

The second excited state seen here is most likely to be an odd-parity state because of the low comparative half-life of the beta transition to this state, and spin 3 is preferred to 2 on account of the Glaubman and Talmi rule. The energy value is just about what is expected.<sup>1</sup>

Ga<sup>74</sup> was produced by bombarding pure germanium metal with the same neutron source. Very many gamma rays with various half-lives were observed after the bombardment, but all could be assigned to some known isotopes produced by fast neutrons on Ge, except for three distinct gamma rays with energies 0.58, 2.3, and 2.6 Mev which decayed with a half-life of about 8 min. All the possible activities expected from neutron irradiation of Ge are known except those of Ga<sup>74</sup> and Ga<sup>76</sup>, and the yield of the latter is expected to be much smaller than that of the former because of the high (*n*,*p*) threshold and the more unfavorable isotopic content. Also, the energy of one of the gamma rays agrees well with that of the first excited state of Ge<sup>74</sup>.<sup>2</sup> Therefore this activity is assigned to Ga<sup>74</sup>.

It is rather difficult to determine the decay scheme from this, but it is noticed that the gamma-ray intensity distribution looks quite similar to that of Ga<sup>72</sup>, just as the Cl<sup>40</sup> decay is similar to the decay of Cl<sup>38</sup>. Also, from the systematics of odd-parity states, odd-parity states can be expected at around 3 Mev. Therefore, the decay scheme of Ga<sup>74</sup> is probably like that given in Fig. 1(d).

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\* Now at the Department of Physics, University of Lund, Lund, Sweden.

<sup>1</sup> H. Morinaga, preceding Letter [Phys. Rev. **103**, 503 (1956)].

<sup>2</sup> Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).

## Bremsstrahlung in High-Energy Nucleon-Nucleon Collisions

R. E. CUTKOSKY

Carnegie Institute of Technology, Pittsburgh, Pennsylvania

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PREVIOUS studies of the bremsstrahlung emitted in high-energy nucleon-nucleon collisions have treated the interaction between the two particles in the Born approximation<sup>1</sup>; this procedure is certainly incorrect for the high-energy gamma rays, where the spectrum shape is modified by the strong interaction in the low-energy final state of the two nucleons. In this note we show that the spectrum, for gamma-ray energies sufficiently close to the end point that the final nucleons are in an *S* state, can be calculated by a

method similar to that used in calculating the spectrum of  $\pi$  mesons produced in nucleon-nucleon collisions.<sup>2</sup> This calculation predicts a peak in the bremsstrahlung spectrum for high-energy gamma rays, a measurement of which would provide interesting information about nucleon-nucleon interactions.

Consider first the neutron-proton bremsstrahlung, with the final state being a <sup>3</sup>S state. As a special case, the final state may be a deuteron, giving a discrete line 2.2 Mev above the end point  $\omega_0$  of the continuous spectrum; the cross section for deuteron formation is determined from the principle of detailed balancing to be

$$d\sigma_0 = 3\omega_0(2M)^{-1}d\sigma_d(\omega_0),$$

where  $M$  is the mass of a nucleon, and  $d\sigma_d$  is the photodisintegration cross section. The energy is supposed to be nonrelativistic but much greater than the deuteron binding energy  $B$ . In general, we may write

$$M_{fi} = -(2\omega)^{-\frac{1}{2}}(\psi_f, J\psi_i), \quad J = \int \mathbf{j}(\mathbf{x}) \cdot \mathbf{e} e^{-ik \cdot \mathbf{x}} d^3x. \quad (1)$$

Let  $J = J_0 + J'$ , where  $J_0$  is the contribution of the ordinary current associated with free particles, and the remainder  $J'$  represents effects such as meson exchange; furthermore, let  $H = H_0 + H'$ , where  $H'$  gives the interaction between the two particles, and let  $\phi$  denote an unperturbed state. Then

$$\begin{aligned} (2\omega)^{\frac{1}{2}} M_{fi} &= -(\psi_f, J'\psi_i) - (\psi_f, J_0\phi_i) \\ &\quad + (\psi_f, J_0[H_0 - E_i - i\epsilon]^{-1}H'\psi_i) \\ &= -(\psi_f, J'\psi_i) \\ &\quad + \sum_n (\psi_f, H'\phi_n)(E_n - E_f - i\epsilon)^{-1}(\phi_n, J_0\phi_i) \\ &\quad + \sum_n (\psi_f, J_0\phi_n)(E_n - E_i - i\epsilon)^{-1}(\phi_n, H'\psi_i). \end{aligned} \quad (2)$$

The contribution of the first two terms depends only on  $\psi_f(x)$  for points  $x$  within the range  $R$  of nuclear forces, and this is also true of the less important third term for sufficiently high-energy gamma rays. While  $\psi_f(x)$

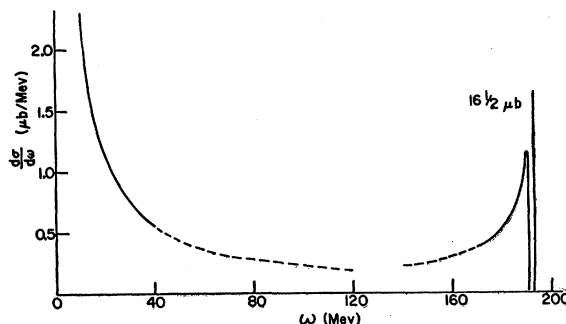


FIG. 1. Predicted bremsstrahlung spectrum (in the c.m. system) for collisions of 400-Mev neutrons with protons. The discrete line with a  $16\frac{1}{2}$ - $\mu$ b cross section corresponds to deuteron formation. The low-energy curve is the semiclassical result, which is known to be correct in the limit; the high-energy peak is predicted by the calculation in the text.

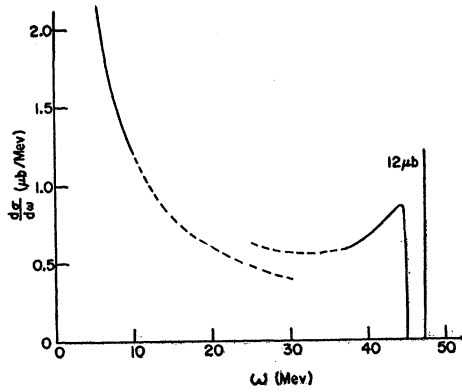


FIG. 2. Predicted bremsstrahlung spectrum (in the c.m. system) for collisions of 90-Mev neutrons with protons.

is unknown for  $r < R$ , we may expect  $\psi_f(x)/R\psi_f(R)$  to be nearly independent of energy over a considerable range of final-state energies. Therefore we can write  $(2\omega)^{1/2}M_{fi} = [R\psi_f(R)]A$ , where the factor  $A$  is a slowly varying function of the initial energy and the gamma-ray frequency; this allows us to relate the part of the high-energy continuous spectrum due to  $^3S$  final states to the cross section for deuteron formation. Since from Eq. (2) it seems plausible that  $A$  depends primarily on the gamma-ray energy,

$$d\sigma_3 = \frac{Mq d\omega |\psi_q^{(3)}(R)|^2}{(2\pi)^2 |\psi_d(R)|^2} d\sigma_0(\omega) \\ = \frac{3}{4\pi} \left( \frac{\omega_0 - \omega}{B} \right)^{1/2} \frac{\omega d\omega / M}{B + \omega_0 - \omega} d\sigma_d(\omega), \quad (3)$$

where  $q = [M(\omega_0 - \omega)]^{1/2}$  is the relative momentum of

the nucleons in the final state, and the zero-range approximation has been used for  $R\psi_f(R)$ .

In an exactly similar way, we find that the contribution of  $^1S$  final states to the spectrum may be expressed as

$$d\sigma_1 = \frac{1}{4\pi} \left( \frac{\omega_0 - \omega}{B^*} \right)^{1/2} \frac{\omega d\omega / M}{B^* + \omega_0 - \omega} d\sigma^*(\omega). \quad (4)$$

Here  $B^*$  is the energy of the virtual singlet state (70 keV), and  $d\sigma^*(\omega)$ , which is defined by the above expression, may be interpreted as the photodisintegration cross section of the (virtual) singlet deuteron. For the purpose of a rough numerical estimate, one may neglect all spin-dependent effects [except in determining  $\psi_q(0)$ ], which gives

$$d\sigma_1 = \frac{1}{3} [|\psi_q^{(1)}(0)|^2 / |\psi_q^{(3)}(0)|^2] d\sigma_3. \quad (5)$$

Equations (3) and (5) were used with the experimental photodisintegration data<sup>3</sup> to calculate the high-energy part of the curves shown in Figs. 1 and 2. The low-energy curves were obtained from the well-known semiclassical formula and the experimental scattering data.<sup>4</sup> Similar results are obtained in proton-proton scattering, although the intensity of gamma radiation is much reduced, owing to the absence of electric dipole radiation. Further discussion of these questions is deferred to a more complete study.

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