tions incorporated:

$$(R^2)_{\text{corr}}=-\frac{1}{8\pi^3}\int dk\frac{\sigma^+(k)-\sigma^-(k)}{\omega}\int dk'\frac{k'^4}{\omega'(\omega'+\omega)^4}$$
$$\times\left\{5+12\frac{\omega}{\omega'}+3\left(\frac{\omega}{\omega'}\right)^2+\frac{2\mu^2}{\omega'^2}\left[5+4\frac{\omega}{\omega'}+\left(\frac{\omega}{\omega'}\right)^2\right]\right\}. \quad (33)$$

The integrations, Eqs. (25) and (33), have been carried out numerically, with cutoffs at k', $k=5$ and 6 μ. The corresponding perturbation-theoretic and higher order contributions to the static pion cloud part of the neutron-electron interaction are given in Table I. For the perturbation term we have used the value $f^2=0.08$ for the coupling constant.[9] This differs from the earlier value 0.058 used by Salzman and accounts for most of the difference between his result and ours (the difference in cutoff functions does not have much effect on the final answer). The important thing, however, is not so much the absolute value of the interaction as the fact that, pretty much independently of cutoff, the higher order corrections are small ($\sim 20\%$) relative to the perturbation result.

TABLE I. Static pion cloud contribution $(V_0)_\pi$ to the neutron-electron interaction, in kev.

Cutoff	Perturbation result	Higher order correction	$(V_0)_\pi$
5 μ	10.9	−2.0	8.9
6 μ	12.4	−2.4	10.0

It therefore appears that the very large discrepancy between the perturbation result and experiment, as regards the static pion cloud part of the neutron-electron interaction, is not appreciably reduced by the higher order mesonic corrections.

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Relativistic Radiative Transitions*†

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We have calculated the summed oscillator strength for a K electron in the Coulomb field of a lead nucleus, using calculations of the oscillator strength for discrete transitions, and the oscillator density for absorption to the continuum. We obtain 86% of the value unity given by the nonrelativistic Thomas-Reiche-Kuhn sum rule. We also use dispersion theory to calculate the forward scattering amplitude as a function of photon energy. Our numerical values for the forward scattering amplitude are in good agreement with Brown's values for a K electron of mercury at 1.7, 3.4, and 6.8 times the binding energy. We also compare with our calculations of the forward scattering amplitude by a nonrelativistic electron in a Coulomb field.

I. INTRODUCTION

IN a previous paper[1] Payne and Levinger presented calculations of the retarded relativistic oscillator strength for radiative transitions from the K level to other discrete states for an electron in a Coulomb field. Numerical results were given for $Z=82$, i.e., for the ion Pb^{81+}. We used these discrete oscillator strengths for three purposes: (1) calculations of x-ray intensities; (2) extrapolation to find the oscillator density at the photoeffect threshold; (3) determination of the summed oscillator strength for a relativistic system.

In this paper we extend our previous work on the summed oscillator strength of Pb^{81+}. In the next section we present the cross sections used in calculating the summed oscillator strength. The major change from A is that we have adopted Brown's proposal[2] of using the cross section σ as the difference of the photoeffect cross section ($\sigma_{\text{P.E.}}$) and the cross section for pair production in which the produced electron would occupy the already filled quantum state ($\sigma_{\text{P.P.}}$).

In Sec. III we use this cross section σ to obtain the finite integrated cross section σ_{int}, or related summed oscillator strength. We also obtain the forward scattering amplitude as a function of photon energy. The real part F of the scattering amplitude is found from the absorption cross section to discrete and continuum states by means of the dispersion integral, while the imaginary part G is directly proportional to the absorption cross section σ. We find good agreement between our values of F and those found by Brown *et al.*[3] Our value $F(\infty)=0.86(e^2/mc^2)$ (for infinite photon energy) corresponds to a summed oscillator strength, or integrated cross section, of 86% of the Thomas-Reiche-Kuhn value. For comparison we also present calcula-

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† A preliminary account was presented by J. S. Levinger and M. L. Rustgi, Bull. Am. Phys. Soc. Ser. II, 1, 84 (1956).
[1] W. B. Payne and J. S. Levinger, Phys. Rev. **101**, 1020 (1956), here denoted by A.
[2] G. E. Brown (private communication); also see A, Sec. IV.
[3] Brenner, Brown, and Woodward, Proc. Roy. Soc. (London) **A227**, 59 (1954); G. E. Brown and D. F. Mayers, Proc. Roy. Soc. (London) **A234**, 387 (1956), and private communication.

TABLE I. Oscillator strengths for discrete transitions by a K electron of lead.

Transition to	η[a]	f_{lead}[b]	f_{NR}
L shell	0.761	0.307	0.416
M shell	0.894	0.063	0.079
N shell	0.941	0.023	0.029
Higher discrete levels	···	0.033	0.041
Integral over continuum	···	0.435	0.435
Summed oscillator strength		0.861	1.000

[a] η is the photon energy/ionization energy for lead.
[b] The oscillator strengths for discrete transitions for lead are taken from reference 1, Table I, there designated as f_{ret}. The sum over the continuum is found from the data of our Table II. Numerical values should be accurate to better than 3%.

tions of F and G found from the nonrelativistic Coulomb absorption cross sections. The assumption by Goldberger *et al.*[4] that $F(\infty)$ equals F for a free particle is of course in agreement with the behavior of the nonrelativistic (NR) system, since in this case the Thomas-Reiche-Kuhn sum-rule holds exactly; but this assumption disagrees with our value of $F(\infty)$ for Pb^{81+} treated relativistically.

II. PHOTOEFFECT CROSS SECTIONS

The oscillator strengths for transitions from the K state of Pb^{81+} to higher discrete states are given in Table I. (See A, Table I for f_{ret}. We have corrected an error in that table for f_{ret} for the L shell. We follow A in taking f_{ret} for the O, P, and higher shells as 80% of the nonrelativistic value; since this percentage holds well for the M, N, and Q shells.)

We found in A that our extrapolation of the "oscillator density" from the discrete side to the threshold gave a threshold cross section of 740 barns, or 23% larger than the value found by Hulme *et al.*[5] In A, we used Hulme's cross sections at threshold and at 0.69 and 2.2 mc^2. In this paper we have changed to using our threshold cross section. We use photoeffect cross sections for Hg by Brown *et al.*[3] at 0.32, 0.64, and 1.28 mc^2, together with Hulme's value at 2.2 mc^2. (We convert cross sections from $_{80}$Hg to $_{82}$Pb assuming a Z^4 dependence of $\sigma_{\text{P.E.}}$ in this energy region.) As stated in A, the summed oscillator strength is not sensitive to whether we use Hulme's or our value for the threshold cross section, changing only by 3% if that change is made.

As stated in the introduction, we have now adopted Brown's proposal of using $\sigma=\sigma_{\text{P.E.}}-\sigma_{\text{P.P.}}$. Subtraction of the cross section $\sigma_{\text{P.P.}}$ for pair production in which the produced electron would occupy the already occupied K state follows from the usual definition of a cross section as that for the lead nucleus with an electron present minus that for a vacuum surrounding the lead nucleus. A second-order perturbation theory calculation of the scattering amplitude is related by dispersion theory to the cross section σ as defined above.

For the photon energy range k from the threshold of 1.8 mc^2 to 3.8 mc^2, we find $\sigma_{\text{P.P.}}$ from Jaeger and Hulme's calculation[6] of the inverse process of single photon annihilation by the K electrons of lead. For high photon energies we find $\sigma_{\text{P.P.}}$ from the Born approximation calculation of single photon annihilation,[7] multiplied by Hall's[8] factor (1/2.2 for lead) for Coulomb corrections. According to Jaeger and Hulme, Hall's factor is equally valid for the calculation of $\sigma_{\text{P.E.}}$ and $\sigma_{\text{P.P.}}$. As stated by Brown,[2] in the extreme relativistic region both $\sigma_{\text{P.E.}}$ and $\sigma_{\text{P.P.}}$ have identical $1/k$ terms, giving σ proportional to k^{-2}. Following Heitler,[9] we have

$$\sigma=\sigma_{\text{P.E.}}-\sigma_{\text{P.P.}}=(1/2.2)(3/4)Z^5\alpha^4(\mu/k)^5\phi_0 \times[f(\gamma_-)-f(\gamma_+)]=(1/2.2)6Z^5\alpha^4 \times(\mu/k)^2\phi_0(1-I/\mu)=15.4(\mu/k)^2 \text{ barns for lead,} \quad (1)$$

where

$$\phi_0=(8/3)\pi(e^2/mc^2)^2=0.665 \text{ barns,}$$

$$f(\gamma)=(\gamma^2-1)^{\frac{3}{2}}\left[\frac{4}{3}+\frac{\gamma(\gamma-2)}{\gamma+1}\left(1-\frac{1}{2\gamma}(\gamma^2-1)^{-\frac{1}{2}} \times\log\frac{\gamma+(\gamma^2-1)^{\frac{1}{2}}}{\gamma-(\gamma^2-1)^{\frac{1}{2}}}\right)\right], \quad (2)$$

and (see Jaeger and Hulme)

$$\gamma_-=(k+\mu-I)/\mu=E_-/\mu, \qquad \gamma_+=(k-\mu+I)/\mu=E_+/\mu. \quad (3)$$

Here k is the photon energy, $\mu=mc^2$, and I is the binding energy of the K electron. γ_- is the total energy E_- of

TABLE II. Cross sections for a K electron of lead, in barns.[a]

k/μ	$\sigma_{\text{P.E.}}$	$\sigma_{\text{P.P.}}$	σ	η	$df/d\eta$
0.2 (threshold)	740[b]	0	740	1.0	0.685
0.32	208[c]	0	208	1.6	0.193
0.64	33.4[c]	0	33.4	3.2	0.0309
1.28	5.78[c]	0	5.78	6.4	0.00535
2.2	2.10[d]	0.07[e]	2.03	11	0.00188
3.0	1.24[f]	0.085[e]	1.16	15	0.00107
5.0	0.665[f]	0.058[e,g]	0.607	25	0.00056
large[g]	$2.4(\mu/k)$	$2.4(\mu/k)$	$15.4(\mu/k)^2$	large	$0.36\eta^{-2}$

[a] σ is the difference of the photoeffect cross section $\sigma_{\text{P.E.}}$ and the cross section $\sigma_{\text{P.P.}}$ for production of a pair in which the electron would occupy the already occupied K state. η = photon energy/ionization energy, $df/d\eta$ is the oscillator density, k/μ = photon energy/mc^2.
[b] Reference 1, Table III.
[c] Reference 3.
[d] Reference 5.
[e] Reference 6.
[f] Reference 8.
[g] See Eq. (1).

[4] Gell-Mann, Goldberger, and Thirring, Phys. Rev. **95**, 1612 (1954).
[5] Hulme, McDougall, Buckingham, and Fowler, Proc. Roy. Soc. (London) **A149**, 131 (1935).
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[7] F. Sauter, Ann. Physik **11**, 454 (1931).
[8] H. Hall, Revs. Modern Phys. **8**, 358 (1936).
[9] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1954), third edition, Sec. 21, Eq. (17); Sec. 27, Eq. (15).

TABLE III. Forward scattering amplitude by a K electron.[a]

	Lead[b]			Mercury[c]		Nonrelativistic[d]		
η	F_c	F	G	F	G	F_c	F	G
0.333	−0.022	−0.113	0	···	···	−0.025	−0.154	0
0.667	−0.121	−1.269	0	···	···	−0.138	−1.878	0
1.11	−0.159	0.822	−0.907	···	···	−0.099	1.143	−1.024
1.70	0.427	0.978	−0.435	0.983	−0.447	0.469	1.197	−0.488
3.4	0.461	0.913	−0.140	0.918	−0.144	0.520	1.117	−0.132
6.8	0.437	0.870	−0.054	0.873	···	0.475	1.048	−0.032
13	0.424	0.851	−0.029	···	···	0.454	1.019	−0.008
25	0.431	0.856	−0.022	···	···	0.439	1.009	−0.002
infinity	0.435	0.860	0	···	···	0.435	1.000	0

[a] η is the photon energy/ionization energy; F is the real (or dispersive) forward scattering amplitude in units of e^2/mc^2; F_c is the part of F found by evaluating the dispersion integral only over continuum absorption; G is the imaginary (or absorptive) forward scattering amplitude.
[b] The calculations of F and F_c for lead are made by numerical evaluation of Eq. (4) using the numerical values of Tables I and II. The results should be accurate to better than 3%.
[c] Reference 3.
[d] Nonrelativistic calculations for Coulomb wave functions for any Z are made by using the Stobbe oscillator density.

the electron emitted in the photoeffect, in units of μ; while γ_+ is the total energy E_+ of a positron emitted in the $\sigma_{\text{P.P.}}$ process, in the same units.

The numerical values used for σ for lead are collected in Table II. The values used fall on a smooth curve; and should be accurate to about 5%. We see that subtracting $\sigma_{\text{P.P.}}$ gives less than a 10% correction up to energies of 5 mc^2; but subtracting $\sigma_{\text{P.P.}}$ changes the asymptotic dependence from the Hall-Sauter k^{-1} dependence to a k^{-2} dependence at very high photon energies.

III. FORWARD SCATTERING AMPLITUDE

The real part F of the forward scattering amplitude is related to the absorption cross section σ by the dispersion theory integral[4]:

$$F(\eta)/r_0 = \mathrm{P}\int_0^\infty \eta^2(df/d\eta')d\eta'/(\eta^2-\eta'^2), \qquad (4)$$

where P represents the Cauchy principal value. Here r_0 is the electron radius e^2/mc^2; we express energies in terms of η=photon energy/I; the oscillator density per unit ionization energy is proportional to the absorption cross section: $df/d\eta' = \sigma/108.7\ I = \sigma/1080$, for our relativistic treatment of lead, where σ is in barns, and I in Mev. We calculate this integral numerically for various values of η taking the oscillator density $df/d\eta'$ for the continuum from Table II, with interpolation; and taking the oscillator strength for discrete transitions from Table I. Our results for lead are given in Table III, together with the calculations of F by Brown *et al.*[2,3] for mercury and calculations for a nonrelativistic K electron.

We have expressed our value of F as the sum of F_c and F_d. The term F_c comes from the integral over the continuum for η' from 1 to infinity; while F_d is the contribution to the forward scattering amplitude from discrete transitions. (We have made this separation since for comparison with experiments on an actual lead atom, the scattering amplitude per K electron will more nearly be given by F_c than by F, since the discrete transitions giving F_d are forbidden by the Pauli principle. Of course, the scattering by L, M, etc., electrons would also have to be included, and calculations of $df/d\eta'$ should be made with screened wave functions.) We have omitted the energy region between $0.667I$ and $1.11I$ since in that region the scattering amplitude F fluctuates violently due to the resonances corresponding to transitions to the various discrete states.

We also present in Table III the imaginary (or absorptive) part of the forward scattering amplitude, here denoted by G. G is proportional to the absorption cross section at that energy, according to the optical theorem

$$G/r_0 = (k/4\pi r_0)\sigma = (\pi/2)\eta(df/d\eta). \qquad (5)$$

We obtain the last expression on the right using the relations: photon wave number $k=I\eta/\hbar c$; and absorption cross section $\sigma=(2\pi^2e^2\hbar/mc)(df/d\eta)/I$. Brown's values of G for mercury are obtained from a private communication in which he gave $\sigma(0.32\ mc^2)=189$ barns, and $\sigma(0.64\ mc^2)=30.4$ barns.

As shown in Table III and Fig. 1, our values of the dispersive forward scattering amplitude F for a K electron of lead check quite well with the three values of F found by Brown *et al.*[3] for mercury, as part of their extensive work on the coherent scattering as a function of angle. We believe that our present value $F(\infty)=0.86r_0$ should be reliable to better than 3%. [This numerical value checks well, but only fortuitously, with the value $F(\infty)=0.85r_0$ given in A.] Thus the forward scattering amplitude does *not* approach the value r_0 for a free electron at very high energies. Similarly the summed oscillator strength, or the corresponding integrated cross section, are not given by the Thomas-Reiche-Kuhn (TRK) value, but are only about 86% as large. This high-energy behavior of F for a relativistically bound electron casts doubt on the validity of the Gell-Mann, Goldberger, and Thirring[4] assumption on the high-energy dependence of F, used for example in their calculation of the integrated cross section for photonuclear reactions.

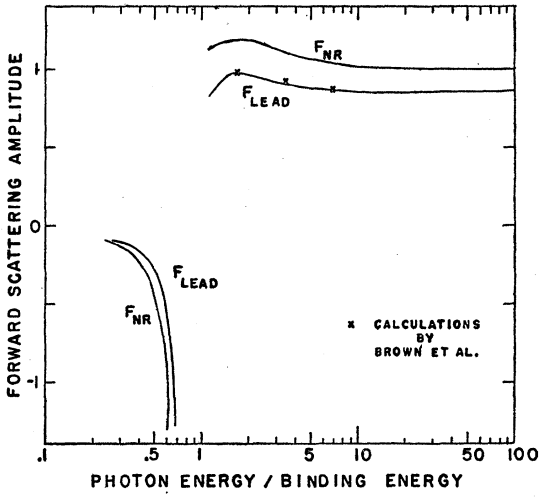

FIG. 1. Numerical values for the dispersive forward scattering amplitude F, in units of e^2/mc^2, are given in Table III. The curve for lead is calculated using relativistic oscillator strengths for a K electron in the Coulomb field of a lead nucleus. The nonrelativistic (NR) curve applies to any nonrelativistic atom. The points marked × show the relativistic calculations by Brown *et al.*, reference 3, for forward scattering by a K electron of mercury.

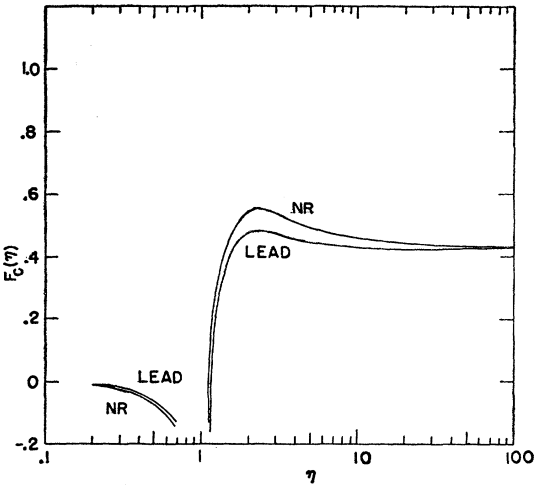

FIG. 2. Here η is the photon energy/binding energy; and F_c is that part of the dispersive forward scattering amplitude that comes from the oscillator density to the continuum for a K electron. See Table III for numerical values. The lead curve is calculated relativistically, while the NR curve applies to any nonrelativistic atom. F_c will be a better approximation than F, drawn in Fig. 1, for the forward scattering amplitude by a K electron in an actual atom. However, screening corrections should be made.

We see that the absorptive term G in the forward scattering amplitude is comparable to the dispersive term F near the K edge, but that G decreases rapidly with increasing photon energy, while F approaches an asymptotic value.

It is also of interest to compare F for the relativistically bound electron in lead with F for a nonrelativistic atom. For the nonrelativistic case we use the Stobbe electric dipole cross section for which $df/d\eta'$ is independent of Z. (See Table I for nonrelativistic oscillator strengths for discrete transitions; and A, Table IV for nonrelativistic oscillator densities based on Stobbe's photoeffect cross section.) First, we see that the nonrelativistic value of $F(\infty)/r_0$ is unity, as it must be. While the relativistic and nonrelativistic treatments differ appreciably on the value of $F(\infty)$, and also for F and G at other photon energies, they are in agreement on the values of $F_c(\infty)$ and in fair agreement on $F_c(\eta)$ as shown in Fig. 2. As discussed above, we would expect F_c to be a better approximation than F in dealing with scattering by K electrons in an actual atom, where the important discrete transitions are forbidden; so we conclude that the K forward scattering in an actual lead atom may not be very different from that given by the nonrelativistic calculation. [The largest difference is at about twice the ionization energy where $(F_c)_{\mathrm{NR}}=0.55$ and $(F_c)_{\mathrm{lead}}=0.48$.] One can also compare the dispersion curves given here with the universal dispersion curves, assuming a power law equation for oscillator density, calculated by Parratt and Hempstead.[10]

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[10] L. G. Parratt and C. F. Hempstead, Phys. Rev. 94, 1593 (1954).