Neutron-Electron Interaction in Cutoff Theory*

S. B. TREIMAN AND R. G. SACHS†
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
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The perturbation-theoretic result for the neutron-electron interaction, as calculated by Salzman in the Chew no-recoil theory, is in serious disagreement with experiment. The main part of the experimental interaction (~4.2 kev) is accounted for by the Foldy term (4.08 kev). But the latter is not included in the no-recoil approximation, where one calculates only the electron interaction with the static pion cloud of the physical neutron. For this part of the interaction the no-recoil perturbation result is too large by a factor of at least twenty. We investigate here the possibility that higher order corrections drastically alter the pion cloud charge distribution. Using the techniques developed by Miyazawa, one can express the higher order corrections in terms of weighted integrals over the pion-nucleon interaction cross sections. It is found that the higher order effects indeed reduce the pion cloud contribution, but only by 20 percent. Because of certain ambiguities involved in treating the cutoff function, it cannot be stated that this represents the true content of Chew's theory; but it seems unlikely that any reasonable treatment of the cutoff could reduce the discrepancy much further.

I. INTRODUCTION

THE neutron-electron interaction has been calculated by a number of workers using the weak-coupling approximation to relativistic pseudoscalar meson theory[1]; and recently the problem has also been investigated in the no-recoil, extended-source approximation.[2,3] The approach taken by Sachs along the latter lines is phenomenological, whereas Salzman[3] has carried out a perturbation calculation using specifically the Chew version of the extended-source theory.[4] In both cases the quantity which is calculated is the interaction of the electrostatic field of the electron with the charged pion cloud surrounding the nucleon core. In effect this amounts to calculating the second moment of the pion cloud charge distribution. We refer to this as the static charge-cloud part of the neutron-electron interaction.

As Salzman has shown, in this no-recoil approximation one completely omits a second contribution to the neutron-electron interaction. This is the Foldy term.[5] The coupling of the neutron magnetic moment with external magnetic fields implies, for reasons of relativistic covariance, an added coupling with external electric fields. The latter term is inversely proportional to the neutron mass and therefore does not appear in the no-recoil approximation (neutron mass $\rightarrow \infty$). Nevertheless, it can be written down exactly in terms of the known anomalous magnetic moment and known (finite) mass of the neutron; and in fact this contribution to the neutron-electron interaction accounts for the bulk of the experimental effect. When expressed in the conventional way, the Foldy interaction has the value 4.08 kev, whereas the experimental value—according to the most recent measurements—is 4.165 ± 0.265 kev.[6] The static charge-cloud term obtained in the no-recoil approximation should therefore contribute an interaction of 0.08 ± 0.27 kev. Instead, Salzman's perturbation calculation leads to a value of ~7.1–8.6 kev (the result depends on the choice of source function and on the coupling constant f; as for the latter, Salzman used $f^2 = 0.058$).

One way to account for this discrepancy has been suggested by Salzman.[3] This is to attribute a finite extension to the charge distribution of the core itself. If the charge distribution of the core is assumed to be proportional to the source function employed in the extended-source theory, one can arrange for a considerable cancellation of charge between the (negative) pion cloud and the (positive) core. There is of course no mathematical basis for this assumption in the present form of the extended-source theory, although the idea is perhaps attractive on physical grounds.

Another possibility, which is the subject of the present investigation, is that the perturbation approximation is inadequate and that higher order mesonic corrections drastically alter the pion cloud charge distribution. To study this, we make use of the elegant techniques developed by Miyazawa[7] in connection with a similar calculation, in the framework of Chew's theory, of higher order contributions to the magnetic moments of nucleons. The higher order terms are in effect expressed in terms of weighted integrals over the pion-nucleon interaction cross sections.

It turns out that the higher order corrections indeed

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† On leave of absence from the University of Wisconsin.
[1] B. D. Fried, Phys. Rev. **88**, 1142 (1952). This paper contains a very lucid application of the weak-coupling approximation, as well as a list of references to the earlier literature on the neutron-electron interaction.
[2] R. G. Sachs, Phys. Rev. **87**, 1100 (1952).
[3] G. Salzman, Phys. Rev. **99**, 973 (1955).
[4] G. F. Chew, Phys. Rev. **95**, 1669 (1954); and references therein. See also the review article by G. C. Wick, Revs. Modern Phys. **27**, 339 (1955).
[5] L. L. Foldy, Phys. Rev. **83**, 688 (1951); **87**, 688 (1952); **87**, 693 (1952).
[6] Melkonian, Rustad, and Havens, Bull. Am. Phys. Soc. Ser. II, 1, 62 (1956). For earlier measurements, see reference 1.
[7] H. Miyazawa, Phys. Rev. **101**, 1564 (1956).

reduce the pion cloud contribution to the neutron-electron interaction; but for a cutoff in the vicinity of 5 or 6μ (μ = meson mass) the reduction amounts to only ~ 20 percent. Because of ambiguities related to the handling of the cutoff, or source function, it cannot be stated that this result represents the true content of the Chew version of the extended-source theory; but it seems unlikely that any reasonable treatment of the cutoff function would serve to appreciably reduce the large discrepancy with experiment—unless, as suggested by Salzman, one amends the theory so as to include a finite extension of the core charge.[7a]

II. NO-RECOIL APPROXIMATION

Insofar as one considers only lowest order (Born approximation) interactions with an external electromagnetic field, one may write the effective Hamiltonian for a Dirac particle in the form[5]

$$H=H_0+V;$$
$$V=-i\gamma_4\sum_{n=0}^{\infty}\left[\epsilon_n\Box^n A_\mu\gamma_\mu-\tfrac{1}{2}\mu_n\Box^n F_{\mu\nu}\gamma_\mu\gamma_\nu\right]; \qquad (1)$$

where H_0 is the free-particle Hamiltonian, A_μ are the electromagnetic potentials, γ_μ are the Dirac matrices, and $F_{\mu\nu}=(\partial A_\nu/\partial x_\mu-\partial A_\mu/\partial x_\nu)$; $\Box$ is the D'Alembertian operator, and the numbers ϵ_n and μ_n are parameters which characterize the internal structure of the particle. This result is based on general Lorentz and gauge invariance considerations. Clearly ϵ_0 and μ_0 represent respectively the static charge and static magnetic moment. The remaining parameters represent higher moments of the internal charge and current distributions of the structured particle.

To describe the interaction of a slow neutron with a quasi-static, weak external field, we need retain only the lowest nonvanishing moments (ϵ_1 and μ_0). Passing, furthermore, to the nonrelativistic limit in which the neutron is represented by a two-component wave function, we find for the effective interaction potential[1]

$$V=-\mu_0\mathbf{H}\cdot\boldsymbol{\sigma}-[\mu_0/2M+\epsilon_1]\nabla\cdot\mathbf{E} \qquad (2)$$

(in units where $\hbar=c=1$; terms which are irrelevant to the present discussion have been freely dropped[1]). Consider now the case where $\mathbf{H}=0$ and where the electric field has as source a static electron charge distribution $\rho_e(\mathbf{r})$, with $\int\rho_e(\mathbf{r})d^3r=-e$. Then, since $\nabla\cdot\mathbf{E}=4\pi\rho_e$,

$$V=-4\pi\rho_e[\mu_0/2M+\epsilon_1].$$

Customarily, this is expressed in terms of an equivalent square well potential $-V_0$ of radius $r_0=e^2/mc^2$. Thus

$$V_0=-(3e/r_0^3)[\mu_0/2M+\epsilon_1]. \qquad (3)$$

The first term is the Foldy interaction which, since it involves known quantities, can be calculated exactly. It is the second term which is calculated in the no-recoil theory where one considers the interaction of a slow electron with the static charge cloud of the physical neutron. If $\rho(r)$ represents the charge density in the cloud, assumed to be spherically symmetric, then as Sachs[2] and Salzman[3] have shown,

$$\epsilon_1=(1/6)\int r^2\rho(\mathbf{r})d^3r\equiv-(e/6)R^2. \qquad (4)$$

We shall assume that the core charge has no appreciable extension, so that R^2 is determined solely by the pion charge cloud. The corresponding contribution to the neutron-electron interaction is denoted by $(V_0)_\pi$:

$$(V_0)_\pi=(e^2/2r_0)(R/r_0)^2. \qquad (5)$$

III. MESON CHARGE DENSITY

The charge density $\rho(\mathbf{r})$ required for Eq. (4) is obtained by taking the expectation value, for the ground state of the physical neutron, of the pion charge density operator $\boldsymbol{\varrho}(\mathbf{r})$. Let $|N\rangle$ be the state vector of the physical neutron. Then

$$eR^2=-\int d^3r r^2\langle N|\boldsymbol{\varrho}(\mathbf{r})|N\rangle, \qquad (6)$$

where

$$\boldsymbol{\varrho}(\mathbf{r})=ie(\pi^*\psi^*-\pi\psi). \qquad (7)$$

In a spherical wave representation,[2]

$$\psi=\sum_{klm}\phi_{klm}(2\omega)^{-\frac{1}{2}}[a_{klm}+(-1)^m b_{kl-m}^*], \qquad (8)$$

$$\pi=i\sum_{klm}\phi_{klm}^*(\omega/2)^{\frac{1}{2}}[a_{klm}^*-(-1)^m b_{kl-m}], \qquad (9)$$

where $\omega=(k^2+\mu^2)^{\frac{1}{2}}$; a_{klm} and a_{klm}^* respectively destroy and create positive mesons in the state specified by the quantum numbers k, l, m; and the operators b_{klm} and b_{klm}^* have a similar meaning for negative pions. The basis functions ϕ_{klm} are spherical wave solutions of the Klein-Gordon equation, normalized according to

$$\phi_{klm}=f_l(kr)Y_l^m,$$
$$\phi_{klm}^*=(-1)^m\phi_{kl-m},$$

and

$$\int drr^2 f_l(kr)f_l(k'r)=\delta_{k'k}. \qquad (10)$$

Since only p waves come into consideration, we hereafter omit the index l. The symbols $\sum_k$ and $\delta_{k'k}$ are to be understood throughout as a shorthand notation with the following meaning:

$$\sum_k\rightarrow(2/\pi)\int dkk^2,$$
$$\delta_{k'k}\rightarrow\delta(k'-k). \qquad (11)$$

Substituting Eqs. (7)–(10) into Eq. (6), and carrying out the integration over space angles, we obtain for R^2

[7a] For an alternative see G. Sandri, Phys. Rev. **101**, 1616 (1956).

the expression

$$R^2=-\tfrac{1}{2}\sum_{kk'}\int drr^4 f(kr)f(k'r)(\omega\omega')^{-\frac{1}{2}}$$
$$\times[(\omega'+\omega)G(k',k)+(\omega'-\omega)F(k',k)], \quad (12)$$

where

$$G(k',k)=\textstyle\sum_m\langle N|a_{k'm}{}^*a_{km}-b_{km}{}^*b_{k'm}|N\rangle, \quad (13)$$

$$F(k',k)=\textstyle\sum_m(-1)^m\langle N|a_{k'm}{}^*b_{k-m}{}^*-a_{km}b_{k'-m}|N\rangle. \quad (14)$$

This can be simplified by observing that $f(kr)$ satisfies the differential equation (recall that $l=1$)

$$-r^2f(kr)=[d^2/dk^2+(2/k)d/dk-2/k^2]f(kr). \quad (15)$$

Making this substitution for $r^2f(kr)$ in Eq. (12), and using the orthogonality relation of Eq. (10), we see that the expression for R^2 has the form

$$\textstyle\sum_k(\sum_{k'}g(k',k)D_k\delta_{k'k}),$$

where D_k is the above differential operator on k and $g(k',k)$ is a function of k' and k. Owing to the delta function this reduces to the single sum

$$\textstyle\sum_k[D_{k'}g(k',k)]_{k'=k}.$$

We also note from Eq. (14) that the real part of $F(k',k)$ is antisymmetric—since a and b commute—and only this part contributes to R^2. This permits us to simplify matters further by carrying out some of the differential operations immediately. The final expression for R^2 is

$$R^2=\tfrac{1}{2}\textstyle\sum_k\{[d^2/dk'^2+(2/k)d/dk'-2/k^2]$$
$$\times[(\omega'+\omega)(\omega\omega')^{-\frac{1}{2}}G(k',k)$$
$$+(2k/\omega^2)dF(k',k)/dk'\}_{k'=k}. \quad (16)$$

IV. CALCULATION OF HIGHER ORDER CORRECTIONS TO R^2

There remains now to determine the functions $G(k',k)$ and $F(k',k)$. The general procedure which we follow is that discussed by Miyazawa[7] and need be indicated therefore only in brief outline.

The interaction Hamiltonian used by Chew is

$$H'=(4\pi)^{\frac{1}{2}}(f_0/\mu)\int\tau_\alpha S(r)\boldsymbol{\sigma}\cdot\nabla\psi_\alpha d^3r, \quad (17)$$

where $S(r)$ is the source function and f_0 is the unrenormalized coupling constant. We now expand the field operators ψ_α in terms of creation and destruction operators, as in Eq. (8), and carry out the integrations over space, obtaining for H' the expression

$$H'=(1/\sqrt{3})(f_0/\mu)\textstyle\sum_k kv(k)(2\omega)^{-\frac{1}{2}}\{2[\tau^+\sigma^+(b_{k-1}{}^*-a_{k1})$$
$$+\tau^+\sigma^-(a_{k-1}-b_{k1}{}^*)+\tau^-\sigma^+(a_{k-1}{}^*-b_{k1})$$
$$+\tau^-\sigma^-(b_{k-1}-a_{k1}{}^*)]+\sqrt{2}[\tau^+\sigma_3(a_{k0}+b_{k0}{}^*)$$
$$+\tau^-\sigma_3(a_{k0}{}^*+b_{k0})+\tau_3\sigma^+(c_{k-1}{}^*-c_{k1})$$
$$+\tau_3\sigma^-(c_{k-1}-c_{k1}{}^*)]+\tau_3\sigma_3(c_{k0}{}^*+c_{k0})\}, \quad (18)$$

where c and c^* are respectively destruction and creation operators for neutral mesons; and $v(k)$ is the Fourier transform of the source function, normalized so that $v(0)=1$. In a shorthand notation, this can be written

$$H'=\sum_{kmt}[V_{kmt}d_{kmt}+V_{kmt}{}^*d_{kmt}{}^*], \quad (19)$$

where d_{kmt} and $d_{kmt}{}^*$ are destruction and creation operators for mesons of momentum k, magnetic quantum number m, and charge t. The important thing is that the momentum dependence of V_{kmt} is completely specified in terms of the source function; namely,

$$V_{kmt}\sim kv(k)/\omega^{\frac{1}{2}}. \quad (20)$$

In the same notation, the free-field Hamiltonian is

$$H_0=\sum_{kmt}\omega d_{kmt}{}^*d_{kmt}. \quad (21)$$

Miyazawa now introduces a complete set $|n\rangle$ of eigenstates representing incoming wave solutions of

$$(H_0+H')|n\rangle=E_n|n\rangle,$$

normalizing so that the energy of the physical nucleon is zero in the ground state. Then, from the commutation relationships of the d_{kmt} and $d_{kmt}{}^*$ with H_0+H', one finds[7–9]

$$\langle N|d_{k'mt}{}^*d_{kmt}|N\rangle=\sum_n\frac{T_{-m-t}{}^*(k',n)T_{-m-t}(k,n)}{(E_n+\omega')(E_n+\omega)}, \quad (22)$$

$$\langle N|d_{k'-m-t}{}^*d_{kmt}{}^*|N\rangle=\sum_n\frac{(-1)^m}{(\omega'+\omega)}\left\{\frac{T_{mt}{}^*(k',n)T_{mt}(k,n)}{(E_n+\omega')}+\frac{T_{-m-t}{}^*(k,n)T_{-m-t}(k',n)}{(E_n+\omega)}\right\}, \quad (23)$$

where

$$T_{mt}(k,n)=\langle n|V_{kmt}|N\rangle, \quad (24)$$

and use has been made of the relationship $V_{kmt}=(-1)^mV_{k-m-t}{}^*$, which follows from Eq. (18). The matrix element $T_{mt}(k,n)$ is related to the scattering matrix in the sense that when $\omega=E_n$ it is the transition matrix element describing the transition into the final state $|n\rangle$ from an initial state consisting of a neutron and a meson with quantum numbers k, m, t.

The four spin and isotopic spin states corresponding to $n=0$ are states of the physical neutron alone (no free pions), and these terms in the above sums give the lowest order perturbation approximations to the expectation values.[7] The corresponding perturbation result for R^2 has already been obtained by Salzman and

[8] G. C. Wick, reference 4.

[9] G. F. Chew and F. Low, Phys. Rev. **101**, 1570, 1579 (1956).

we merely note the result

$$(R^2)_{\text{pert}}=\frac{1}{\pi}\left(\frac{f}{\mu}\right)^2\int dk\frac{k^4}{\omega^7}\times\left[2\omega^4\left(\frac{dv}{dk}\right)^2+5(3\omega^2-2k^2)v^2\right],\quad (25)$$

where f is the renormalized coupling constant.

The remaining terms $n>0$ give us the higher order corrections to the expectation values. By using Eq. (20), we extract the k'-dependent factors from the matrix elements; namely,

$$T^*(k',n)T(k,n)=T^*(k,n)T(k',n)=\frac{k'v(k')}{kv(k)}\left(\frac{\omega}{\omega'}\right)^{\frac{1}{2}}|T(k,n)|^2.\quad (26)$$

Also, it is convenient to decompose the matrix elements into elements T_{ij} which refer to states of definite isotopic spin $i/2$ and ordinary spin $j/2$. Carrying out these operations, and then explicitly performing the differentiations required in Eq. (16), we obtain the following expression for the higher order correction to R^2:

$$(R^2)_{\text{corr}}=-\tfrac{1}{3}\sum_k\sum_{n>0}U(E_n,\omega)\{2|T_{33}(k,n)|^2+|T_{31}(k,n)|^2-2|T_{13}(k,n)|^2-|T_{11}(k,n)|^2\},\quad (27)$$

where

$$U(E_n,\omega)=\frac{1}{(E_n+\omega)^2}\left[\frac{2\omega^2+3\mu^2}{\omega^4}-\frac{2}{v}\frac{d^2v}{dk^2}-2\left(\frac{3\omega^2+\mu^2}{k\omega^2v}\right)\frac{dv}{dk}\right]+\frac{1}{(E_n+\omega)^3}\left[2\left(\frac{3\omega^2+2\mu^2}{\omega^3}\right)+\frac{4k}{\omega v}\frac{dv}{dk}\right]-\frac{4}{(E_n+\omega)^4}\frac{k^2}{\omega^2}.\quad (28)$$

The final step consists in observing that the total cross sections for interaction of positive and negative pions respectively on unpolarized protons can be written

$$\sigma^+(k')=8\pi^2(\omega'/k')\sum_{n>0}\delta(E_n-\omega')\times\{2|T_{33}(k',n)|^2+|T_{31}(k',n)|^2\},$$

$$\sigma^-(k')=(8\pi^2/3)(\omega'/k')\sum_{n>0}\delta(E_n-\omega')\times\{2|T_{33}(k',n)|^2+|T_{31}(k',n)|^2+4|T_{13}(k',n)|^2+2|T_{11}(k',n)|^2\}.\quad (29)$$

Once again, we extract from the matrix elements their k'-dependent factors, writing

$$\sigma^+(k')-\sigma^-(k')=\frac{16}{3}\pi^2\frac{\omega'\omega}{k^2}\frac{v^2(k')}{v^2(k)}\sum_{n>0}\delta(k_n-k')|T(k,n)|^2,\quad (30)$$

where

$$|T(k,n)|^2=2|T_{33}(k,n)|^2+|T_{31}(k,n)|^2-2|T_{13}(k,n)|^2-|T_{11}(k,n)|^2\quad (31)$$

is the factor appearing in Eq. (27).

It then follows that we may substitute into Eq. (27) the following expression:

$$\sum_k\sum_{n>0}|T(k,n)|^2U(E_n,\omega)=\frac{3}{8\pi^3}\int dk\frac{k^4v^2(k)}{\omega}\int dk'\frac{\sigma^+(k')-\sigma^-(k')}{\omega'v^2(k')}U(\omega',\omega),\quad (32)$$

where we have made use of Eq. (11).

V. DISCUSSION

So far our procedure has been exact, within the framework of the Chew theory. A difficulty arises, however, in connection with the factor $v^2(k')$ which appears in the denominator of Eq. (32). One would want to choose a cutoff function v which starts out with unit value for $k=0$ and falls off to zero for large k. But this would lead to a divergence at large k, since the experimental pion-nucleon interaction cross sections certainly remain finite up to the largest known energies. Of course, insofar as the present analysis is based on the Chew approximation, the cross sections σ which should be used are the ones which follow from the theory. However, if the theory has any merit, we expect these cross sections to agree with the experimental ones, at least at low energies where $v\approx 1$. At large energies where $v\rightarrow 0$, we cannot identify σ with the experimental cross sections. We therefore simply assume that the theoretical ratios σ/v^2 fall off sufficiently rapidly beyond some "cutoff" value of k so that their contributions beyond the cutoff are negligible.

In practice then, we set $v=1$ in the above integrals, identify σ with the experimental cross sections, but simply cut off the integrations in the vicinity of 5 or 6 μ. It is a question whether we should also neglect the terms containing derivatives of v in Eqs. (25) and (28). We think this is more nearly consistent with the present approach than any other procedure would be. At any rate, for reasonably smooth cutoff functions, the terms involving derivatives of v do not contribute appreciably to the final numerical result if the integrals are extended only up to a cutoff of 5 or 6 μ.

As a matter of fact, essentially the entire contribution to the integral over the cross sections comes in the vicinity of the large (3,3) resonance at ~200-Mev lab energy. The experimental values of the cross sections are summarized by Anderson *et al.*[10] We record here the final expression for $(R^2)_{\text{corr}}$, obtained after some algebraic manipulations and with the above approxima-

[10] Anderson, Davidon, and Kruse, Phys. Rev. **100**, 339 (1955).

tions incorporated:

$$(R^2)_{\mathrm{corr}}=-\frac{1}{8\pi^3}\int dk\frac{\sigma^+(k)-\sigma^-(k)}{\omega}\int dk'\frac{k'^4}{\omega'(\omega'+\omega)^4}$$
$$\times\left\{5+12\frac{\omega}{\omega'}+3\left(\frac{\omega}{\omega'}\right)^2+\frac{2\mu^2}{\omega'^2}\left[5+4\frac{\omega}{\omega'}+\left(\frac{\omega}{\omega'}\right)^2\right]\right\}. \quad (33)$$

The integrations, Eqs. (25) and (33), have been carried out numerically, with cutoffs at k', $k=5$ and 6 μ. The corresponding perturbation-theoretic and higher order contributions to the static pion cloud part of the neutron-electron interaction are given in Table I. For the perturbation term we have used the value $f^2=0.08$ for the coupling constant.[9] This differs from the earlier value 0.058 used by Salzman and accounts for most of the difference between his result and ours (the difference in cutoff functions does not have much effect on the final answer). The important thing, however, is not so much the absolute value of the interaction as the fact that, pretty much independently of cutoff, the higher order corrections are small ($\sim$20%) relative to the perturbation result.

TABLE I. Static pion cloud contribution $(V_0)_\pi$ to the neutron-electron interaction, in kev.

Cutoff	Perturbation result	Higher order correction	$(V_0)_\pi$
5 μ	10.9	-2.0	8.9
6 μ	12.4	-2.4	10.0

It therefore appears that the very large discrepancy between the perturbation result and experiment, as regards the static pion cloud part of the neutron-electron interaction, is not appreciably reduced by the higher order mesonic corrections.

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Relativistic Radiative Transitions*†

J. S. LEVINGER AND M. L. RUSTGI
Louisiana State University, Baton Rouge, Louisiana
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We have calculated the summed oscillator strength for a K electron in the Coulomb field of a lead nucleus, using calculations of the oscillator strength for discrete transitions, and the oscillator density for absorption to the continuum. We obtain 86% of the value unity given by the nonrelativistic Thomas-Reiche-Kuhn sum rule. We also use dispersion theory to calculate the forward scattering amplitude as a function of photon energy. Our numerical values for the forward scattering amplitude are in good agreement with Brown's values for a K electron of mercury at 1.7, 3.4, and 6.8 times the binding energy. We also compare with our calculations of the forward scattering amplitude by a nonrelativistic electron in a Coulomb field.

I. INTRODUCTION

IN a previous paper[1] Payne and Levinger presented calculations of the retarded relativistic oscillator strength for radiative transitions from the K level to other discrete states for an electron in a Coulomb field. Numerical results were given for $Z=82$, i.e., for the ion Pb^{81+}. We used these discrete oscillator strengths for three purposes: (1) calculations of x-ray intensities; (2) extrapolation to find the oscillator density at the photoeffect threshold; (3) determination of the summed oscillator strength for a relativistic system.

In this paper we extend our previous work on the summed oscillator strength of Pb^{81+}. In the next section we present the cross sections used in calculating the summed oscillator strength. The major change from A is that we have adopted Brown's proposal[2] of using the cross section σ as the difference of the photoeffect cross section ($\sigma_{\mathrm{P.E.}}$) and the cross section for pair production in which the produced electron would occupy the already filled quantum state ($\sigma_{\mathrm{P.P.}}$).

In Sec. III we use this cross section σ to obtain the finite integrated cross section σ_{int}, or related summed oscillator strength. We also obtain the forward scattering amplitude as a function of photon energy. The real part F of the scattering amplitude is found from the absorption cross section to discrete and continuum states by means of the dispersion integral, while the imaginary part G is directly proportional to the absorption cross section σ. We find good agreement between our values of F and those found by Brown *et al.*[3] Our value $F(\infty)=0.86(e^2/mc^2)$ (for infinite photon energy) corresponds to a summed oscillator strength, or integrated cross section, of 86% of the Thomas-Reiche-Kuhn value. For comparison we also present calcula-

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† A preliminary account was presented by J. S. Levinger and M. L. Rustgi, Bull. Am. Phys. Soc. Ser. II, 1, 84 (1956).
[1] W. B. Payne and J. S. Levinger, Phys. Rev. **101**, 1020 (1956), here denoted by A.
[2] G. E. Brown (private communication); also see A, Sec. IV.
[3] Brenner, Brown, and Woodward, Proc. Roy. Soc. (London) **A227**, 59 (1954); G. E. Brown and D. F. Mayers, Proc. Roy. Soc. (London) **A234**, 387 (1956), and private communication.