Electromagnetic Waves in an Ionized Gas*

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A study is made of the effect of space-dependent electric fields upon the complex conductivity of a low-pressure ionized gas. The case of a standing wave is considered first and it is found that the diffusion constant for the electrons becomes space dependent. Finally the case of a traveling wave in an infinite ionized medium is studied. For the latter situation, the spatial variation of the field has no effect on the conductivity.

A. STANDING WAVE

It has been customary in the literature[1,2] to calculate the complex conductivity of a low-pressure ionized gas at high frequency by using the formula

$$\sigma_c=\left[\frac{e^2}{m}\frac{\nu}{\nu^2+\omega^2}-i\frac{e^2\omega}{m(\nu^2+\omega^2)}\right]n(\mathbf{r}). \tag{1}$$

Here ω is angular frequency of the field, ν is the collision frequency of electrons with gas molecules and $n(\mathbf{r})$ is the number of electrons per unit volume at position $\mathbf{r}$. Formula (1) assumes ν to be constant; it is based on the work of Margenau,[3] whose explitic formula refers to a constant mean free path. The procedure that has been used to calculate $n(\mathbf{r})$[1,2] is to solve the diffusion equation using the ambipolar diffusion coefficient. The physical process involved is simply a balance between diffusion of electrons and ions to the walls and the generation of the electrons and ions in the gas. More recently Allis and Rose[4] have considered the diffusion problem in a more general manner. They take the diffusivities and mobilities of the electrons and ions to be constant and obtain differential equations for the current densities and the electric field due to space charge under conditions of equal positive and negative current densities.

The question we wish to raise is the following. Formula (1) is based on a derivation in which it was assumed that both n and the microwave electric field were not dependent on position. Also the method for calculating $n(\mathbf{r})$ involves using constant diffusivities and mobilities. How valid are the preceding methods when both n and the microwave field vary with position?

In order to answer the foregoing question, we consider a simple model of a microwave standing wave of the form $E_x=E_0\cos\beta z\cos\omega t$ between infinite parallel plates at $z=0$ and $z=L$. We take the ionization to be maintained by radiation such that the rate of generation of electrons per unit volume is uniform and equal to Q.

We consider with Margenau[3] only elastic encounters of electrons with molecules. We neglect encounters of electrons with electrons, assuming[5] that the ratio of electrons to molecules is less than 10^{-3}. For the electrons we must solve the Boltzmann transport equation for the distribution function $f(z,\mathbf{v},t)$

$$\frac{\partial f}{\partial t}+v_z\frac{\partial f}{\partial z}+\frac{eE_0}{m}\cos\beta z\cos\omega t\frac{\partial f}{\partial v_x}+\frac{eE'(z)}{m}\frac{\partial f}{\partial v_z}=\left(\frac{\delta f}{\delta t}\right)_e, \tag{2}$$

where $E'(z)$ is the unknown space charge field and $(\delta f/\delta t)_e$ is the rate of change of f due to encounters. Since the ionization is caused by radiation we can assume that the ionization electrons have a distribution of velocities which is almost isotropic. Thus

$$(\delta f/\delta t)_e=(\delta f/\delta t)_{\text{elastic}}+U(v),$$

where

$$\int_0^\infty U(v)4\pi v^2dv=Q,$$

and[6,7]

$$\left(\frac{\delta f}{\delta t}\right)_{\text{elastic}}=Nv\int[f(v,\theta',\varphi')-f(v,\theta,\varphi)]\sigma(v,\chi)d\Omega'$$
$$+\frac{m}{M}\frac{N}{v^2}\frac{\partial}{\partial v}\left[v^4\int(1-\cos\chi)f(v,\theta',\varphi')\sigma(v,\chi)d\Omega'\right]. \tag{3}$$

In Eq. (3), N is the number density of molecules; $\sigma(v,\chi)$ is the differential scattering cross section for elastic scattering through angle χ; v, θ, φ are spherical coordinates in velocity space ($\cos\theta=v_x/v$, $\tan\varphi=v_y/v_z$); and M is the mass of a molecule.

Van Zandt[6] has shown that if the distribution function is expanded in the Fourier spherical harmonic series:

$$f=\sum_{l,m,n}f_{\pm l,m,n}(v,z)\begin{Bmatrix}\cos l\omega t\\ \sin l\omega t\end{Bmatrix}Y_n{}^m(\theta,\varphi)$$

($f_{-l,m,n}(v,z)$ going with $\sin l\omega t$), and if the external force is invariant under the product of transformations $x\to -x$ and $t\to t+(p+\frac{1}{2})(2\pi/\omega)$ (p integral), then the Boltzmann transport equation is invariant and the

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[1] E. Everhart and S. C. Brown, Phys. Rev. **76**, 839 (1949).
[2] Allis, Brown, and Everhart, Phys. Rev. **84**, 519 (1951).
[3] H. Margenau, Phys. Rev. **69**, 508 (1946).
[4] W. P. Allis and D. J. Rose, Phys. Rev. **93**, 84 (1954).
[5] J. H. Cahn, Phys. Rev. **75**, 293 (1949).
[6] T. E. Van Zandt, Ph.D. dissertation, Yale University, 1954 (unpublished).
[7] Morse, Allis, and Lamar, Phys. Rev. **48**, 412 (1935).

only terms in the above infinite series which do not vanish are those for which $l+m+n$ is even.

Still following reference 3 we will consider terms for which $l\leqslant 1$, $n\leqslant 1$ and we obtain

$$f=f_{000}(v,z)+f_{011}(v,z)\sin\theta\cos\varphi +[f_{101}(v,z)\cos\omega t+g_{101}(v,z)\sin\omega t]\cos\theta. \quad (4)$$

No term in $Y_1^{-1}(\theta,\varphi)$ appears since f must be invariant to the transformation $y\to -y$. Using Eq. (4) $(\delta f/\delta t)_{\text{elastic}}$ becomes[6]

$$\left(\frac{\delta f}{\delta t}\right)_{\text{elastic}}=-\nu[f_{101}\cos\omega t+g_{101}\sin\omega t]\cos\theta$$
$$-\nu f_{011}\sin\theta\cos\varphi+\frac{m}{M}\frac{1}{v^2}\frac{\partial}{\partial v}\left[v^3\nu f_{000}+\frac{kT}{m}v^2\nu\frac{\partial f_{000}}{\partial v}\right], \quad (5)$$

where ν is the collision frequency

$$\nu=2\pi Nv\int_{-1}^{+1}\sigma(v,\chi)(1-\cos\chi)d(\cos\chi).$$

We are now prepared to solve the Boltzmann transport equation by insertion of Eqs. (4) and (5) into Eq. (2). We are thus led to the following set of equations, using the method of Margenau and Hartman[8]

$$\frac{-m}{Mv^2}\frac{\partial}{\partial v}\left[v^3\nu f_{000}+\frac{kT}{m}v^2\nu\frac{\partial f_{000}}{\partial v}\right]+\frac{v}{3}\frac{\partial f_{011}}{\partial z}$$
$$+\frac{eE_0}{6m}\frac{\cos\beta z}{v^2}\frac{\partial}{\partial v}(v^2f_{101})+\frac{eE'(z)}{3m}\frac{1}{v^2}\frac{\partial}{\partial v}v^2f_{011}-U(v)=0, \quad (6)$$

$$\nu f_{011}+v\frac{\partial f_{000}}{\partial z}+\frac{eE'(z)}{m}\frac{\partial f_{000}}{\partial v}=0, \quad (7)$$

$$\nu f_{101}+\omega g_{101}+\frac{eE_0}{m}\cos\beta z\frac{\partial f_{000}}{\partial v}=0, \quad (8)$$

$$\nu g_{101}-\omega f_{101}=0. \quad (9)$$

We first consider the pair of equations, (8) and (9), and obtain from these

$$f_{101}=-\frac{eE_0}{m}\cos\beta z\frac{\nu}{\nu^2+\omega^2}\frac{\partial f_{000}}{\partial v}, \quad (10)$$
$$g_{101}=(\omega/\nu)f_{101}.$$

Since the microwave current density j_x is

$$j_x=\frac{4\pi}{3}e\int_0^\infty[f_{101}\cos\omega t+g_{101}\sin\omega t]v^3dv, \quad (11a)$$

[8] H. Margenau and L. M. Hartman, Phys. Rev. **73**, 309 (1948).

we find that

$$\sigma_c=-\frac{e^2}{m}\left[\frac{4\pi}{3}\int_0^\infty\frac{\partial f_{000}}{\partial v}\frac{\nu}{\nu^2+\omega^2}v^3dv -i\frac{4\pi}{3}\int_0^\infty\frac{\partial f_{000}}{\partial v}\frac{\omega v^3dv}{\nu^2+\omega^2}\right]. \quad (11b)$$

For constant collision frequency we thus obtain Eq. (1). We notice that this result is unaffected by the spatial variation of the microwave field. However if the microwave field were responsible for inelastic collisions the collision term $(\delta f/\delta t)_e$ might have a contribution in Eqs. (8) and (9) and Eq. (1) would be modified.

Let us now consider the equation that determines f_{000} which in turn determines

$$n=4\pi\int_0^\infty f_{000}v^2dv.$$

We find that f_{000} satisfies

$$\frac{m}{Mv^2}\frac{\partial}{\partial v}\left[\nu v^3f_{000}+\frac{kT}{m}\nu v^2\frac{\partial f_{000}}{\partial v}\right]$$
$$+\left(\frac{eE_0}{m}\right)^2\frac{\cos^2\beta z}{6v^2}\frac{\partial}{\partial v}\left(\frac{\nu v^2}{\nu^2+\omega^2}\frac{\partial f_{000}}{\partial v}\right)$$
$$+\frac{v^2}{3\nu}\frac{\partial^2 f_{000}}{\partial z^2}+\frac{e}{3m}\frac{v}{\nu}\frac{\partial}{\partial z}\left(E'(z)\frac{\partial f_{000}}{\partial v}\right)$$
$$+\frac{eE'}{3mv^2}\frac{\partial}{\partial v}\left(\frac{v^3}{\nu}\frac{\partial f_{000}}{\partial z}\right)$$
$$+\left(\frac{eE'}{m}\right)^2\frac{1}{3v^2}\frac{\partial}{\partial v}\left(\frac{v^2}{\nu}\frac{\partial f_{000}}{\partial v}\right)+U(v)=0. \quad (12)$$

If Eq. (12) is multiplied by $4\pi v^2$ and integrated over all velocities the result is

$$\frac{d^2}{dz^2}\left[\frac{4\pi}{3}\int_0^\infty\frac{v^4}{\nu}f_{000}dv\right]+\frac{d}{dz}\left[\frac{4\pi e}{3m}E'\int_0^\infty\frac{v^3}{\nu}\frac{\partial f_{000}}{\partial v}dv\right]$$
$$=-4\pi\int_0^\infty U(v)v^2dv=-Q. \quad (13)$$

Now the diffusion current is

$$j_z=\frac{4\pi e}{3}\int_0^\infty f_{011}v^3dv.$$

Because of Eq. (9), we have

$$j_z=-\frac{4\pi e}{3}\frac{d}{dz}\int_0^\infty\frac{v^4f_{000}}{\nu}dv-\frac{4\pi e^2}{3m}E'\int_0^\infty\frac{\partial f_{000}}{\partial v}\frac{v^3}{\nu}dv.$$

If we consider the number current density $\Gamma_z = j_z/e$, we find the following from Eq. (13):

$$d\Gamma_z/dz = Q,$$

where

$$\Gamma_z = -\frac{d}{dz}\left[\frac{4\pi}{3}\int_0^\infty \frac{v^4}{\nu} f_{000}dv\right] - \left[\frac{4\pi e}{3m}\int_0^\infty \frac{\partial f_{000}}{\partial v}\frac{v^3}{\nu}dv\right]E'(z).$$

We now can define a space-dependent electron diffusivity and a mobility by the relations

$$D_e(z) = \frac{4\pi}{3}\int_0^\infty \frac{v^4}{\nu} f_{000}dv \bigg/ 4\pi\int_0^\infty v^2 f_{000}dv, \tag{14a}$$

$$\mu_e(z) = -\frac{4\pi}{3m}|e|\int_0^\infty \frac{\partial f_{000}}{\partial v}\frac{v^3}{\nu}dv \bigg/ 4\pi\int_0^\infty v^2 f_{000}dv. \tag{14b}$$

Thus the effect of spatial variation in the microwave field and of diffusion to the walls under conditions of varying space charge fields is to make f_{000} [see Eq. (12)] as well as the electron diffusivity and mobility depend on z. Under conditions of constant collision frequency, $\mu_e = \text{constant} = |e|/m\nu$. However, D_e is still a function of z. For simplicity, we consider the case of constant collision frequency and we have

$$\Gamma_z = -\frac{d}{dz}D_e(z)n(z) - \mu_e n(z)E'(z). \tag{15}$$

If f_{000} is Maxwellian, then the only condition which leads to constant D_e is that the average energy extracted from microwave and space-charge field between collisions shall be small compared with kT. We return later to an approximate solution of Eq. (12) so that we may calculate $D_e(z)$.

Let us consider the companion equation to Eq. (15) for the positive ions. Since the ions have much greater masses than the electrons, we make little error in taking Γ_{zp} = ion number current density $= -D_p(d/dz)P + \mu_p PE'$, where P is the density of positive ions. It is a good approximation to assume that μ_p and D_p are constant unless the space charge fields are very large. Theoretical formulas for the mobility for the ions are given by Kihara[9] and Maxfield and Benedict.[10] For the diffusivity we can use the well-known relation[11]:

$$D_p/\mu_p = kT/|e|.$$

We must therefore solve the following set of equations in a manner similar to the work of Allis and Rose

$$\Gamma_{zp} = -D_p dP/dz + \mu_p PE', \tag{16a}$$

$$\Gamma_z = -d[nD_e(z)]/dz - \mu_e nE', \tag{16b}$$

$$\Gamma_z = \Gamma_{zp} \quad \text{in steady state}, \tag{16c}$$

$$d\Gamma_z/dz = Q, \tag{16d}$$

$$dE'/dz = |e|(P-n)/\epsilon_0 \quad \text{(Poisson's equation)}. \tag{16e}$$

The boundary conditions are $n=0$, $P=0$, and $dE'/dz=0$ at $z=0$ and L. For Γ_z we have

$$\Gamma_z = Q(z-L'),$$

where L' is the distance at which Γ_z is zero. Because of the space-dependent D_e, there is no symmetry and L' is not $L/2$. The distance L' must be determined by solution of the problem. It is easily seen that the set of equations (16) leads to

$$n = -\frac{D_p\epsilon_0}{|e|[D_p+(\mu_p/\mu_e)D_e]}\frac{dE'}{dz} + \frac{\mu_p\epsilon_0[E'^2 - E_0'^2]}{2|e|[D_p+(\mu_p/\mu_e)D_e]} - \frac{[1+(\mu_p/\mu_e)]Q(\frac{1}{2}z^2 - L'z)}{D_p+(\mu_p/\mu_e)D_e}, \tag{17a}$$

where

$$L' = \frac{L}{2} - \frac{\mu_p\epsilon_0}{2|e|}\frac{(E_L'^2 - E_0'^2)}{[1+(\mu_p/\mu_e)]QL} \tag{17b}$$

and

$$E_L' = E_{z=L}', \quad E_0' = E_{z=0}'.$$

When Eq. (17a) is combined with Eq. (16b) it is apparent that we must solve a nonlinear eigenvalue equation in E' for which the fields at the boundaries E_0', E_L' are the eigenvalues.

Because of the difficulty of solving the general problem we confine ourselves here to the free-diffusion case, in which the space-charge field can be neglected. It is then convenient to start with Eqs. (16b) and (16d) with $E'=0$. This yields

$$d^2nD_e/dz^2 = -Q,$$

whose solution is

$$n = \frac{Q}{2D_e}(Lz - z^2). \tag{18}$$

One can estimate the magnitude of the electron density for free diffusion by taking the Debye shielding distance for the electron[4] to be greater than the length L. Thus, for free diffusion,

$$\frac{\epsilon_0 D_e}{n|e|\mu_e} > L^2, \tag{19}$$

where we take n and D_e to have an average value.

We now wish to calculate D_e when E' is neglected. Under the latter condition, for constant ν Eq. (12) becomes

$$\frac{m\nu}{M}\frac{\partial}{\partial v}(v^3 f_{000}) + \nu\left[\frac{kT}{M} + \left(\frac{eE_0}{m}\right)^2\frac{\cos^2\beta z}{6(\nu^2+\omega^2)}\right]\frac{\partial}{\partial v}\left(v^2\frac{\partial f_{000}}{\partial v}\right) + \frac{v^4}{3\nu}\frac{\partial^2 f_{000}}{\partial z^2} + v^2U(v) = 0. \tag{20}$$

[9] T. Kihara, Revs. Modern Phys. **25**, 844 (1953).
[10] F. A. Maxfield and R. R. Benedict, *Theory of Gaseous Conduction and Electronics* (McGraw-Hill Book Company, Inc., New York, 1941).
[11] L. R. Loeb, *Fundamental Processes of Electrical Discharge in Gases* (John Wiley and Sons, Inc., New York, 1939).

We consider the solution of Eq. (20) when the first two terms are much larger than the last two. In other words, we consider the Maxwellian distribution

$$f_{000}=\frac{n(z)}{[\pi\alpha(z)]^{\frac{3}{2}}}\exp[-v^2/\alpha(z)], \tag{21}$$

where

$$\alpha(z)=\frac{2}{m}\left[kT+M\left(\frac{eE_0}{m}\right)^2\cos^2\beta z/6(\nu^2+\omega^2)\right].$$

This result is Margenau's formula[3] for constant ν and for field amplitude of $E_0\cos\beta z$. Following Brown and MacDonald[12] we take the effective dc field $E_f=E_0\nu/[2(\nu^2+\omega^2)]^{\frac{1}{2}}$, so that the diffusivity, D_e, for the preceding distribution is

$$D_e=\frac{\alpha(z)}{2\nu}=\frac{1}{m\nu}\left[kT+\frac{Me^2E_f^2}{3\nu^2m^2}\cos^2\beta z\right]. \tag{22}$$

We will now determine the conditions under which the distribution (21) is valid. We note that the neglect of the last two terms of Eq. (20) means that the effects of diffusion on the distribution are small. If the number of electrons created in one collision time in a unit volume is small compared with the number of electrons present then diffusional effects will be small. Thus we require

$$\frac{Q}{\nu}\ll n \quad\text{or}\quad \frac{1}{\nu}\frac{d^2(D_en)}{dz^2}\ll n.$$

Using Eqs. (18) and (22), we have

$$\frac{2kT}{m\nu^2L^2}+\frac{2Me^2}{3m\nu^4L^2}E_f^2\cos^2\beta z\ll\frac{z}{L}-\left(\frac{z}{L}\right)^2.$$

If we consider those points for which $(z/L)-(z/L)^2>10^{-2}$, and take $\cos^2\beta z\cong 1$, the requirement is

$$\frac{2kT}{m\nu^2L^2}+\frac{2}{3}\frac{Me^2}{mL^2}\frac{E_f^2}{\nu^4}\ll 10^{-2}. \tag{23}$$

Let us now assume that the microwave field is the main factor controlling diffusion. Thus

$$D_e\cong\frac{M}{3m}\frac{e^2}{m^2}\frac{E_f^2}{\nu^3}\cos^2\beta z,$$

and

$$n\cong\frac{Q(Lz-z^2)}{[\frac{2}{3}(M/m)(e^2/m^2)(E_f^2/\nu^3)]\cos^2\beta z}. \tag{24}$$

For this case the electron density will have strong maxima approximately when $\cos\beta z=0$ or when $\beta z=\pi/2,\ 3\pi/2,\ 5\pi/2,\ \cdots$, depending on the mode $\beta L=\pi/2,\ 3\pi/2,\ 5\pi/2,\ \cdots$. For the fundamental there are no such maxima between the plates. For the second mode, however, a maximum exists at $z=L/3$.

The solution, Eq. (24), is valid when

$$\frac{2}{3}\frac{M}{m}\frac{e^2}{m^2L^2}\frac{E_f^2}{\nu^4}\ll 10^{-2}, \tag{25}$$

and also when [see Eq. (19)]

$$n<\epsilon_0 ME_f^2/(3m^2L^2\nu^2). \tag{26}$$

For hydrogen, $\nu\cong 6\times 10^9 p$(mm of Hg). If we take $L=10^{-2}$ meter, we obtain, for the foregoing inequalities,

$$\frac{E_f(\text{volt/meter})}{p^2(\text{mm Hg})^2}\ll 5\times 10^5,$$

$$n\left(\frac{\text{electrons}}{\text{m}^3}\right)<3\times 10^6\frac{E_f^2(\text{volt/m})^2}{p^2(\text{mm Hg})^2}.$$

B. TRAVELING WAVE

We shall now consider the case of the traveling wave of the form $E_x=E_0\cos(\beta z-\omega t)$ in an ionized gas of infinite extent. In this case we need not concern ourselves with the balance of electron-ion production and diffusion. We therefore must solve the equation

$$\frac{\partial f}{\partial t}+v_z\frac{\partial f}{\partial z}+\frac{eE_0}{m}\cos(\beta z-\omega t)\frac{\partial f}{\partial v_x}=\left(\frac{\delta f}{\delta t}\right)_{\text{elastic}}. \tag{27}$$

It is easily seen that the force $\mathbf{i}eE_0\cos(\beta z-\omega t)$ is invariant to the product of transformations $x\rightarrow -x$ and $t\rightarrow t+(p+\frac{1}{2})(2\pi/\omega)$. Thus the same considerations which led to Eq. (4) apply and the form of the distribution function is given by that equation. Following the same procedure that led to Eqs. (6)–(9), we obtain the following pertinent equations:

$$-\frac{m}{Mv^2}\frac{\partial}{\partial v}\left[v^3\nu f_{000}+\frac{kT}{m}v^2\nu\frac{\partial f_{000}}{\partial v}\right]+\frac{v}{3}\frac{\partial f_{011}}{\partial z}$$

$$+\frac{eE_0}{6mv^2}\cos\beta z\frac{\partial}{\partial v}(v^2f_{101})+\frac{eE_0}{6mv^2}\sin\beta z\frac{\partial}{\partial v}(v^2g_{101})=0, \tag{28}$$

$$\nu f_{101}+\omega g_{101}+\frac{eE_0}{m}\cos\beta z\frac{\partial f_{000}}{\partial v}=0, \tag{29}$$

$$\nu g_{101}-\omega f_{101}+\frac{eE_0}{m}\sin\beta z\frac{\partial f_{000}}{\partial v}=0, \tag{30}$$

$$\nu f_{011}+v\frac{\partial f_{000}}{\partial z}=0. \tag{31}$$

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[12] S. C. Brown and A. D. MacDonald, Phys. Rev. 76, 1629 (1949).

From Eqs. (29) and (30), we find

$$f_{101}=\frac{eE_0}{m}\left(\frac{\omega\sin\beta z-\nu\cos\beta z}{\nu^2+\omega^2}\right)\frac{\partial f_{000}}{\partial v}, \quad (32)$$

and

$$g_{101}=\frac{eE_0}{m}\left(\frac{\nu\sin\beta z-\omega\cos\beta z}{\nu^2+\omega^2}\right)\frac{\partial f_{000}}{\partial v}. \quad (33)$$

To find the spherically symmetric f_{000}, we must solve

$$\frac{m}{M}\frac{\partial}{\partial v}(v^3\nu f_{000})+\frac{kT}{M}\frac{\partial}{\partial v}\left(v^2\nu\frac{\partial f_{000}}{\partial v}\right)+\left(\frac{eE_0}{m}\right)^2\frac{1}{6}\frac{\partial}{\partial v}\left(\frac{v^2\nu}{\nu^2+\omega^2}\frac{\partial f_{000}}{\partial v}\right)+\frac{v^4}{3\nu}\frac{\partial^2 f_{000}}{\partial z^2}=0. \quad (34)$$

Now because the force on the electrons is invariant to the transformation $z\rightarrow z+(2\pi/\beta)$, the distribution function must also be. Since β does not appear in Eq. (34), it follows that if f_{000} is invariant to the above transformation it must be independent of z. We are thus led to the familiar result[3]

$$\log f_{000}=-\int_0^{v^2}\frac{\frac{1}{2}md(v^2)}{kT+M(eE_0/m)^2/6(\nu^2+\omega^2)}+\text{const.} \quad (35)$$

Finally we use Eqs. (11a), (32), and (33) to obtain

$$j_x=-\frac{4\pi e^2E_0}{3}\left[\cos(\beta z-\omega t)\int_0^\infty\frac{\partial f_{000}}{\partial v}\frac{v^3\nu}{\nu^2+\omega^2}dv-\sin(\beta z-\omega t)\int_0^\infty\frac{\partial f_{000}}{\partial v}\frac{v^3\omega}{\nu^2+\omega^2}dv\right]. \quad (36)$$

Since $j_x=\mathrm{Re}\{\sigma_cE_0e^{-i(\beta z-\omega t)}\}$, we obtain the familiar result given by Eq. (11b) for σ_c. We thus see that the conductivity is independent of position.

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Radiochemical Studies of the Interaction of Lead with Protons in the Energy Range 0.6 to 3.0 Bev*

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Lead targets have been bombarded with protons of several energies between 0.6 and 3.0 Bev and formation cross sections have been determined for about 30 nuclides of $A<140$ produced in these bombardments. The excitation functions, both for the lightest products studied ($A<35$) and for neutron-deficient barium isotopes, rise steeply with increasing energy. For intermediate-mass products ($50<A<120$), the changes in formation cross sections with increasing energy are much smaller and may largely be interpreted as a shift towards more neutron-deficient products and as a broadening of the yield-mass distribution. At 3 Bev the spallation and fission product regions have merged, and the cross sections for forming all mass numbers below the target mass are equal within an order of magnitude. The changes in yield pattern above about 0.4 Bev are shown to be associated with increasing probabilities of very large energy transfers (of the order of 1 Bev) from the incident proton to the struck nucleus, and this trend is explained in terms of an energy transfer mechanism involving the production, scattering, and reabsorption of pions. Besides the well-known modes of de-excitation—particle evaporation (spallation) and fission—a new mode termed fragmentation is postulated to account for some of the observed products, especially the light fragments. Fragmentation is thought to be associated with the short mean free paths of pions in nuclear matter which cause local heating and can thus lead to dissociation of the nucleus into fragments in a time short compared to that required for equipartition of energy.

RADIOCHEMICAL studies of a wide variety of products resulting from the bombardment of bismuth with 340-Mev,[1] 480-Mev,[2] and 660-Mev[3] protons have been reported from other laboratories. At these energies the products fall quite distinctly into two mass regions, generally referred to as the spallation and fission regions, respectively. The so-called spallation products comprise nuclides within 30 or 40 units of A of

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[1] W. F. Biller, University of California Radiation Laboratory Report UCRL-2067 (unpublished).

[2] Vinogradov, Alimarin, Baranov, Lavrukhina, Baranova, Pavlotskaya, Bragina, and Yakovlev, Conference of the Academy of Sciences, U.S.S.R., on Peaceful Uses of Atomic Energy, July 1–5, 1955; Session of Division of Chemical Sciences, p. 97 and p. 132. (English translation by Consultants Bureau, New York, U. S. Atomic Energy Commission Rept. TR-2435, Pt. 2, 1956, pp. 65 and 85.)

[3] Murin, Preobrazhensky, Yutlandov, and Yakimov, Conference of the Academy of Sciences, U.S.S.R., on Peaceful Uses of Atomic Energy, July 1–5, 1955; Session of Division of Chemical Sciences, p. 160. (English translation by Consultants Bureau, New York, U. S. Atomic Energy Commission Rept. TR-2435, Pt. 2, 1956, p. 101.)