weightless plates necessary for satisfactory pulse analyses could be easily achieved.

*Polonium.*—Polonium was chemically plated on a silver disk immersed in a dilute nitric acid solution of the uranium target.

*Methods of Radiation Detection.*—The methods used to determine quantitatively the presence of a particular isotope included beta-particle decay, alpha decay, alpha pulse-height distribution analysis, and parent-daughter separations. Table VII lists in some detail these methods as they apply to the specific nuclide.

### ACKNOWLEDGMENTS

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# Electron Double Scattering by Nuclear Magnetic Moments*

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Polarization effects in the scattering of high-energy electrons by charges and magnetic moments are discussed. A double scattering experiment using targets with lined-up nuclear spins should be a sensitive means of obtaining information about a possible difference in the charge and magnetic moment distributions of nuclei.

## I. INTRODUCTION

FOR some time now electron scattering by nuclei has served as a tool for the determination of the shape of the nuclear charge distribution.[1] So sensitive has this technique become that it has begun to yield information about the shape of the magnetic moment distribution as distinct from that of the charge. Although in the only case measured so far, the proton, no difference between the two distributions was detected,[2] one might, on the basis of the shell model, expect the two distributions to be significantly different for nuclei other than those of hydrogen.

The purpose of the present note is to exhibit the results one could expect from another scattering experiment more specifically suited to the direct measurement of the effects of the nuclear magnetic moment. Such an experiment would involve the detection of the polarizing effect of a scattering by nuclei whose magnetic moments point in a prescribed direction. Unfortunately, no feasible experimental techniques are at present available for lining up nuclear spins for such a purpose. Presumably, however, it is but a question of time (and perhaps not a very long one, at that) that they will be. In that event the experiment here proposed would become possible.

## II. GENERAL DISCUSSION

We can very easily get an idea of the relative order of magnitude of the magnetic moment effects in scattering. While the measure of the strength of the Coulomb interaction is $eZ$, $e$ being the electronic charge and $Z$, the atomic number of the nucleus, the measure of the magnetic moment interaction is $|\mathbf{p}|\mu$, where $\mathbf{p}$ is the momentum of the scattered electron and $\mu$, the magnetic moment of the nucleus. The relative size of the two effects will therefore be of the order of magnitude

$$\frac{|\mathbf{p}|\mu}{eZ}=\frac{1}{2}\frac{|\mathbf{p}|}{Mc}\cdot\frac{\mu/\mu_N}{Z},$$

where $\mu_N$ is the nuclear magneton, and $M$ is the proton mass. The energies used now or shortly to be used in electron scattering, of the order of one Bev or higher, are therefore large enough to produce sizable magnetic effects. It is also clear then that the effects are biggest for small values of $Z$.

The specific effects of the magnetic moment interaction which cannot be produced by the Coulomb field, are due to its noncentral nature, i.e., the fact that it contains a preferred direction. They are therefore not obtainable from a target whose nuclear magnetic moments are oriented at random. Those results (antisymmetries, etc.) which are obtainable from unoriented scatterers are also, qualitatively, obtainable from a pure Coulomb field and are therefore much harder to disentangle from those of the latter alone. Since the aim is to pin down experimentally the specific magnetic moment phenomena, we will be primarily interested in

* Supported in part by the National Science Foundation.
[1] Particularly the experiments by the group at Stanford. See Hahn, Ravenhall, and Hofstadter, Phys. Rev. 101, 1131 (1956), and previous papers mentioned there.
[2] R. Hofstadter and R. W. McAllister, Phys. Rev. 98, 217 (1955); R. Hofstadter and E. E. Chambers, Bull. Am. Phys. Soc. Ser. II, 1, 10 (1956).

those features which disappear if the scatterers are randomly oriented.

The agent of the special orientation effects we are looking for is of course the electron's spin. Let $u_+$ and $u_-$ be (normalized) Dirac wave functions for spins pointing, respectively, forward and backward. Then

$$u=au_+ + bu_-,\quad |a|^2+|b|^2=1,\quad b/a=(\tan\tfrac{1}{2}\chi)e^{i\omega},$$

can be interpreted as the wave function of an electron whose spin axis has the orientation $(\chi,\omega)$ in its rest system.[3] Such a wave function always represents an entirely polarized beam; a less than completely polarized beam forms a mixed state. The appropriate means for its representation is a density matrix.[4] We shall, however, be able to avoid the use of the latter.

In a coordinate system in which the electron is not at rest, the eigenvalues of $\boldsymbol{\sigma}\cdot\mathbf{n}$ are constants of the motion only if $\mathbf{n}$ points in a direction parallel to the momentum. Thus the physical significance of $|a|$ and $|b|$ is much more immediate than that of their relative phase:

$$\eta\equiv|a|^2-|b|^2=2|a|^2-1=u^\dagger\boldsymbol{\sigma}\cdot\mathbf{p}u|\mathbf{p}|^{-1} \qquad (1)$$

is the difference between the probabilities of finding the spin forward and backward. We shall call $\eta$ the $s$-polarization; when $\eta=0$, we call the beam $s$-unpolarized. The $s$-polarization is therefore a property of a wave function; it refers to a pure state. In addition, the wave function contains the relative phase of $a$ and $b$, which we shall take into account without explicit reference to polarization.

If a scattering of an unpolarized beam results in a polarization, or $s$-polarization, then the proper means for analyzing it is a second scattering experiment. Any dependence of the second on the first scattering is evidence for a polarization phenomenon, or, equivalently, for an $s$-polarization possibly together with an interference effect due to coherence between the scatterings. It is in the interference that the relative phase of $a$ and $b$ is important. Rather than use an explicit polarization formalism, we shall directly set up a double scattering formulation, in which coherence effects are automatically taken into account. It will turn out that the interference terms are quite negligible at high energies, and we can restrict ourselves to the explicit consideration of the simpler $s$-polarization. The latter, we shall see, is entirely due to the magnetic moment interaction. This must, indeed, be rigorously true and cannot be a feature of the approximation. The $s$-polarization is a pseudoscalar quantity; no such object can be formed from two vectors, the incoming and outgoing momenta, alone. Therefore, a spherically symmetric potential such as the Coulomb field or randomly oriented magnetic moments, cannot produce an $s$-polarization.

We shall first set up an exact expression for the double scattering cross section which includes coherence phenomena. We are then going to calculate, in the first Born approximation, the differential cross section for double scattering by nuclei which consist of static charge and magnetic moment distributions. The use of the Born approximation is justified by the fact that the phenomena of interest will be most pronounced for low values of $Z$. In agreement with Mott,[3] we obtain no coherence effects as a result of the pure (spherical) charge distributions. Those will occur but in a higher approximation and are therefore relatively small.

Owing to the magnetic moment, we obtain both an interference term and an $s$-polarization term. The former, however, is quite negligible at the energies considered (just as is Mott's interference term for the Coulomb field). The only "memory" of the first scattering is therefore carried by the $s$-polarization, which is given as a function of energy, scattering angle, and magnetic moment direction. These dependencies are exhibited in special graphs which can be used to facilitate experimental detection of a difference between the charge and magnetic moment distributions.

## III. CALCULATION

We consider an electron in a fixed external electromagnetic field. Since we neglect the nuclear recoil, the results will be applicable only to nuclei heavier than the proton. Let[5]

$$H=(1+\mathcal{H}G_0)^{-1}\mathcal{H} \qquad (2)$$

where[6]

$$\mathcal{H}=-e\gamma A,\quad G_0{}^{-1}=\gamma p+m. \qquad (3)$$

Then the wave function which "evolves" from an incoming plane wave $\psi_{0p\lambda}$ of momentum $\mathbf{p}$ and spin $\lambda$, can be written

$$\begin{aligned}\psi_{p\lambda}(\mathbf{r})&=\psi_{0p\lambda}(\mathbf{r})-(m+\gamma_0p_0+i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla})\\&\quad\times\int\int(dr')^3(dr'')^3(\mathbf{r}|(m^2+p^2)^{-1}|\mathbf{r}')\\&\quad\times(\mathbf{r}'|H|\mathbf{r}'')\psi_{0p\lambda}(\mathbf{r}'')\underset{r\to\infty}{\longrightarrow}\psi_{0p\lambda}(\mathbf{r})\\&\quad-(m+\gamma_0p_0-\boldsymbol{\gamma}\cdot\mathbf{q})\frac{e^{ipr}}{r}\frac{1}{4\pi}\int\int(dr')^3\\&\qquad\times(dr'')^3e^{-i\mathbf{q}\cdot\mathbf{r}'}(\mathbf{r}'|H|\mathbf{r}'')\psi_{0p\lambda}(\mathbf{r}'')\\&=u_\lambda(\mathbf{p})e^{i\mathbf{p}\cdot\mathbf{r}}-\frac{e^{ipr}}{r}\cdot\frac{(2\pi)^3}{(4\pi)}(m-\gamma q)(\mathbf{q}|H|\mathbf{p})u_\lambda(\mathbf{p}),\end{aligned}$$

where $\mathbf{q}=|\mathbf{p}|\mathbf{r}/r$. Hence at large distances from the

[3] N. F. Mott, Proc. Roy. Soc. (London) **A124**, 425 (1929), and **A135**, 429 (1932).

[4] H. A. Tolhoek and S. R. De Groot, Physica **17**, 1 (1951); also H. Mendlowitz and K. M. Case, Phys. Rev. **97**, 33 (1955) and **100**, 1551 (1955).

[5] See R. G. Newton, Phys. Rev. **94**, 1773 (1954).

[6] We use natural units, with $\hbar=c=1$. The $\gamma$ matrices are skew-Hermitian; $\gamma_0$ is Hermitian.

scattering center, the state of the beam observed in the direction $\mathbf{q}$ is described by the amplitude

$$u(\mathbf{q})=-2\pi^2(m-\gamma q)(\mathbf{q}|H|\mathbf{p})u_\lambda(\mathbf{p}). \tag{4}$$

If the initial amplitude is normalized so that $u_\lambda^\dagger(\mathbf{p})u_\lambda(\mathbf{p})=1$, then the differential cross section is (with $\bar{u}=u^\dagger\gamma_0$)

$$\sigma_\lambda(\mathbf{q},\mathbf{p})=|u(\mathbf{q})|^2=8\pi^4 p_0\bar{u}_\lambda(\mathbf{p})(\mathbf{p}|\gamma_0 H^\dagger\gamma_0|\mathbf{q})\times(m-\gamma q)(\mathbf{q}|H|\mathbf{p})u_\lambda(\mathbf{p}).$$

We obtain from this the differential cross section for double scattering via an intermediate momentum $\mathbf{Q}$ (which points from the first to the second scattering center) by setting $\mathbf{p}=\mathbf{Q}$ and substituting (4) after replacing there $\mathbf{q}$ by $\mathbf{Q}$. The result is restricted to situations in which the distance between scatterings is large compared to the effective regions of interaction. Then[7]

$$\sigma_\lambda(\mathbf{q},\mathbf{Q},\mathbf{p})=32\pi^{16}p_0\bar{u}_\lambda(\mathbf{p})(\mathbf{p}|\gamma_0 H_1^\dagger\gamma_0|\mathbf{Q})\times(m-\gamma Q)(\mathbf{Q}|\gamma_0 H_2^\dagger\gamma_0|\mathbf{q})(m-\gamma q)(\mathbf{q}|H_2|\mathbf{Q})\times(m-\gamma Q)(\mathbf{Q}|H_1|\mathbf{p})u_\lambda(\mathbf{p}).$$

Since $(m-\gamma p)\gamma_0/2p_0$ is the projection onto positive energy states, the double scattering cross section for an unpolarized beam is

$$\sigma(\mathbf{q},\mathbf{Q},\mathbf{p})=16\pi^{16}\,\mathrm{tr}(m-\gamma p)(\mathbf{p}|\gamma_0 H_1^\dagger\gamma_0|\mathbf{Q})\times(m-\gamma Q)(\mathbf{Q}|\gamma_0 H_2^\dagger\gamma_0|\mathbf{q})(m-\gamma q)(\mathbf{q}|H_2|\mathbf{Q})\times(m-\gamma Q)(\mathbf{Q}|H_1|\mathbf{p}). \tag{5}$$

So far this result is exact. In the first Born approximation now $H=\mathcal{H}$, and in the case of a point charge and a point magnetic moment

$$\mathcal{H}=-e\gamma A=(e^2Z\gamma_0+e\boldsymbol{\gamma}\cdot\boldsymbol{\mu}\times\boldsymbol{\nabla})r^{-1}=e^2Z(\gamma_0+\lambda|\mathbf{p}|^{-1}\mu^{-1}\boldsymbol{\gamma}\cdot\boldsymbol{\mu}\times\boldsymbol{\nabla})r^{-1},$$

where

$$\lambda=\tfrac{1}{2}(|\mathbf{p}|/Mc)(\mu/\mu_N)Z^{-1}. \tag{6}$$

If we write

$$\mathbf{k}_1=\boldsymbol{\mu}_1\times(\mathbf{Q}-\mathbf{p})/\mu_1|\mathbf{p}|,\quad \mathbf{k}_2=\boldsymbol{\mu}_2\times(\mathbf{q}-\mathbf{Q})/\mu_2|\mathbf{p}|, \tag{7}$$

then the double scattering cross section becomes in the first Born approximation

$$\sigma(\mathbf{q},\mathbf{Q},\mathbf{p})=(\mathcal{R}^2/8p_0^4)\tfrac{1}{4}\,\mathrm{tr}(m-\gamma p)(\gamma_0+i\lambda_1\boldsymbol{\gamma}\cdot\mathbf{k}_1)\times(m-\gamma Q)(\gamma_0+i\lambda_2\boldsymbol{\gamma}\cdot\mathbf{k}_2)(m-\gamma q)\times(\gamma_0-i\lambda_2\boldsymbol{\gamma}\cdot\mathbf{k}_2)(m-\gamma Q)(\gamma_0-i\lambda_1\boldsymbol{\gamma}\cdot\mathbf{k}_1), \tag{8}$$

where $\mathcal{R}$ is the Rutherford cross section.

For an extended charge and magnetic moment distribution, the only modification is the multiplication of $\mathcal{R}$ and $\lambda$ by form factors. If the radii of the two distributions are $R_c$ and $R_m$, and their radial shapes described by $\rho_c$ and $\rho_m$ with

$$\int_0^1 dvv^2\rho_c(v)=\int_0^1 dvv^2\rho_m(v)=1,$$

then the form factors are

$$S^{(c)}=\int_0^1 dvv^2\rho_c(v)\frac{\sin vKR_c}{vKR_c},\qquad S^{(m)}=\int_0^1 dvv^2\rho_m(v)\frac{\sin vKR_m}{vKR_m}, \tag{9}$$

where

$$K=2|\mathbf{p}|\sin(\tfrac{1}{2}\theta), \tag{10}$$

and $\theta$ is the scattering angle. The modifications in (8) are

$$\mathcal{R}^2\rightarrow(\mathcal{R}S_1^{(c)}S_2^{(c)})^2, \tag{11}$$

$$\lambda\rightarrow\lambda S,\quad S=S^{(m)}/S^{(c)}. \tag{12}$$

After evaluation of the trace, (8) becomes

$$\sigma(\mathbf{q},\mathbf{Q},\mathbf{p})=\mathcal{R}^2\{[1-\beta^2\sin^2(\tfrac{1}{2}\theta_1)+4\lambda_1^2\beta^2\Theta_1\sin^2(\tfrac{1}{2}\theta_1)]\times[1-\beta^2\sin^2(\tfrac{1}{2}\theta_2)+4\lambda_2^2\beta^2\Theta_2\sin^2(\tfrac{1}{2}\theta_2)]+\lambda_1\lambda_2\beta^4(\mathbf{k}_1\cdot\mathbf{p}_1\times\mathbf{p}_2)(\mathbf{k}_2\cdot\mathbf{p}_2\times\mathbf{p}_3)+\lambda_1\lambda_2\beta^2(1-\beta^2)\mathbf{k}_1\times(\mathbf{p}_2-\mathbf{p}_1)\cdot\mathbf{k}_2\times(\mathbf{p}_3-\mathbf{p}_2)\}, \tag{13}$$

where $\beta=|\mathbf{p}|/p_0=v/c$,

$$\Theta=\cos^2(\tfrac{1}{2}\theta)\cos^2(\boldsymbol{\mu},\mathbf{p}_i\times\mathbf{p}_f)+\sin^2(\tfrac{1}{2}\theta)\sin^2(\boldsymbol{\mu},\mathbf{p}_i-\mathbf{p}_f), \tag{14}$$

and $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ are unit vectors in the directions $\mathbf{p}$, $\mathbf{Q}$, and $\mathbf{q}$.

The first term in the brace in (13), which consists of a product of two brackets, represents the product of the two cross sections for single scattering of an unpolarized beam. The second term is the modification due to the $s$-polarization effects of the scattering. It can be shown directly[8] that the cross sections $\sigma_{+-}$, $\sigma_{++}$, $\sigma_{-+}$, $\sigma_{--}$ for single scattering from backward spin to forward spin, forward to forward, etc., are

$$\sigma_{+-}=\sigma_{-+}=\mathcal{R}(1-\beta^2)\sin^2(\tfrac{1}{2}\theta),$$
$$\sigma_{++}=\mathcal{R}[\cos^2(\tfrac{1}{2}\theta)+4\lambda^2\beta^2\Theta\sin^2(\tfrac{1}{2}\theta)+\beta\lambda(\mathbf{k}\cdot\mathbf{p}_i\times\mathbf{p}_f)],$$
$$\sigma_{--}=\mathcal{R}[\cos^2(\tfrac{1}{2}\theta)+4\lambda^2\beta^2\Theta\sin^2\tfrac{1}{2}\theta-\beta\lambda(\mathbf{k}\cdot\mathbf{p}_i\times\mathbf{p}_f)]. \tag{15}$$

The last term in (13), finally, is due to interference between the two scatterings; it is a true polarization phenomenon.

We observe that the last term in (13) contains the factor $(1-\beta^2)=(mc^2/p_0)^2$ and is therefore extremely small at energies over 100 Mev. Consequently, in the

[7] The subscripts 1 and 2 on $H$ refer to the possibility that the two scattering centers are not identical.

[8] We merely insert the projections $\frac{1}{2}(1\pm\boldsymbol{\sigma}\cdot\mathbf{p}/|\mathbf{p}|)$ onto forward and backward spin states into the trace in the expression for the single-scattering cross section.

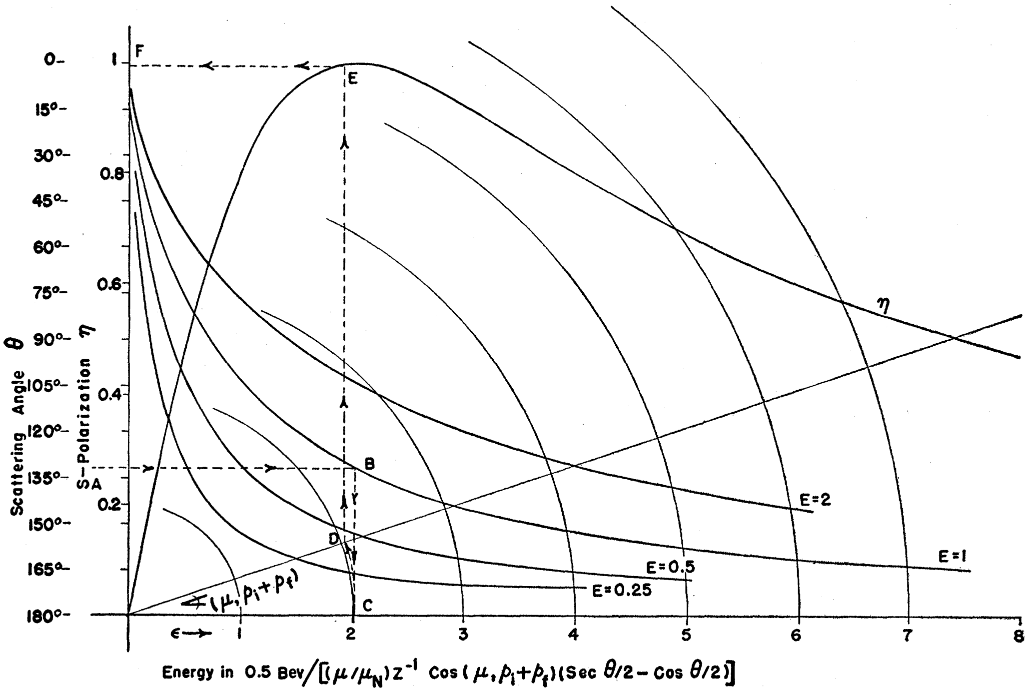



FIG. 1. The curve marked $\eta$ represents the $s$-polarization due to a magnetic moment in the scattering plane, as a function of $\mathcal{E}$, the indicated energy angle combination. The other curves represent the scattering angle $\theta$ as a function of $\mathcal{E}$, for $\cos(\boldsymbol{\mu}, \mathbf{p}_i+\mathbf{p}_f)=1$ and various values of $E$, the energy parameter. The dashed line $A$–$F$ indicates a reading of the $s$-polarization for given scattering angle, magnetic moment direction, and energy.

relativistic limit (in which we set $\beta=1$), the only correlation between the two scatterings is due to $s$-polarization:

$$\begin{aligned}\sigma(\mathbf{q},\mathbf{Q},\mathbf{p}) \underset{\text{rel.}}{\longrightarrow} \mathcal{R}^2\{&[\cos^2(\tfrac{1}{2}\theta_1)+4\lambda_1^2\Theta_1 \sin^2(\tfrac{1}{2}\theta_1)] \\ &\times[\cos^2(\tfrac{1}{2}\theta_2)+4\lambda_2^2\Theta_2 \sin^2(\tfrac{1}{2}\theta_2)] \\ &+\lambda_1\lambda_2(\mathbf{k}_1\cdot \mathbf{p}_1\times \mathbf{p}_2)(\mathbf{k}_2\cdot \mathbf{p}_2\times \mathbf{p}_3)\} \\ &=\sigma_{0+}^{(2)}\sigma_{+0}^{(1)}+\sigma_{0-}^{(2)}\sigma_{-0}^{(1)} \\ &=\sigma_{00}^{(2)}\sigma_{00}^{(1)}(1+\eta_1\eta_2),\end{aligned} \tag{16}$$

where

$$\sigma_{0\pm}=\sigma_{+\pm}+\sigma_{-\pm}, \quad \sigma_{\pm0}=\tfrac{1}{2}(\sigma_{\pm+}+\sigma_{\pm-}),$$

and the $s$-polarization

$$\eta=\frac{\sigma_{+0}-\sigma_{-0}}{\sigma_{+0}+\sigma_{-0}}=\frac{4\beta\lambda \cos(\frac{1}{2}\theta) \sin^2(\frac{1}{2}\theta) \cos(\boldsymbol{\mu}, \mathbf{p}_i+\mathbf{p}_f)}{1-\beta^2 \sin^2(\frac{1}{2}\theta)+4\lambda^2\beta^2\Theta \sin^2(\frac{1}{2}\theta)}$$

$$\underset{\text{rel.}}{\longrightarrow}\frac{4\lambda \cos(\frac{1}{2}\theta) \cos(\boldsymbol{\mu}, \mathbf{p}_i+\mathbf{p}_f)}{\cot^2(\frac{1}{2}\theta)+4\lambda^2\Theta}. \tag{17}$$

Since $\eta$ changes sign if $\boldsymbol{\mu}$ is replaced by $-\boldsymbol{\mu}$, it can be isolated from the double scattering cross section by comparing two double scatterings which differ only by having one of their magnetic moments reversed. Let us call the corresponding double scattering cross sections $\sigma$ and $\sigma'$. Then

$$\eta_1\eta_2=(\sigma-\sigma')/(\sigma+\sigma')\equiv\Delta. \tag{18}$$

If one, finally, chooses the second scattering (in the double scattering) a replica of the first, with the same kind of target and similar angles, then

$$\eta=\Delta^{\frac{1}{2}}. \tag{19}$$

## IV. DISCUSSION OF RESULT

Let us consider expression (17) for the $s$-polarization. A glance at (14) shows that, other angles being equal, $\eta$ is largest when the magnetic moment lies in the plane of the scattering. We shall therefore assume that to be the case from now on. We can then write:

$$\eta=\mathcal{E}/(1+\tfrac{1}{4}\mathcal{E}^2), \tag{20}$$

where

$$\mathcal{E}=SE[\sec(\tfrac{1}{2}\theta)-\cos(\tfrac{1}{2}\theta)] \cos(\boldsymbol{\mu}, \mathbf{p}_i+\mathbf{p}_f), \tag{21}$$

$$E=2(p_0/Mc)(\mu/\mu_N)Z^{-1}. \tag{22}$$

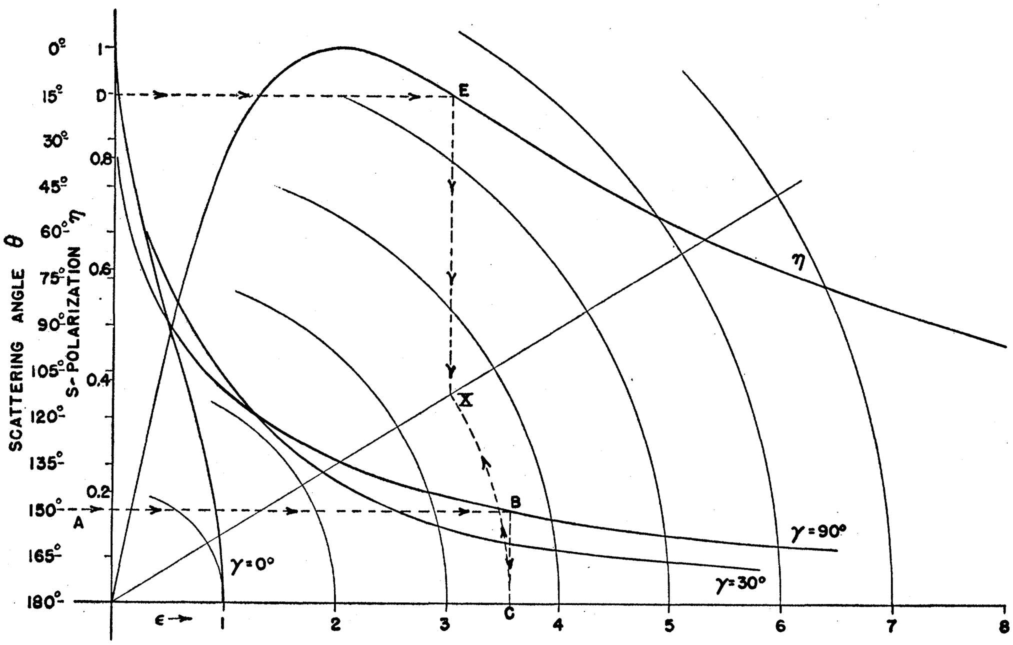
In Fig. 1 we have plotted $\eta$ and $\theta$ against $\mathcal{E}$, for

$$S=1, \quad \cos(\boldsymbol{\mu}, \mathbf{p}_i+\mathbf{p}_f)=1, \quad \text{and} \quad E=0.25,\ 0.5,\ 1,\ 2.$$

For an arbitrary scattering angle $\theta$ and arbitrary angle between the magnetic moment and the line bisecting the scattering angle, one merely draws a line at the latter angle with the abscissa and proceeds as indicated in Fig. 1 by the dashed line from $A$ to $F$. The point $B$, of course, lies on the curve appropriate to the electron energy. The curve $CD$ is a circular arc with center at the origin.

In order to obtain information about $S$, the ratio of the charge and magnetic moment form factors, from an experimental knowledge of $\eta$ [via (18)] as a function of the energy, one adopts an arbitrary energy scale on

FIG. 2. The curve marked $\eta$ represents the $s$-polarization due to a magnetic moment in the scattering plane, as a function of $\mathcal{E}$. The other curves represent the scattering angle $\theta$ as a function of $\mathcal{E}$, for $E=1$ and various values of $\gamma=\measuredangle(\boldsymbol{\mu},\mathbf{p}_i)$. The dashed lines $A$–$X$ and $D$–$X$ indicate a way of plotting the locus of $X$, a straight line for equal charge and magnetic moment distributions, if $\eta$ is given experimentally as a function of $\theta$.




the abscissa of Fig. 1, proceeds as from $F$ to $E$[9] and plots the points $D$, the intersection of the vertical from $E$ with the circle through $C$. If the two distributions in the nucleus are equal, then the points $D$ must form a straight line through the origin. Any deviation from such a straight line indicates unequal charge and magnetic moment distributions.

Figure 2 contains a plot of $\eta$ against $\mathcal{E}$, and various graphs of $\theta$ against $\mathcal{E}$ with $SE=1$ and fixed angle $\gamma$ between the magnetic moment $\boldsymbol{\mu}$ and the initial momentum $\mathbf{p}_i$, for several values of $\gamma$. This figure can be used if $\eta$ is experimentally given, at a fixed energy, as a function of the scattering angle $\theta$. One then selects the appropriate $\mathcal{E}-\theta$ curve for the angle $\gamma$ used and proceeds as indicated by the dashed line[9] in Fig. 2: from $A$ to $C$ and from $D$ to $E$. One then plots the points $X$, the intersection of the circular arc through $C$ with the vertical through $E$. The points $X$ must again fall on a straight line through the origin if $S=1$.

Finally, a word about modifications if the scattering magnetic moments are not perfectly lined up. In that case $\cos(\boldsymbol{\mu},\ \mathbf{p}_i+\mathbf{p}_f)$ is replaced by

$$\int d\Omega_\mu f(\boldsymbol{\mu}) \cos(\boldsymbol{\mu},\ \mathbf{p}_i+\mathbf{p}_f).$$

The only complication arises from the fact that if the magnetic moment does not lie exactly in the plane of the scattering, $\eta$ cannot be written quite as simply as in (19). The graphs must then be somewhat modified. The principle, however, will be unchanged.

[9] Care must be taken to be sure on which part of the $\eta$-curve $E$ ought to be. That depends on whether $\eta$ is found to increase or decrease with the energy.