3s_{1/2}$	$1g_{7/2}$	607	5.6×10^{-3}	7.0×10^{-3}	1.3	42–44
${}_{76}Os_{115}{}^{191}$	$3p_{3/2}$	$1h_{9/2}$	74.2	$6\ \times10^{7}$	5.5×10^{4}	0.001	47, 48

fected by this approximation. Secondly, they had three adjustable constants to work with in matching the magnetic moment data and so the strength of any one operator, in particular $\mathbf{M}_3$ which corresponds to the dipole operator used here, can be varied over a considerable range and still give agreement with the data. The smaller value of the strength is somewhat hopeful, since phenomenological magnetic moment calculations based on a strict two-body interaction interpretation require strengths much larger than those deduced from two-body scattering data.[32] Even though our viewpoint, that the effective interaction moment has a many-body as well as a two-body origin, does not demand that the strengths be deduced from two-body scattering data, we nevertheless feel that the strengths should not be too different from the strengths so deduced.

IV. RESULTS

Experimental lifetime data for forbidden magnetic isomeric transitions exist for sixteen dipole and three octupole transitions.[33–48] The results are summarized in Table I. The theoretical lifetimes recorded therein for dipole transitions are those calculated on the Fermi model and for these calculations the integral I_F, Eq. (14), is not evaluated but is arbitrarily equated to that value which gives the best agreement with the experimental data. As already observed, octupole calculations can be made only on the shell model.

The results for the dipole transitions in Zn^{67} and Ta^{181} and the octupole transition in Os^{191} strongly indicate that these transitions are of a different nature from the others. Meyerhof, Mann, and West[33] did the experimental work on Zn^{67} and pointed out that the measured lifetime is much too long for an $M1$ transition. They postulated that this might be due to the l forbiddenness, but this is not tenable without further modification in view of the results presented here insofar as the other transitions are l forbidden and are not reduced to the same extent. As far as Os^{191} is concerned, the level assignments given by Goldhaber and Hill[49] in their review article are $i_{13/2}$ and $7/2+$. More recently, Mihelich and Goldhaber[47] have given the assignments of $3/2-$ and $9/2-$ to the levels, and it is our assumption that these are single-particle levels which leads to the conclusion that the transition is forbidden octupole.

Ta^{181} is a particularly interesting nucleus in that it

[32] A. Kerman, Phys. Rev. **92**, 1176 (1953). See footnote 13 of this article.
[33] Meyerhof, Mann, and West, Phys. Rev. **92**, 758 (1953).
[34] R. W. Hayward and D. D. Hoppes, Phys. Rev. **98**, 1172 (1955).
[35] R. L. Graham and R. E. Bell, Can J. Phys. **31**, 377 (1953).
[36] This transition has recently been observed in Coulomb excitation experiments. See L. W. Fagg *et al.*, Phys. Rev. **100**, 1299 (1955).
[37] Hill, Scharff-Goldhaber, and McKeown, Phys. Rev. **84**, 382 (1951); M. Langevin, Compt. rend. **238**, 1310 (1954).
[38] C. H. Pruett and R. G. Wilkinson, Phys. Rev. **96**, 1340 (1954); T. R. Gerholm and H. de Waard, Physica **21**, 601 (1955).
[39] J. Jones, Jr., and E. Jensen, Phys. Rev. **97**, 1031 (1955); Ambler, Hudson, and Temmer, Phys. Rev. **97**, 1212 (1955); H. de Waard and T. R. Gerholm, Physica **21**, 599 (1955).
[40] R. L. Graham and J. Walker, Phys. Rev. **94**, 794(A) (1954); M. C. Lee and R. J. Katz, Phys. Rev. **93**, 155 (1954).
[41] G. M. Temmer and N. P. Heydenberg, Phys. Rev. **94**, 1399 (1954); G. M. Temmer (private communication).
[42] F. K. McGowan, Phys. Rev. **93**, 163 (1954).
[43] Alaga, Alder, Bohr, and Mottelson, Kgl. Danske Videnskab, Mat.-fys. Medd. **29**, No. 9 (1955).
[44] Burson, Blair, Keller, and Wexler, Phys. Rev. **83**, 62 (1951).
[45] F. R. Metzger and W. B. Todd, Phys. Rev. **95**, 627(A) (1954); H. W. Wilson and S. C. Curran, Phil. Mag. **42**, 762 (1951); H. de Waard, Phys. Rev. **99**, 1045 (1955).
[46] E. der Mateosian and M. Goldhaber, Phys. Rev. **83**, 843 (1951).
[47] J. W. Mihelich and M. Goldhaber, Phys. Rev. **98**, 1185 (1955).
[48] Rose, Goertzel, and Swift (privately circulated tables).
[49] M. Goldhaber and R. D. Hill, Revs. Modern Phys. **24**, 179 (1952).

seems to have both a forbidden dipole and forbidden octupole transition. The 48-kev transition is a mixed multipole one with the ratio[42] $M1/E2=1$. The $E2$ part has been studied recently by Sunyar[50] who finds it to be "a glaring exception to the general trend of $E2$ transitions." The results of this work indicate the same conclusion with regard to the $M1$ part. There is some question as to the level assignments of the excited states in this nucleus, McGowan's[42] being very different from those of Alaga *et al.*,[43] but all evidence agrees with the assignment of $g_{7/2}$ to the ground state, and it is difficult to account for this on the Mayer model according to which the $g_{7/2}$ should be "buried" for this nucleus. Mention should be made here that there is a rather wide divergence in the lifetime and K-conversion coefficient reported for the Eu^{153} transition.[42,40] Temmer and Heydenberg[41] have made a detailed study of the energy levels which correspond to the 69.1-kev transition here considered. For this nucleus, as well as for Ta^{181} which also has been thoroughly investigated,[43] the conclusion is that there are two kinds of energy levels, one being the Bohr-Mottelson levels, the other presumably the one-particle levels.

Three of the dipole transitions listed are mixed $E2+M1$. In two cases, Ta^{181} and Tl^{203}, the ratio is known and the lifetime given in Table I is just the $M1$ lifetime. For Cs^{135}, the available evidence indicates that the amount of $E2$ is small; the total γ lifetime is the one given.

In addition to the transitions listed in Table I there are a score of nuclei having dipole l-forbidden transitions for which no experimental lifetime data are available. These nuclei are: V^{51} (0.61 Mev); Co^{59} (0.191); Rb^{85} (0.150); Mo^{95} (0.73) (?); Mo^{97} (0.665) (?); Ru^{101} (0.307); Pd^{105} (0.063) (?), (0.32) (?), (0.154); Cd^{111} (0.340); In^{115} (0.460); Sn^{117} (0.162); Sn^{119} (0.024); Te^{121} (0.213); Sb^{123} (0.153); Xe^{129} (0.038); Pr^{143} (0.290) (?), (057) (?); Re^{185} (0.056); Au^{193} (0.038); Au^{195} (0.061); Hg^{195} (0.037); Pt^{195} (0.031) (?), (0.126) (?); Hg^{201} (0.168), the energies being given in Mev. A question mark indicates that there is doubt about the level assignments or the multipolarity of the transition. Data are taken from the Goldhaber and Hill[49] review article and material which has appeared in the literature since then. It is significant that none of these levels has been found by Coulomb excitation.

It has been noted that a few of the transitions listed in Table I are mixed $M1+E2$. It is possible to measure the relative phases of the two multipoles in a mixture, and this has been done in several cases but not for any transitions considered here. This raises the interesting possibility that the nature of the magnetic part of the mixture might be determined from a knowledge of the sign of δ. The sign of the matrix element cannot be determined from transition data which involve squares, but it can be found from magnetic moment data and has been done essentially for $M1$ (interaction).

If $M1$ (ordinary) and $M1$ (interaction) have different signs, then a determination of the sign of δ will uniquely determine the nature of the transition. However, this implies that the sign of $E2$ is known and for odd-neutron nuclei, on the single-particle model, and only contribution to an electric transition is from core recoil and this is too small to explain observed $E2$ lifetimes. Even for odd-proton nuclei, the Weisskopf formula[51] does not give results which agree very well with experiments. Nevertheless, it would be highly suggestive if we could find a case for which the observed sign of δ cannot be explained assuming the ordinary magnetic moment operator but could be explained using the interaction moment operator.[52]

V. CONCLUSION

We have carried out an investigation to see to what extent so-called forbidden magnetic dipole and octupole isomeric transitions can be explained by the interaction contributions to the magnetic moment operators. In order to make the calculations manageable, simplifying assumptions have been made regarding both the operators and wave functions. The dipole calculations were carried out using a Fermi model of the nucleus and it was found that all dipole transitions can be expressed in terms of one integral which was then arbitrarily set equal to that value which gives the best agreement with the data. Because the Fermi model cannot give results for higher multipoles, a shell model calculation was carried out in order to evaluate the octupole transition probabilities.

Even if the transitions in Zn and Os and the dipole transition in Ta are omitted from further consideration, it is seen in Table I that there is no exact quantitative agreement with the data. The effect of considering the radial nodes which are ignored on the Fermi model would be expected to reduce somewhat the value of the radial integral, and this is confirmed by the shell model calculations which require a somewhat larger value for the strength of the arbitrary function to get agreement with the data. If the nodes were somehow taken into account on the Fermi model, the form of the radial integral would be changed, the value of the integral being decreased, the more so the greater the number of nodes. This would necessitate a renormalization. Since the matrix elements for transitions with more nodes would be decreased more than those with fewer nodes, the calculated lifetimes for $3s_{1/2}\leftrightarrow 2d_{3/2}$ transitions would be increased (matrix elements decreased), whereas for $2d_{5/2}\leftrightarrow 1g_{7/2}$ transitions the lifetimes would be decreased. This would make agreement with the data somewhat poorer than that shown in Table I.

[50] A. W. Sunyar, Phys. Rev. 98, 653 (1955).

[51] Reference 29, pp. 595 ff.

[52] The possibility of obtaining information about the nuclear model from the sign of δ has been considered independently by S. Frankel and C. Greifinger (private communication).

Qualitatively, the most striking observation to be made in Table I is that dipole transitions in odd-proton nuclei are about an order of magnitude slower than those in odd-neutron nuclei, and it will be noticed that if the value of the integral I_F had been chosen separately for neutrons and protons, the calculated and observed transition probabilities would agree to within a factor of two. The inhibition of transition probabilities in odd-proton nuclei was observed by Graham and Bell[35] and a possible theoretical explanation has been given by Wallace[53] who pointed out that because of the Coulomb force, proton transitions will affect the nuclear wave functions over a greater distance than will neutron transitions where only specifically nuclear forces are involved. Thus in proton transitions the wave functions of many nucleons are involved and the cumulative effect may be enough to account for the inhibited transition probabilities. Other factors which have been neglected and which should be taken into account seem all to give corrections to the transition probabilities which are in the wrong direction to account for the inhibition of odd proton transitions. The effect of considering the radial nodes has already been considered above where it was found that the $g_{7/2}\leftrightarrow d_{5/2}$ transitions, which happen to be odd-proton ones, are not particularly slowed down. The effect on the Fermi model of the small difference in the value of P (the maximum momentum in the care) between neutrons and protons has been calculated only in the case of a square well potential and is found to give a correction in the wrong direction. Another possibility is another effect of the Coulomb repulsion. It has been shown[54] that the radius of the proton distribution in the nucleus is contracted somewhat by the Coulomb repulsion and this means that in an odd-neutron nucleus the odd neutron, which tends to be at the outside, has fewer protons with which to interact. This would be expected to decrease somewhat the transition probability of an odd-neutron nucleus, again in the wrong direction to improve agreement with the data.

Only odd-even nuclei have been considered here. Transitions in even-even nuclei seem to be most readily explained[55] in terms of the Bohr-Mottelson rotational level pattern. The situation in odd-odd nuclei is usually rather complicated owing to the fact that either of the outer nuclei can change state in an isomeric transition. In none of the odd-odd nuclei cases which we have looked into have we found any good evidence that a single nucleon undergoes a forbidden transition.

In view of the fact that no thorough analysis of all possible operators has been attempted, the results presented here are not intended to be conclusive. Nevertheless, the results do seem to show that the interaction effects can explain the dipole data, provided some mechanism can be found to account for the difference between odd-proton and odd-neutron nuclei. It is impossible to draw any conclusions regarding the forbidden octupole transitions owing to the paucity of data, although the calculations do suggest that the interaction moments may possibly account for these also.

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[53] P. R. Wallace, Phys. Rev. 98, 1205 (A) (1953).
[54] M. H. Johnson and E. Teller, Phys. Rev. 93, 357 (1954).
[55] G. Scharff-Goldhaber and J. Weneser, Phys. Rev. 98, 212 (1955).