Reorientation Effect in Coulomb Excitation*

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The semiclassical treatment of the effect of the reorientation of the spin of the target nucleus during a collision resulting in Coulomb excitation is presented. It is found that the effect vanishes for zero excitation, but that it may be appreciable for finite excitation. The effect on the angular distribution of the photons is similarly found to vanish for head-on collisions. Three different types of experiments involving the measurement of the angular distribution of the photons are discussed. Typical numerical results for comparison with experiment are presented and the possibilities offered by bombardment with heavy ions are noted.

I. INTRODUCTION

THE fact that finite-amplitude effects can be appreciable in Coulomb excitation has been pointed out by Breit and Lazarus.[1] In the present note, one of the effects is calculated in the semiclassical approximation for a $0 \to 2$ transition. The effect under consideration consists in the reorientation of the nuclear axis caused by the electric field of the bombarding particle after it has excited the nucleus. The change in the nuclear spin direction affects the angular distribution of the γ rays. The latter distribution is affected by other finite-amplitude effects entering in the same order of the calculation such as the excitation $0 \to 4$ followed by de-excitation $4 \to 2$. Since this effect depends on higher (2^4) multipole action, it appears reasonable to neglect it in a preliminary survey of possibilities. The transition chain $0 \to 2 \to 2'$ will also affect the γ angular distribution. In the usual case of Coulomb excitation from ground states of even-even nuclei, the order of the rotational levels is 0, 2, and 4 while levels with $I=2$ occur at higher energies and transitions to them involve changes in quantum numbers additional to the rotational one. For both reasons, their effects may be expected to be less serious although they should be eventually taken into account.

The principal interest in the reorientation effect lies in the possibility which it offers of ascertaining static nuclear quadrupole moments in excited states. The other available method[2] employing intermolecular field involves the theory of solids and complexities regarding the motion of a recoil nucleus through a solid. The reorientation effect in Coulomb excitation appears to be relatively free of such complications, the orbit of the projectile being well defined. It should be pointed out that screening corrections for the influence of atomic electrons are minor for the Coulomb excitation reorientation effect while they are present in the method of intermolecular fields. Since values of quadrupole moments of ground states are usually affected by screening corrections[3] and since the latter are hard to estimate reliably,[4] there may be a special value in the Coulomb excitation reorientation effect for the measurement of these moments.

The calculations presented below are made by means of the semiclassical treatment (SCT) which employs classical mechanics for the relative motion of the target and projectile. Since one expects the reorientation effect to be more serious for the heavier projectiles, this approximation is likely to be satisfactory for a preliminary survey. The calculations are presented with reference to three possible experiments. In the first the angular distribution of the γ rays is measured in the usual manner, the only reference line being the incident charged particle beam. In the second the inelastically scattered charged particles are counted in coincidence with the γ rays, no attempt being made to define the charged particle orbit. In the third the incident beam and the inelastically scattered particle directions are used to define the orbit in coincidence with γ counting. In the third type of experiment there is a maximum possibility of obtaining checks on the parameters entering the interpretation. It is probably the one most seriously affected by the inexactness of the SCT.

The present paper is confined to consideration of the first nonvanishing order in the finite amplitude effects. It is thus concerned with the calculation of cross-term effects arising from the second order probability amplitude effects. Terms quadratic in the second-order probability amplitude corrections are omitted since their inclusion would require the consideration of third order effects in the probability amplitudes. With these approximations, it is found that some special circumstances cause the reorientation effect on the γ-angular distribution to vanish for zero excitation energy. These considerations do not exclude the possibility of detection of finite-amplitude effects for small excitation energies by studies of inelastic scattering. For nonvanishing excitation energies, there remains an effect which, while somewhat affected by the special circumstances which

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[4] C. Schwartz, Phys. Rev. 97, 380 (1955).

make it vanish for zero excitation energy, appears to be large enough for observation.

An additional special circumstance arises for the case of finite excitation by head-on collisions. The total cross section shows an appreciable effect, but the angular distribution remains unaffected in the higher order calculation. Since the large probabilities of excitation occur for head-on collisions, this circumstance makes the reorientation effect smaller than it might have been otherwise. Nevertheless the second-order effects rise rapidly as the impact parameter increases from its vanishing value for head-on collisions.

List of Notation

The following is a list of the more frequently occurring symbols and their definitions in the approximate order of their appearance.

Z_1, Z_2 = nuclear charge of the incident, target particles respectively.

v = velocity of the incident particle.

$\mathbf{r}$, $\mathbf{r}_t$ = displacement of the incident particle from the target nucleus in the frame of reference in which the x axis bisects the orbit hyperbola. The plane of the orbit is the xy plane and the direction of the orbit is such that the incident particle moves from the fourth to the first quadrant of the xy plane in its trajectory.

Φ = azimuth of $\mathbf{r}$ with respect to the x axis.

$\mathbf{r}_p$; r_p, θ_p, φ_p = displacement and components in spherical coordinates of the equivalent nuclear proton in the same frame of reference as that used for $\mathbf{r}$.

θ_{pt} = angle between $\mathbf{r}$ and $\mathbf{r}_p$.

Y_{lm} = spherical harmonic of order l and magnetic quantum number m.

$R_i(r_p)$, $R_f(r_p)$ = radial wave function of the equivalent nuclear proton in the ground and excited states of the target nucleus.

$\mu=2, 1, 0, -1, -2$ = magnetic sublevels of an $I=2$ state of the excited target nucleus.

$H'(t)$ = time-dependent quadrupole interaction term between the incident particle and the proton in the target nucleus.

$E_{fi}=\hbar\omega_{fi}=\hbar\omega$ = excitation energy of the excited state of the target nucleus.

$H'_{\mu i}(t)$ = matrix element of $H'(t)$ between the target nucleus ground state ($I=0$) and the excited state ($I=2$) sublevel with magnetic quantum number μ.

$H'_{\mu\mu'}(t)$ = matrix element of $H'(t)$ between sublevels μ and μ' of the excited ($I=2$) state of the target nucleus.

$c_0(t)$, $c_\mu(t)$ = time-dependent amplitudes of the nuclear wave function corresponding to the ground-state ($I=0$) and excited-state ($I=2$) sublevels given by μ.

$c=c^{(0)}+c^{(1)}+c^{(2)}$ = separation of the time-dependent amplitudes into terms corresponding to the power of the interaction H', as given in the superscript.

$\langle r^2\rangle_{ff}$, $\langle r^2\rangle_{fi}$ = radial matrix element of r_p^2 between the states R_f, R_f and R_f, R_i respectively.

$2a'$ = closest distance of approach of the incident particle to the target nucleus for head-on collision.

ϵ = eccentricity of the hyperbolic orbit.

$\xi=\eta_f-\eta_i$, $\eta_i=Z_1Z_2e^2/\hbar v_i$, $\eta_f=Z_1Z_2e^2/\hbar v_f$.

$S_\mu^{(2)}$ = amplitude for direct quadrupole Coulomb excitation to the sublevel μ, apart from ϵ- and ξ-independent factors.

S_μ = quantity which replaces $S_\mu^{(2)}$ after inclusion of the reorientation effect.

(θ,φ) = polar angles of the direction of the photon emission in the same coordinate system as that used for $\mathbf{r}$.

$\mathbf{c}$, $\mathbf{s}$ = $\cos\theta$, $\sin\theta$ respectively.

$\delta=\tan^{-1}(\epsilon^2-1)^{\frac{1}{2}}$. The angle $\pi-2\delta$ is the scattering angle of the incident particle.

$\varphi'=\varphi+\delta$ = azimuthal angle of the photon emission direction for the x-axis directed along the negative of the initial particle velocity and the z axis the same as for φ.

$\mathbf{1}_s$ = unit vector in the direction of polarization of the photon.

$\mathbf{k}$ = propagation vector of the photon.

$F_2(kr)=(\pi/2kr)^{\frac{1}{2}}J_1(kr)$.

θ' = angle of $\mathbf{k}$ with respect to the negative of the incident particle direction.

J = angular distribution, apart from constant factors, of the emitted photons.

$\langle J\rangle$ = average of J with respect to a rotation of the orbit plane about the incident beam direction.

$a_{2\epsilon}(\xi)$, $a_{4\epsilon}(\xi)$ = angular distribution coefficients of the Legendre polynomial of order 2 and 4 for a given orbit eccentricity, ϵ.

$a_2(\xi)$, $a_4(\xi)$ = averages over orbit eccentricity of $a_{2\epsilon}(\xi)$, $a_{4\epsilon}(\xi)$.

$T_0=\frac{1}{2}S_0^{(2)}$.

$T_c=-(S_2^{(2)}+S_{-2}^{(2)})/2\sqrt{6}$.

$T_s=(S_{-2}^{(2)}-S_2^{(2)})/2\sqrt{6}$.

$\mathcal{T}_0=\frac{1}{2}S_0$.

$\mathcal{T}_c=-(S_2+S_{-2})/2\sqrt{6}$.

$\mathcal{T}_s=(S_{-2}-S_2)/2\sqrt{6}$.

$\lambda=Z_1e^2\langle r^2\rangle_{ff}/[7\hbar a'^2v]$, parameter relating magnitude of the reorientation effect to direct Coulomb excitation.

$D+\delta D$ = separation of the θ'-independent term in $(1/4)\langle J\rangle$ into the result of neglecting reorientation and the change due to reorientation.

$N_2+\delta N_2$ = separation of the coefficient of $P_2(\theta')$ in $(56/5)\langle J\rangle$ into the result of neglecting reorientation and the change due to reorientation.

$N_4+\delta N_4$ = separation of the coefficient of $P_4(\theta')$ in $(7/2)\langle J\rangle$ into the result of neglecting reorientation and the change due to reorientation.

$C_1, \cdots C_7$ = coefficients of the angle-dependent terms in a measurement of the correlation between the directions of the scattered particle and the photon. The C's are defined in Eqs. (15), (14.2), (14.3), (14.4), (14.5).

II. PROBABILITY AMPLITUDES

In the interests of simplicity, the calculation will be presented as though there were only one nuclear proton present. As is well known, the generality of application is not affected by doing so provided the results are expressed in terms of appropriate transition and static quadrupole moments. The interaction Hamiltonian is taken as

$$H'(t)=Z_1e^2(r_p{}^2/r_t{}^3)P_2(\cos\theta_{pt}), \tag{1}$$

the nuclear proton quantities being distinguished by subscript p, the trajectory quantities by subscript t, and θ_{pt} being the angle between the proton and projectile directions as viewed from the origin. For simplicity the discussion will be carried on as though the bombarded nucleus were infinitely heavy. The nuclear wave function is represented as

$$w_\mu(\mathbf{r}_p)=R_f(r_p)Y_{2\mu}(\theta_p,\varphi_p) \tag{1.1}$$

in the state with $I=2$ and as

$$v_0(\mathbf{r}_p)=R_i(r_p)Y_{00}(\theta_p,\varphi_p) \tag{1.2}$$

in the state with $I=0$. The radial functions are normalized by

$$\int_0^\infty R^2(r_p)r_p{}^2dr_p=1; \tag{1.3}$$

the latter equation being meant to apply to both radial functions. The wave function is expanded as

$$\psi=c_0v_0+\Sigma_\mu c_\mu w_\mu; \tag{1.4}$$

all other nuclear states being omitted in view of the simplifying assumptions mentioned in the introduction. Employing the zero-order approximation

$$c_0{}^{(0)}=\exp(-iE_0t/\hbar) \tag{1.5}$$

and the equation

$$\frac{\hbar}{i}\frac{d}{dt}[c_\mu\exp(iE_\mu t/\hbar)]+\{H'_{\mu i}(t)c_0+\Sigma_{\mu'}H'_{\mu\mu'}(t)c_{\mu'}\}\times\exp(iE_\mu t/\hbar)=0, \tag{1.6}$$

one obtains

$$c_\mu=c_\mu{}^{(1)}+c_\mu{}^{(2)} \tag{2}$$

with

$$c_\mu{}^{(1)}=-(i/\hbar)\exp(-iE_\mu t/\hbar)\int_{-\infty}^t H'_{\mu i}(t)\times\exp(iE_{\mu i}t/\hbar)dt, \tag{2.1}$$

$$c_\mu{}^{(2)}=-(i/\hbar)\exp(-iE_\mu t/\hbar)\times\int_{-\infty}^t\exp(iE_\mu t/\hbar)[H'_{\mu i}(t)c_0{}^{(1)}+\Sigma_{\mu'}H'_{\mu\mu'}(t)c_{\mu'}{}^{(1)}(t)]dt, \tag{2.2}$$

where $c_0{}^{(1)}$ is the first-order correction to c_0, the designation of the order being in terms of the power of H' involved. One obtains also

$$c_0{}^{(1)}=-(i/\hbar)\exp(-iE_0t/\hbar)\times\int_{-\infty}^t H'_{i\mu}(t)\exp(iE_0t/\hbar)c_\mu{}^{(1)}(t)dt. \tag{2.3}$$

Since the c_μ are being calculated by an iteration procedure which gives perfect normalization when completely carried out, a correction for normalization need not be made and Eq. (2) should be therefore good enough for obtaining $|c_\mu|^2$ correct up to and including terms in H'^3. It may be of interest to note that the normalization sum resulting from the inclusion of $c_\mu{}^{(2)}$ and without taking $c_0{}^{(1)}$ into account is already good enough for calculating H'^3 effects on $|c_\mu|^2$ without the consideration of what happens in higher iterations as may be seen as follows. In a given order, the iteration procedure gives only an approximately normalized solution. Calculation of

$$|c_0|^2+\Sigma_\mu|c_\mu|^2$$

shows the presence of a correction factor of the form

$$1+\mathcal{O}(H'^2)$$

to all probability amplitudes. Since the intention is only that of obtaining c_μ to within order H'^2, one may omit this factor and similarly, since $c_0{}^{(1)}=\mathcal{O}(H'^2)$, the first term in brackets in Eq. (2.2) is $\mathcal{O}(H'^3)$ so that it may be dropped. There results

$$c_\mu(t)=c_\mu{}^{(1)}(t)-(i/\hbar)\exp(-iE_\mu t/\hbar)\int_{-\infty}^t\exp(iE_\mu t/\hbar)\times\Sigma_{\mu'}H'_{\mu\mu'}(t)c_{\mu'}{}^{(1)}(t)dt+\mathcal{O}(H'^3). \tag{2.4}$$

From now on, the correction $\mathcal{O}(H'^3)$ will be omitted. The quantity c_μ being needed only for $t=\infty$, it suffices to consider

$$c_\mu(\infty)=c_\mu{}^{(1)}(\infty)-\hbar^{-2}\exp(-i\omega_f t)\int_{-\infty}^{+\infty}dt''\times\int_{t''}^\infty dt'\Sigma_{\mu'}H'_{\mu\mu'}(t')H'_{\mu' i}(t'')\exp(i\omega_{fi}t'') \tag{3}$$

with the notation

$$\omega_f=E_f/\hbar,\quad \omega_i=E_i/\hbar,\quad \omega_{fi}=E_{fi}/\hbar. \tag{3.1}$$

Employing standard manipulations, one finds

$$\Sigma_{\mu'}H'_{\mu\mu'}(t)H'_{\mu' i}(t')=\frac{5}{(4\pi)^{\frac{3}{2}}}AI_\mu/[r^3(t)r^3(t')] \tag{3.2}$$

with

$$A=Z_1{}^2e^4\left\{\int_0^\infty r_p{}^4R_f{}^2(r_p)dr_p\right\}\times\left\{\int_0^\infty r_p{}^4R_f(r_p)R_i(r_p)dr_p\right\} \tag{3.3}$$

and

$$I_\mu = \int Y_{2\mu}^*(p)P_2(pp')P_2(pt)P_2(p't')d\Omega_p d\Omega_{p}'$$

$$= (4\pi/5)\int Y_{2\mu}^*(p)P_2(pt)P_2(pt')d\Omega_p. \quad (3.4)$$

Here p stands for θ_p and φ_p, pt for θ_{pt}, etc. The quantities I_μ can be conveniently evaluated in a coordinate system with z-axis perpendicular to the plane tt'. It is found that

$$I_\mu = (4\pi/5)^{\frac{1}{2}} i_\mu/7, \quad (3.5)$$

$$i_\mu = \tfrac{1}{2}(\tfrac{3}{2})^{\frac{1}{2}}(\tau^{-2}+\tau'^{-2}),\ 0,\ 2-3\cos^2(\Phi-\Phi'),\ 0,$$

and

$$\tfrac{1}{2}(\tfrac{3}{2})^{\frac{1}{2}}(\tau^2+\tau'^2) \quad (3.6)$$

for $\mu=2, 1, 0, -1$, and -2, respectively, with

$$\tau = \exp(i\Phi), \quad \tau' = \exp(i\Phi') \quad (3.7)$$

and Φ, Φ' standing for azimuthal angles of $\mathbf{r}_t$ and $\mathbf{r}_{t'}$ respectively.

If one denotes the integrals in (3.3) as $\langle r^2\rangle_{ff}$ and $\langle r^2\rangle_{fi}$, respectively, Eq. (3) becomes

$$c_\mu^{(2)}(\infty) = -\frac{Z_1^2 e^4}{7\sqrt{5}}\langle r^2\rangle_{ff}\langle r^2\rangle_{fi}\frac{\exp(i\omega_f t)}{\hbar^2} \times \iint_{t>t'} \frac{i_\mu(t,t')\exp(i\omega_{fi}t')}{r^3 r'^3} dt dt' \quad (4)$$

with r and r' standing for values of r at times t and t', respectively. Strictly speaking, the quantity in this equation is not exactly $c_\mu^{(2)}$ in the sense of Eq. (2.2) but is obtained by omitting the first term in brackets in (2.2), causing an error of order H'^3. Introducing the parametric representation[5]

$$x = a'(\epsilon+\cosh w), \quad y = a'(\epsilon^2-1)^{\frac{1}{2}}\sinh w,$$
$$r = a'(1+\epsilon\cosh w), \quad t = (a'/v)(w+\epsilon\sinh w), \quad (4.1)$$

where

$$a' = (Z_1 Z_2 e^2/Mv^2), \quad (4.1')$$

the quantity M being the reduced mass, one finds that

$$c_\mu^{(1)}(\infty) = -\frac{(4\pi)^{\frac{1}{2}} i}{5\hbar} Z_1 e^2 \langle r^2\rangle_{fi}\exp(-i\omega_f t) \times \int_{-\infty}^{+\infty} r^{-3} Y_{2\mu}^*(t)\exp(i\omega_{fi}t)dt \quad (4.2)$$

has the value

$$c_\mu^{(1)}(\infty) = \frac{iZ_1 e^2}{2(\sqrt{5})\hbar a'^2 v}\langle r^2\rangle_{fi}\exp(-i\omega_f t)S_\mu^{(2)} \quad (4.3)$$

with

$$S_{\pm 2}^{(2)} = -\left(\frac{3}{2}\right)^{\frac{1}{2}}\int_{-\infty}^{+\infty} e^{i\xi(w+\epsilon\sinh w)} \times \frac{[\epsilon+\cosh w \mp i(\epsilon^2-1)^{\frac{1}{2}}\sinh w]^2}{(1+\epsilon\cosh w)^4}dw, \quad (4.4)$$

$$S_0^{(2)} = \int_{-\infty}^{+\infty}\frac{e^{i\xi(w+\epsilon\sinh w)}}{(1+\epsilon\cosh w)^2}dw. \quad (4.5)$$

The notation $S_\mu^{(2)}$ is that used by Alder and Winther.[6] The quantity ξ is taken to be

$$\xi = \eta_f - \eta_i \quad (4.6)$$

rather than $\omega_{fi}a'/v$ in order to obtain better agreement with the quantum-mechanical treatment of first-order Coulomb excitation. Here

$$\eta_i = Z_1 Z_2 e^2/\hbar v_i, \quad \eta_f = Z_1 Z_2 e^2/\hbar v_f. \quad (4.7)$$

For $E_{fi}=0$, the integrals in Eq. (4) are simplified. Thus one finds in this case

$$\frac{1}{2}\left(\frac{3}{2}\right)^{\frac{1}{2}}\iint_{>t'}\frac{\tau^{-2}+\tau'^{-2}}{r^3 r'^3}dt dt' = -\tfrac{1}{2}S_2^{(2)}S_0^{(2)}/(a'^4 v^2), \quad (E_{fi}=0), \quad (5)$$

where the vanishing of contributions from $\sin 2\Phi+\sin 2\Phi'$ can be inferred by a consideration of the transformation $(t,t')\to(-t', -t)$. Since the integral is symmetric, it is possible to remove the condition $t>t'$ and to express the result in terms of an integral over the whole t, t' plane. Substitution in terms of the $S_\mu^{(2)}$ through a comparison with (4.2) and (4.3) yields then the right side of (5). Similarly,

$$\iint_{t>t'}[2-3\cos^2(\Phi-\Phi')]/(r^3 r'^3)dt dt' = [\tfrac{1}{4}(S_0^{(2)})^2 - \tfrac{1}{2}(S_2^{(2)})^2]/(a'^4 v^2), \quad (E_{fi}=0). \quad (5.1)$$

Substitution in Eq. (4) and comparison with Eq. (4.3) gives

$$c_\mu^{(2)}(\infty)/c_\mu^{(1)}(\infty) = \frac{iZ_1 e^2}{7\hbar a'^2 v}\langle r^2\rangle_{ff} \times (-S_0^{(2)}, 0, \tfrac{1}{2}S_0^{(2)} - (S_2^{(2)})^2/S_0^{(2)}, 0, -S_0^{(2)}), \quad (E_{fi}=0) \quad (5.2)$$

the order being again for $\mu=2, 1, 0, -1$, and -2, respectively. The ratio of second-order to first-order effects is seen to be pure imaginary since the $S_\mu^{(2)}$ are real. The gamma-ray intensities depend on the probabilities of emission from the states μ, which in turn are proportional to $|c_\mu|^2$. According to Eq. (5.2), the

[5] K. A. Ter-Martirosyan, J. Exptl. Theoret. Phys. (U.S.S.R.) 22, 284 (1952).

[6] K. Alder and A. Winther, Phys. Rev. 91, 1578 (1953).

population of the excited levels is affected only to the second order of $c_\mu^{(2)}(\infty)/c_\mu^{(1)}(\infty)$ on account of the purely imaginary value of the ratio, and the effect on the angular distribution disappears in the order worked with here.

The consideration just given is incomplete on account of the presence of interference effects between γ rays emitted from different sublevels μ. A consideration of their effect shows that in the lowest order they appear in the combination

$$\left|-\mathbf{cs}S_0^{(2)}+\frac{1}{\sqrt{6}}\mathbf{cs}(S_2^{(2)}e^{2i\varphi}+S_{-2}^{(2)}e^{-2i\varphi})\right|^2 \quad (5.3)$$

with

$$\mathbf{c}=\cos\theta,\quad \mathbf{s}=\sin\theta, \quad (5.4)$$

where (θ,φ) are the polar angles of the direction of photon emission. From Eq. (4.4), it follows that

$$S_2^{(2)}=S_{-2}^{(2)},\quad (E_{fi}=0), \quad (5.5)$$

and hence, according to Eq. (5.2) the effect of the second-order correction, may be represented by

$$S_2^{(2)}\rightarrow(1+i\alpha_2)S_2^{(2)},\quad S_{-2}^{(2)}\rightarrow(1+i\alpha_2)S_{-2}^{(2)},$$
$$S_0^{(2)}\rightarrow(1+i\alpha_0)S_0^{(2)}, \quad (5.6)$$

and the $S_\mu^{(2)}$ as well as α_0, α_2 are real. The cross-product terms arising from (5.2) with the $S_\mu^{(2)}$ corrected for second order effects are seen to be

$$-(4/6^{\frac{1}{2}})\mathbf{c}^2\mathbf{s}^2$$
$$\times\mathrm{Re}\{S_2^{(2)*}S_0^{(2)}(1-i\alpha_2)(1+i\alpha_0)\cos2\varphi\}. \quad (5.7)$$

The α_0 and α_2 survive only in the combination $\alpha_0\alpha_2$ and the angular distribution effect is therefore of a higher order than the effects considered here.

For head-on collisions, which correspond to $\epsilon=1$, one has

$$S_2^{(2)}=-(3/2)^{\frac{1}{2}}S_0^{(2)},\quad (\epsilon=1, E_{fi}=0), \quad (6)$$

so that

$$\tfrac{1}{2}S_0^{(2)}-(S_2^{(2)})^2/S_0^{(2)}=-S_0^{(2)},\quad (\epsilon=1, E_{fi}=0). \quad (6.1)$$

According to Eq. (5.2), the $c_\mu^{(2)}/c_\mu^{(1)}$ are all changed in the same ratio and there can be no change in the γ-ray angular distributions emitted for these orbits even apart from the fact that the changes in the c_μ are 90° out of phase with the first order effects.

III. GAMMA ANGULAR DISTRIBUTIONS

The angular distribution of γ rays for quadrupole radiation can be obtained by employing the interaction energy with the transverse electromagnetic field in the $-\int\mathbf{j}\cdot\mathbf{A}d\mathbf{r}$ form, which makes the transition matrix element appear as

$$H''=C\int w_\mu^*\left(\frac{\mathbf{r}}{r}\cdot\mathbf{1}_s\right)e^{i\mathbf{k}\cdot\mathbf{r}}(dv_0/dr)d\mathbf{r}, \quad (7)$$

with C standing for a constant the value of which is immaterial for immediate purposes. The term $\mathbf{1}_s\cdot(\mathbf{r}/r)$ arises from the particle current, which contains $(\hbar/i)\nabla v_0$. Here $\mathbf{k}$ is the propagation vector of the photon while $\mathbf{1}_s$ is its polarization vector. In Eq. (7), effects of all multipoles are included. For the transition in question, there can be no electric or magnetic dipole effects on account of the ΔL selection rule. The electric quadrupole effect arises from the $3iP_1(\cos\theta)F_2(kr)/(kr)$ part of the expansion of $e^{i\mathbf{k}\cdot\mathbf{r}}$ and gives rise therefore to

$$\int Y_{2\mu}^*(\theta_p,\varphi_p)\left[\left(\frac{\mathbf{r}}{r}\right)_p\cdot\mathbf{1}_s\right]P_1\left(\frac{\mathbf{k}}{k}\cdot\left[\frac{\mathbf{r}}{r}\right]_p\right)d\Omega_p \quad (7.1)$$

as the angle-dependent factors in the matrix elements. Employment of relations between rates of change of matrices and their values replaces the dv_0/dr of Eq. (7) by v_0 giving rise to standard forms of quadrupole matrix elements with the same angular factors. The term in $P_3((\mathbf{k}/k)\cdot(\mathbf{r}/r))$ in the expansion of $e^{i\mathbf{kr}}$ also gives a nonvanishing contribution, but since it is multiplied by $F_3(kr)/kr$ it corresponds to a 2^4 pole and will be omitted. It is convenient to introduce the polarization vectors with direction cosines as follows:

$$\mathbf{1}_a=(\mathbf{c}\cos\varphi,\ \mathbf{c}\sin\varphi,\ -\mathbf{s}),$$
$$\mathbf{1}_b=(-\sin\varphi,\ \cos\varphi,\ 0), \quad (7.2)$$

the first of which is in the plane through the z axis and the photon direction while the second is perpendicular to that plane. The direction cosines of the nuclear proton are

$$(\mathbf{r}/r)_p=(\sin\theta_p\cos\varphi_p,\ \sin\theta_p\sin\varphi_p,\ \cos\theta_p). \quad (7.3)$$

Substitution in Eq. (7.1) gives

$$\int Y_{2,\mu}\cdot\left(\left(\frac{\mathbf{r}}{r}\right)_p\cdot\mathbf{1}_a\right)P_1\left(\frac{\mathbf{k}}{k}\cdot\left[\frac{\mathbf{r}}{r}\right]_p\right)d\Omega_p$$
$$=\left(\frac{4\pi}{5}\right)^{\frac{1}{2}}\frac{1}{\sqrt{6}}\mathbf{cs}\exp(\mu i\varphi),\quad (\mu=2,-2) \quad (7.4)$$

$$\int Y_{2,0}\cdot\left(\left(\frac{\mathbf{r}}{r}\right)_p\cdot\mathbf{1}_a\right)P_1\left(\frac{\mathbf{k}}{k}\cdot\left[\frac{\mathbf{r}}{r}\right]_p\right)d\Omega_p$$
$$=-\left(\frac{4\pi}{5}\right)^{\frac{1}{2}}\mathbf{cs}, \quad (7.5)$$

$$\int Y_{2,\mu}\cdot\left(\left(\frac{\mathbf{r}}{r}\right)_p\cdot\mathbf{1}_b\right)P_1\left(\frac{\mathbf{k}}{k}\cdot\left[\frac{\mathbf{r}}{r}\right]_p\right)d\Omega_p$$
$$=i^{\mu/2}\left(\frac{4\pi}{5}\right)^{\frac{1}{2}}\frac{\mathbf{s}}{\sqrt{6}}\exp(\mu i\varphi),\quad (\mu=2,-2) \quad (7.6)$$

while the other matrix elements vanish. Since the emission of a γ ray leaves the nucleus always in the same state v_0, the γ rays emitted from the sublevels $\mu=2$, 0,

-2 are coherent[7] and one should add the amplitudes obtainable from Eqs. (7.4), (7.5), and (7.6) before taking the square of absolute values. Since the $S_\mu^{(2)}$ are real, it does not matter in the calculation of lowest order effects (i.e., on assumption of infinitesimal amplitude of excitation) whether one uses the quantities in (7.4) and (7.6) or their complex conjugates in the calculation of γ-emission amplitudes. For finite-amplitude effects, however, the $S_\mu^{(2)}$ are replaced in the c_μ by complex numbers and care must be taken therefore regarding having the gamma emission matrix element multiplied by the correct excitation amplitude. The whole process takes place in the order of transitions $i0\rightarrow f\mu\rightarrow i0$, the first arrow occurring by Coulomb excitation and the second by gamma emission. Since, according to (4.2), the $c_\mu^{(1)}$ contains $Y_{2\mu}^*$ under the integral, the element in (7.1) corresponds to absorption of a γ ray and the quantities listed in (7.4), (7.5), and (7.6) should be used multiplied by corresponding $S_\mu^{(2)}$ for the same μ. The relative amplitudes for emission, applicable also in the case of complex quantity replacements for the $S_\mu^{(2)}$, are

$$\begin{aligned}
&\mathbf{c}^2\mathbf{s}^2|-S_0+(1/\sqrt{6})(S_2e^{2i\varphi}+S_{-2}e^{-2i\varphi})|^2\\
&\quad+\tfrac{1}{6}\mathbf{s}^2|S_2e^{2i\varphi}-S_{-2}e^{-2i\varphi}|^2\\
&=\mathbf{c}^2\mathbf{s}^2|S_0|^2+\tfrac{1}{6}\mathbf{s}^2(\mathbf{c}^2+1)(|S_2|^2+|S_{-2}|^2)\\
&\qquad-(2/\sqrt{6})\mathbf{c}^2\mathbf{s}^2\,\mathrm{Re}S_0^*(S_2e^{2i\varphi}+S_{-2}e^{-2i\varphi})\\
&\qquad\qquad-\tfrac{1}{3}\mathbf{s}^4\,\mathrm{Re}(S_2S_{-2}^*e^{4i\varphi})\equiv(2/15)J,
\end{aligned}\tag{7.7}$$

where the omission of the superscript (2) on $S_\mu^{(2)}$ means that it is replaced by its corrected value in such a way that substitution of S_μ for $S_\mu^{(2)}$ on the right side of Eq. (4.3) causes a change of $c_\mu^{(1)}$ on the left to $c_\mu^{(1)}+c_\mu^{(2)}$. The quantity J in Eq. (7.7) contains the angular correlation of the γ rays with the direction of the incident particle; the factor 2/15 turns out to be convenient in later expressions, but has no other significance.

In the calculation carried out above, the parametric representation of Eq. (4.1) has been used. This corresponds to the x axis directed along the major axis of the hyperbolic orbit. It is more convenient to have the results referred to axes such that the initial particle velocity is along the negative direction of the x axis. This is accomplished by the transformation

$$x'+iy'=(x+iy)e^{i\delta},\quad \delta=\tan^{-1}(\epsilon^2-1)^{\frac{1}{2}},\tag{7.8}$$

where (x',y',z) are the new axes. The azimuthal angle of the γ ray with respect to the (x',y',z) axes will be called φ'. It is related to φ by

$$\varphi'=\varphi+\delta.\tag{8}$$

In the γ-ray intensity formulas one should substitute therefore $\varphi'-\delta$ for φ.

The relative intensities will first be obtained by averaging over rotations around x'. Such intensities are needed for an experiment in which no coincidences with inelastically scattered particles are used. The intensities so obtained can also be used for an experiment in which the inelastically scattered particles are counted in a cone corresponding to a given scattering angle $\pi-2\delta$ of the inelastically scattered particle. The latter type of experiment yields more information than the former since it gives the angular distribution corresponding to a given orbit eccentricity ϵ. In the evaluation of averages of Eq. (7.7) one needs the following averages over rotations of orbit planes around Ox'.

$$\begin{aligned}
\langle(\mathbf{c}^2+1)\mathbf{s}^2\rangle&=(4/5)+(2/7)P_2-(3/35)P_4,\\
\langle\mathbf{c}^2\mathbf{s}^2\rangle&=(2/15)-(1/21)P_2-(3/35)P_4,\\
\langle\mathbf{c}^2\mathbf{s}^2\exp(2i\varphi')\rangle&=(1/7)(P_2-P_4),\\
\langle\mathbf{s}^4\exp(4i\varphi')\rangle&=P_4.
\end{aligned}\tag{8.1}$$

The argument of the Legendre function P_L is $\cos\theta'$, with θ' standing for the angle between the photon propagation vector and the negative of the incident beam direction. The averaging is readily performed by introducing the coordinate system of the orbit plane, transforming to a coordinate system fixed with respect to the laboratory, and working in Cartesian coordinates until one is ready to introduce the angle of rotation of the orbit plane by a simple transformation for z and y'. Substitution of these averages in (7.7) gives

$$\begin{aligned}
\langle J\rangle=\mathrm{Re}\{&|S_2|^2+|S_0|^2+|S_{-2}|^2\\
&+(5/14)[|S_2|^2-|S_0|^2+|S_{-2}|^2\\
&-(\sqrt{6})S_0^*(S_2e^{-2i\delta}+S_{-2}e^{2i\delta})]P_2\\
&-(3/28)[|S_2|^2+6|S_0|^2+|S_{-2}|^2\\
&-(10/3)(\sqrt{6})S_0^*(S_2e^{-2i\delta}+S_{-2}e^{2i\delta})\\
&+(70/3)S_2S_{-2}^*e^{-4i\delta}]P_4\}.
\end{aligned}\tag{8.2}$$

The quantity $\langle J\rangle$ consists of two contributions, the first of which corresponds to replacing the S_μ by their first-order values $S_\mu^{(2)}$. This part will be referred to as $\langle J^{(1)}\rangle$, the superscript on J referring to the order of the calculation rather than to the order of the multipole. The second part consists of terms one order higher than $\langle J^{(1)}\rangle$ and is obtained by collecting cross-product terms of the first-order terms and the correction terms to the $S_\mu^{(2)}$. The contribution to $\langle J\rangle$ due to these terms will be called $\langle J^{(2)}\rangle$. Thus

$$\langle J\rangle=\langle J^{(1)}\rangle+\langle J^{(2)}\rangle+\cdots.\tag{8.3}$$

One finds from Eq. (8.2)

$$\begin{aligned}
\langle J^{(2)}\rangle=-(5/14)(\sqrt{6})[&S_0^{(2)}(S_2^i-S_{-2}^i)\\
&-S_0^i(S_2^{(2)}-S_{-2}^{(2)})](P_2-P_4)\sin2\delta\\
&-(5/2)(S_2^iS_{-2}^{(2)}-S_2^{(2)}S_{-2}^i)\\
&\qquad\times P_4\sin4\delta+\delta_r\langle J^{(1)}\rangle,
\end{aligned}\tag{8.4}$$

where δ_r means the change caused by changing the $S_\mu^{(2)}$ in $J^{(1)}$ to $\mathrm{Re}\{S_\mu\}$, and where

$$S_\mu^i=\mathrm{Im}S_\mu,\tag{8.4'}$$

[7] See, for example, G. Breit, Revs. Modern Phys. 5, 91 (1933), Part VII.

while

$$\sin 2\delta = 2(\epsilon^2-1)^{\frac{1}{2}}/\epsilon^2, \quad \sin 4\delta = 4(2-\epsilon^2)(\epsilon^2-1)^{\frac{1}{2}}/\epsilon^4. \quad (8.5)$$

When one employs the last equation, the first-order contribution is

$$\begin{aligned}\langle J^{(1)}\rangle &= (S_2^{(2)})^2+(S_0^{(2)})^2+(S_{-2}^{(2)})^2 \\ &+\frac{5}{14}\bigg\{(S_2^{(2)})^2-(S_0^{(2)})^2+(S_{-2}^{(2)})^2 \\ &+(\sqrt{6})\left(1-\frac{2}{\epsilon^2}\right)S_0^{(2)}(S_2^{(2)}+S_{-2}^{(2)})\bigg\}P_2 \\ &-\frac{3}{28}\bigg[(S_2^{(2)})^2+6(S_0^{(2)})^2 \\ &+(S_{-2}^{(2)})^2+\frac{10}{3}6^{\frac{1}{2}}\left(1-\frac{2}{\epsilon^2}\right)S_0^{(2)}(S_2^{(2)}+S_{-2}^{(2)}) \\ &+\frac{70}{3}\left(1-\frac{8}{\epsilon^2}+\frac{8}{\epsilon^4}\right)S_2^{(2)}S_{-2}^{(2)}\bigg]P_4, \quad (9)\end{aligned}$$

in agreement with Alder and Winther[6] provided the sign of the term in P_4 is changed as noted by Breit, Ebel, and Benedict.[8] In this comparison the explicit values of Alder and Winther's $B_2=5/14$, $B_4=8/7$ applying to the present $0\rightarrow 2$ case will be found helpful. In the evaluation of $c_\mu^{(2)}$, it is convenient to make use of the symmetry of various parts of the integrand in reducing the region $t>t'$ to the region $t>t'>0$. Introducing the abbreviations

$$P_0(w)=\frac{1}{(1+\epsilon\cosh w)^2}, \quad P_c(w)=P_0(w)\cos 2\varphi,$$
$$P_s(w)=P_0(w)\sin 2\varphi, \quad (9.1)$$

$$Q_0(w)=\int_0^w P_0(w)dw, \quad Q_c(w)=\int_0^w P_c(w)dw,$$
$$Q_s(w)=\int_0^w P_s(w)dw, \quad (9.2)$$

$$R_0=Q_0(\infty), \quad R_c=Q_c(\infty), \quad R_s=Q_s(\infty), \quad (9.3)$$

$$\begin{aligned} T_0&=\int_0^\infty P_0(w)\cos\omega t dw, \\ T_c&=\int_0^\infty P_c(w)\cos\omega t dw, \quad (9.4) \\ T_s&=\int_0^\infty P_s(w)\sin\omega t dw,\end{aligned}$$

and defining

$$\begin{aligned} A&=\int_0^\infty (P_0Q_c+P_cQ_0)\sin\omega t dw, \\ B&=\int_0^\infty (P_0Q_s+P_sQ_0)\cos\omega t dw, \quad (9.5) \\ C&=\int_0^\infty (-P_0Q_0+3P_cQ_c+3P_sQ_s)\sin\omega t dw,\end{aligned}$$

one finds

$$S_0^{(2)}=2T_0, \quad \tfrac{1}{2}(S_2^{(2)}+S_{-2}^{(2)})=-(\sqrt{6})T_c,$$
$$\tfrac{1}{2}(S_{-2}^{(2)}-S_2^{(2)})=(\sqrt{6})T_s, \quad (9.6)$$

while

$$S_0=2\mathcal{T}_0, \quad \tfrac{1}{2}(S_2+S_{-2})=-(\sqrt{6})\mathcal{T}_c,$$
$$\tfrac{1}{2}(S_{-2}-S_2)=(\sqrt{6})\mathcal{T}_s, \quad (9.7)$$
$$S_2=-(\sqrt{6})(\mathcal{T}_c+\mathcal{T}_s), \quad S_{-2}=(\sqrt{6})(\mathcal{T}_s-\mathcal{T}_c) \quad (9.7')$$

with

$$\begin{aligned}\mathcal{T}_0&=T_0+\lambda\{3R_sT_s+i(R_0T_0-3R_cT_c)-C\}, \\ \mathcal{T}_c&=T_c+\lambda\{-i(R_cT_0+R_0T_c)-A\}, \quad (9.8) \\ \mathcal{T}_s&=T_s+\lambda\{-R_sT_0-iR_0T_s+B\}\end{aligned}$$

and

$$\lambda=Z_1e^2\langle r^2\rangle_{ff}/[7\hbar a'^2v]. \quad (9.9)$$

In terms of these quantities, one finds on substitution into Eq. (8.2)

$$\langle J\rangle=4\,\mathrm{Re}\{D+\delta D+(5/14)(N_2+\delta N_2)P_2$$
$$+(8/7)(N_4+\delta N_4)P_4\}, \quad (10)$$

with

$$D+\delta D=\mathrm{Re}\{3|\mathcal{T}_c|^2+3|\mathcal{T}_s|^2+|\mathcal{T}_0|^2\}, \quad (10.1)$$

$$N_2+\delta N_2=\mathrm{Re}\{3|\mathcal{T}_c|^2+3|\mathcal{T}_s|^2-|\mathcal{T}_0|^2$$
$$+6\mathcal{T}_0^*(\mathcal{T}_c\cos 2\delta-i\mathcal{T}_s\sin 2\delta)\}, \quad (10.2)$$

$$N_4+\delta N_4=-\tfrac{3}{32}\,\mathrm{Re}\{3|\mathcal{T}_c|^2+3|\mathcal{T}_s|^2+6|\mathcal{T}_0|^2$$
$$+20\mathcal{T}_0^*(\mathcal{T}_c\cos 2\delta-i\mathcal{T}_s\sin 2\delta)$$
$$-35[|\mathcal{T}_s|^2-|\mathcal{T}_c|^2+2i\,\mathrm{Im}(\mathcal{T}_c\mathcal{T}_s^*)]e^{-4i\delta}\}. \quad (10.3)$$

Here D, N_2, N_4 correspond to $J^{(1)}$ and δD, δN_2, δN_4 to $J^{(2)}$ of Eq. (8.4). Explicitly

$$D=3T_c^2+3T_s^2+T_0^2, \quad (10.4)$$

$$N_2=3T_c^2+3T_s^2-T_0^2+6T_0T_c\cos 2\delta,$$
$$\cos 2\delta=(2/\epsilon^2)-1, \quad (10.5)$$

$$N_4=-\tfrac{9}{32}(T_c^2+T_s^2)-\tfrac{9}{16}T_0^2-(15/8)T_0T_c\cos 2\delta$$
$$+(105/32)(T_s^2-T_c^2)\cos 4\delta. \quad (10.6)$$

[8] Breit, Ebel, and Benedict, Phys. Rev. 100, 429 (1955); see also Breit, Ebel, and Russell, Phys. Rev. 101, 1504 (1956).

The parts corresponding to $J^{(2)}$ are

$$\delta D=\lambda[-6AT_c+6BT_s-2CT_0], \tag{10.7}$$

$$\begin{aligned}\delta N_2=\lambda\{&-6A[T_c+T_0\cos 2\delta]\\&-12R_sT_0T_s+18R_sT_cT_s\cos 2\delta\\&+[-12R_0T_0T_s+18R_cT_cT_s]\sin 2\delta\\&+6BT_s+C[2T_0-6T_c\cos 2\delta]\},\end{aligned} \tag{10.8}$$

$$\begin{aligned}\delta N_4=\lambda\Big\{&A[\tfrac{9}{16}T_c+(15/8)T_0\cos 2\delta+(105/16)T_c\cos 4\delta]\\&+B[-\tfrac{9}{16}T_s+(105/16)T_s\cos 4\delta]\\&+C[(9/8)T_0+(15/8)T_c\cos 2\delta]-(45/16)R_sT_0T_s\\&-(45/8)R_sT_cT_s\cos 2\delta+\frac{15}{4}\frac{(\epsilon^2-1)^{\frac{1}{2}}}{\epsilon^2}\\&\times[(-3R_cT_c+2R_0T_0)T_s-7R_cT_0T_s\cos 2\delta]\\&-(105/16)R_sT_0T_s\cos 4\delta\Big\}.\end{aligned} \tag{10.9}$$

For $\epsilon=1$, one has $\Phi=0$ and according to Eqs. (9.1)$\cdots$(9.5), $P_s=Q_s=R_s=T_s=0$, $P_c=P_0$, $Q_c=Q_0$, $R_c=R_0$, $T_c=T_0$, $B=0$, $A=C$. Consequently, in Eq. (9.8) one has

$$\mathcal{T}_0=T_0-\lambda(A+2iR_0T_0)=\mathcal{T}_c,\ \mathcal{T}_s=0,\ \delta=0,\ (\epsilon=1) \tag{11}$$

and hence

$$\langle J\rangle=4|\mathcal{T}_0|^2[4+(20/7)P_2-(48/7)P_4],\quad(\epsilon=1). \tag{11.1}$$

Since $|\mathcal{T}_0|^2$ enters as a common factor, there is no effect on the γ-angular distribution coefficients usually called $a_2(\xi)$ and $a_4(\xi)$. In this case ($\epsilon=1$), the second-order effects are not detectable by an ordinary γ-angular distribution measurement although there is an effect on the absolute value entering through $|\mathcal{T}_0|^2$. A special case of this relationship has already been pointed out for $\epsilon=1$, $E_{fi}=0$ in connection with Eq. (6.1). On account of the vanishing of second-order effect in the γ-angular distribution coefficients $a_{2\epsilon}(\xi)$ and $a_{4\epsilon}(\xi)$ for $\epsilon=1$, the nearly head-on collisions are also especially unfavorable for the detection of the reorientation effects. This circumstance makes the reorientation effect smaller than might have been expected, because for head-on collisions the excitation effects are largest.

An interesting consequence of the linear variation of $\langle J^{(2)}\rangle$ with ξ is that the reorientation effect is, in first approximation, independent of bombarding energy. This can be seen from Eqs. (4.1′), (4.6), and (9.9) since the product of λ and ξ is

$$\lambda\xi\cong\frac{Z_1e^2}{\hbar va'^2}\frac{\langle r^2\rangle_{ff}}{7}\frac{\omega_{fi}a'}{v}=\frac{M\omega_{fi}\langle r^2\rangle_{ff}}{7Z_2\hbar}$$

and since in the range of ξ for which the linear approximation holds the reorientation effect depends on ξ only through $\lambda\xi$.

Since according to Eq. (10) the spherically symmetric part of $\langle J\rangle$ is represented by $D+\delta D$, the angular distribution coefficients are obtainable as

$$a_s(\xi)=(N_s+\delta N_s)/(D+\delta D)=N_s/D+\delta N_s/D-N_s(\delta D)/D^2,\quad(s=2,4). \tag{12}$$

According to Eq. (9.8), the number of excitations for $\epsilon=1$ is proportional to

$$|\mathcal{T}_0|^2=T_0^2\left[1-(4\lambda/T_0)\int_0^\infty P_0Q_0(\sin\omega t)dw\right],\quad(\epsilon=1). \tag{13}$$

The integral in this formula can be expressed as follows:

$$A/2=\int_0^\infty P_0Q_0(\sin\omega t)dw=\frac{4}{3}\int_1^\infty\frac{u}{(1+u)^4}\left(1-\frac{2(1+3u)}{(1+u)^3}\right)(\sin\omega t)du \tag{13.1}$$

with

$$\omega t=\xi\left[\ln u+\frac{1}{2}\left(u-\frac{1}{u}\right)\right]. \tag{13.2}$$

In terms of it one finds

$$\left(\frac{D+\delta D}{D}\right)_{\epsilon=1}=1-\frac{2A}{T_0}=1-4\lambda\frac{\int_0^\infty P_0Q_0\sin\omega t dw}{\int_0^\infty P_0\cos\omega t dw}. \tag{13.3}$$

By means of Eq. (13.1), this can be expressed as

$$\left(\frac{D+\delta D}{D}\right)_{\epsilon=1}=1-\frac{4\lambda}{3}\left\{\int_1^\infty\frac{u}{(1+u)^4}\left(1-\frac{2(1+3u)}{(1+u)^3}\right)\times(\sin\omega t)du\right\}\Big/\int_1^\infty\frac{u\cos\omega t}{(1+u)^4}du. \tag{13.4}$$

For 5-Mev protons on ${}_{78}$Pt, with an assumed $Q=7\times10^{-24}$ cm^2, $\Delta E=330$ kev, one has $\xi=0.1915$, $\lambda=-0.100$, and the effect on the collision for inelastic scattering may be estimated from (13.3) to be $\pm3.8\%$. Here the quadrupole moment Q is connected with the radial matrix element $\langle r^2\rangle_{ff}$ by the equation $|Q|=(4/7)\langle r^2\rangle_{ff}$.

By means of Eq. (12) combined with Eqs. (10.4), (10.5), (10.6), (10.7), (10.8), and (10.9), one obtains the change in the angular distribution of γ rays for a fixed value of ϵ but averaged over rotations of orbit planes around the direction of the incident beam. The change in the values of $a_2(\xi)$ and $a_4(\xi)$ is obtained by integration of the γ intensity in any direction over ϵ,

taking the number of collisions as $2\pi v p dp = 2\pi v a'^2 \epsilon d\epsilon$. The formulas described so far suffice therefore for the calculation of γ-angular distributions in experiments on the angular distributions without coincidences and also with coincidences in which the inelastically scattered particles are collected in a cone with symmetry axis along the incident beam. Some information is lost in this type of experiment because of the averaging over orbit plane orientations.

This information is retained if the orbit plane is defined by the observation of the inelastically scattered particle and the direction of the γ ray is observed in coincidence. The angular distribution under these conditions is obtainable from Eq. (7.7) which gives J before this quantity is averaged over directions of orbit planes. Expressing the S_μ in terms of $\mathcal{T}_0$, $\mathcal{T}_c$, $\mathcal{T}_s$ by means of Eqs. (9.6) and (9.7), one obtains

$$
\begin{aligned}
(2/15)J = {} & 4\mathbf{c}^2\mathbf{s}^2|\mathcal{T}_0|^2 + 2\mathbf{s}^2(\mathbf{c}^2+1)[|\mathcal{T}_c|^2+|\mathcal{T}_s|^2] \\
& + 8\mathbf{c}^2\mathbf{s}^2 \operatorname{Re}\{\mathcal{T}_0^*[\mathcal{T}_c \cos 2\varphi + i\mathcal{T}_s \sin 2\varphi]\} \\
& + 2\mathbf{s}^4 \operatorname{Re}\{[|\mathcal{T}_s|^2 - |\mathcal{T}_c|^2 + 2i \operatorname{Im}(\mathcal{T}_s^*\mathcal{T}_c)]e^{4i\varphi}\}, \quad (14)
\end{aligned}
$$

and the division into first- and second-order parts is obtainable from Eq. (9.8). Averaging the first line over all directions and taking account of the factor 4 of Eq. (10) in the definition of D, one obtains $D+\delta D$ in agreement with Eq. (10.1).

The remainder of the first line contributes to $J/4$ the quantity

$$
\begin{aligned}
(5/7)[|\mathcal{T}_0|^2 - 3|\mathcal{T}_c|^2 - 3|\mathcal{T}_s|^2]P_2 & \\
- (6/7)[|\mathcal{T}_c|^2 + |\mathcal{T}_s|^2 + 2|\mathcal{T}_0|^2]P_4. & \quad (14.1)
\end{aligned}
$$

This quantity is related to but is not the same as the θ-dependent but δ-independent part of Eq. (10). The reason for the difference is that in the present consideration the directions of orbit planes have not been averaged over. *It should be observed that the argument of P_2 and P_4 in Eq. (14.1) is $\cos\theta$, with θ standing for the angle between the γ ray and the normal to the orbit plane while through all of the formulas for $\langle J \rangle$ the argument of P_2 and P_4 is the cosine of the angle with the incident wave.* The analysis of the angular distribution in the coincidence experiment is in principle capable of giving the coefficients of $\cos 2\varphi$, $\sin 2\varphi$, $\cos 4\varphi$, and $\sin 4\varphi$ in Eq. (14). It should be possible to verify the dependence of these coefficients on θ which appears in this equation. By doing so, the interpretation of the experiment in terms of the mechanism described in this paper would be more certain than in the axially symmetric type of coincidence experiment discussed in relation to $\langle J^{(2)} \rangle$. It is probably important to have such a verification in order to be sure that second-order effects caused by transitions to other levels have been sufficiently corrected for, and it may be helpful to have such a verification as a check on the experimental procedure. It would appear that for these reasons the coincidence experiment defining the orbit should be more informative than the axially symmetric type of experiment or the observation of the γ-intensity distribution without coincidences. A further reason for believing that the orbit-defining experiment is preferable is that some of the angle-independent quantities entering the coefficients of $\sin 2\varphi, \cdots \cos 4\varphi$ can be obtained in more than one way in this arrangement, providing an additional check as will be discussed presently. Before doing so, it should be pointed out, however, that the validity of the semiclassical theory has never been tested to the degree implied in the coincidence experiments. Since in the orbit-defining geometry lack of large diffuseness resulting from diffraction effects is presupposed both in the orbit plane and in a direction perpendicular to it, one may expect this circumstance to be more serious in the orbit-defining arrangement. The expressions derived in the present paper are applicable only approximately on account of the semiclassical nature of the treatment. The errors due to this cause are probably smaller, however, for bombardment by heavy ions such as that by N^{14} than for light particle projectiles such as protons.

In the coefficient of $\cos 2\varphi$, one has available

$$
\begin{aligned}
C_4 &\equiv \operatorname{Re}(\mathcal{T}_0^*\mathcal{T}_c) \\
&= T_0T_c + \lambda[-AT_0 + (3R_sT_s - C)T_c]; \quad (14.2)
\end{aligned}
$$

in that of $\sin 2\varphi$ there is present

$$
C_5 \equiv \operatorname{Im}(\mathcal{T}_0^*\mathcal{T}_s) = \lambda(3R_cT_c - 2R_0T_0)T_s. \quad (14.3)
$$

Similarly there is available in the coefficient of $\sin 4\varphi$

$$
C_6 \equiv \operatorname{Im}(\mathcal{T}_s^*\mathcal{T}_c) = -\lambda R_cT_0T_s \quad (14.4)
$$

and in the coefficient of $\cos 4\varphi$

$$
\begin{aligned}
C_7 &\equiv \operatorname{Re}[|\mathcal{T}_s|^2 - |\mathcal{T}_c|^2] \\
&= T_s^2 - T_c^2 + 2\lambda[AT_c + (B - R_sT_0)T_s]. \quad (14.5)
\end{aligned}
$$

Another way of stating the possibilities of the orbit-defining coincidence experiment is to observe that the coefficients of P_0, P_2, and P_4 of the φ independent parts of J give the combinations

$$
\begin{aligned}
C_1 &\equiv |\mathcal{T}_0|^2 + 3|\mathcal{T}_c|^3 + 3|\mathcal{T}_s|^2, \\
C_2 &\equiv |\mathcal{T}_0|^2 - 3|\mathcal{T}_c|^2 - 3|\mathcal{T}_s|^2, \quad (15) \\
C_3 &\equiv 2|\mathcal{T}_0|^2 + |\mathcal{T}_c|^2 + |\mathcal{T}_s|^2,
\end{aligned}
$$

and that through these quantities there is available $|\mathcal{T}_0|^2$, $|\mathcal{T}_c|^2+|\mathcal{T}_s|^2$ with a check on experimental values through $7C_1+5C_2=6C_3$. The quantities

$$
\Delta|\mathcal{T}_0|^2 = 2\lambda T_0(C - 3R_sT_s), \quad \Delta|\mathcal{T}_s|^2 + \Delta|\mathcal{T}_c|^2,
$$

with

$$
\Delta|\mathcal{T}_s|^2 = 2\lambda T_s(B - R_sT_0), \quad \Delta|\mathcal{T}_c|^2 = -2\lambda AT_c, \quad (15.1)
$$

are thus available, and from the coefficient of $\cos 4\varphi$ there is available $\Delta|\mathcal{T}_s|^2 - \Delta|\mathcal{T}_c|^2$ so that $\Delta|\mathcal{T}_s|^2$ and $\Delta|\mathcal{T}_c|^2$ can both be obtained from experiment.

TABLE I. Values of ϵ and δ used for numerical computation.

ϵ	δ	ϵ	δ	ϵ	δ
1.000	0°	1.250	36.9°	2.000	60°
1.015	10°	1.414	45°	2.500	66.4°
1.064	20°	1.550	49.8°	3.000	70.5°
1.100	24.6°	1.700	54.0°	3.864	75°
1.155	30°				

The angles used in the description of the orbit-defining experiment are as in Fig. 1.

As has been mentioned in the discussion immediately preceding Eq. (14.2), the reorientation effect has to be separated from other second-order effects such as the change in the angular distribution caused by transitions to other levels. It may be pointed out, however, that in the usual 0, 2, 4 sequence of levels a quadrupole transition $0\rightarrow4$ is forbidden, and hence the populations of sublevels of $I=2$ are not affected by the $0\rightarrow4\rightarrow2$ sequence of excitations if higher multipole effects are neglected. There may be effects caused by $0\rightarrow2\rightarrow2$ sequences, but the intermediate level has to belong to another configuration of nucleons such as would be obtained by changing the vibrational quantum number. It may be expected to lie at a higher energy than the $I=4$ state and to be consequently less important in its effect on the γ-ray angular distribution; the transition quadrupole moments to states involving a change of vibrational quantum number are presumably also somewhat smaller than those in the normal $0\rightarrow2\rightarrow4$ sequence.

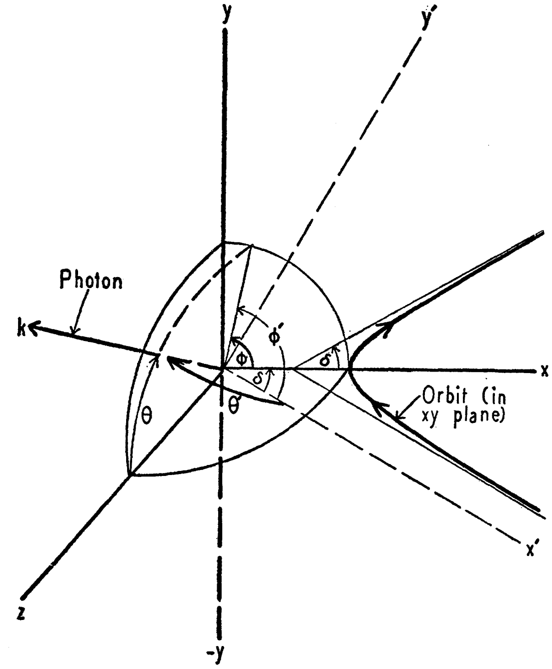

FIG. 1. Diagram of coordinate systems used, showing particle trajectory and direction of emitted photon.

It appears desirable to mention that even though the regular sequence of excitations $0\rightarrow2\rightarrow4$ gives effects in populating the $I=4$ level, the associated depletion of the $I=2$ level does not produce an effect on the

TABLE II. Values of $\mathcal{T}_0$, $\mathcal{T}_c$, and $\mathcal{T}_s$ as a function of ξ and ϵ.

ξ	ϵ	$10\mathcal{T}_0$	$10\mathcal{T}_c$	$10\mathcal{T}_s$
0.0000	1.000	$3.33-(4.06\xi+2.22i)\lambda$	$3.33-(4.06\xi+2.22i)\lambda$	0
	1.015	$3.27-(3.65\xi+2.06i)\lambda$	$3.23-(3.86\xi+2.12i)\lambda$	$1.70\xi+(0-0.56i\xi)\lambda$
	1.064	$3.09-(2.56\xi+1.64i)\lambda$	$2.94-(3.30\xi+1.82i)\lambda$	$3.16\xi+(0-0.98i\xi)\lambda$
	1.155	$2.79-(1.23\xi+1.09i)\lambda$	$2.50-(2.51\xi+1.40i)\lambda$	$4.18\xi+(0-1.17i\xi)\lambda$
	1.414	$2.15+(0.27\xi-0.373i)\lambda$	$1.67-(1.25\xi+0.715i)\lambda$	$4.63\xi+(0-0.99i\xi)\lambda$
	2.000	$1.32+(0.549\xi-0.035i)\lambda$	$0.83-(0.372\xi+0.220i)\lambda$	$3.75\xi+(0-0.491i\xi)\lambda$
	3.864	$0.47+(0.158\xi+0.0068i)\lambda$	$0.22-(0.034\xi+0.0208i)\lambda$	$1.95\xi+(0-0.091i\xi)\lambda$
0.1915	1.000	$2.81-(0.534+1.88i)\lambda$	$2.81-(0.534+1.88i)\lambda$	0
	1.015	$2.76-(0.481+1.75i)\lambda$	$2.74-(0.508+1.79i)\lambda$	$0.21-(0.033+0.069i)\lambda$
	1.064	$2.59-(0.346+1.42i)\lambda$	$2.51-(0.435+1.54i)\lambda$	$0.39-(0.057+0.121i)\lambda$
	1.155	$2.32-(0.179+0.974i)\lambda$	$2.16-(0.333+1.18i)\lambda$	$0.52-(0.068+0.144i)\lambda$
	1.414	$1.74+(0.007-0.375i)\lambda$	$1.50-(0.171+0.610i)\lambda$	$0.56-(0.057+0.120i)\lambda$
	2.000	$1.00+(0.046-0.069i)\lambda$	$0.80-(0.054+0.189i)\lambda$	$0.43-(0.028+0.057i)\lambda$
	3.864	$0.28+(0.0101-0.0033i)\lambda$	$0.24-(0.0063+0.0175i)\lambda$	$0.19-(0.0050+0.0086i)\lambda$
0.4028	1.000	$1.97-(0.673+1.31i)\lambda$	$1.97-(0.673+1.31i)\lambda$	0
	1.015	$1.92-(0.610+1.23i)\lambda$	$1.92-(0.641+1.25i)\lambda$	$0.25-(0.064+0.083i)\lambda$
	1.064	$1.79-(0.445+1.02i)\lambda$	$1.78-(0.552+1.08i)\lambda$	$0.47-(0.111+0.145i)\lambda$
	1.155	$1.58-(0.243+0.732i)\lambda$	$1.56-(0.425+0.832i)\lambda$	$0.61-(0.131+0.171i)\lambda$
	1.414	$1.13-(0.016+0.320i)\lambda$	$1.12-(0.220+0.430i)\lambda$	$0.65-(0.107+0.139i)\lambda$
	2.000	$0.58+(0.037-0.078i)\lambda$	$0.62-(0.070+0.130i)\lambda$	$0.47-(0.048+0.061i)\lambda$
	3.864	$0.11+(0.0061-0.0056i)\lambda$	$0.16-(0.0069+0.0101i)\lambda$	$0.15-(0.0062+0.0069i)\lambda$
0.6882	1.000	$1.07-(0.546+0.716i)\lambda$	$1.07-(0.546+0.716i)\lambda$	0
	1.015	$1.05-(0.497+0.680i)\lambda$	$1.05-(0.521+0.684i)\lambda$	$0.19-(0.069+0.064i)\lambda$
	1.064	$0.97-(0.369+0.580i)\lambda$	$0.99-(0.450+0.592i)\lambda$	$0.36-(0.120+0.111i)\lambda$
	1.155	$0.84-(0.210+0.436i)\lambda$	$0.89-(0.348+0.458i)\lambda$	$0.46-(0.139+0.129i)\lambda$
	1.414	$0.56-(0.030+0.209i)\lambda$	$0.66-(0.181+0.235i)\lambda$	$0.47-(0.108+0.102i)\lambda$
	2.000	$0.25+(0.016-0.055i)\lambda$	$0.35-(0.055+0.067i)\lambda$	$0.30-(0.044+0.040i)\lambda$
	3.864	$0.03+(0.0019-0.0029i)\lambda$	$0.06-(0.0038+0.0037i)\lambda$	$0.06-(0.0035+0.0029i)\lambda$

TABLE III. Values of the coefficients, $C_1 \cdots C_7$, of the angular dependent terms available in a coincidence type experiment which defines the eccentricity and the plane of the orbit of the incident particle, in coincidence with the γ ray. The quantities C_i are defined in Eqs. (15), (14.2), (14.3), (14.4), (14.5).

ξ	ϵ	$100\,C_1$	$100\,C_2$	$100\,C_3$	$100\,C_4$	$100\,C_5$	$100\,C_6$	$100\,C_7$
0.0000	1.000	$44.4-108\xi\lambda$	$-22.2+54\xi\lambda$	$33.3-81\xi$	$11.11-27.1\xi\lambda$	0	0	$-11.11+27.1\xi\lambda$
	1.064	$35.5-74\xi\lambda$	$-16.4+42\xi\lambda$	$27.8-51\xi\lambda$	$9.10-17.7\xi\lambda$	$2.2\xi\lambda$	$-2.9\xi\lambda$	$-8.66+19.4\xi\lambda$
	1.414	$12.9-11.4\xi\lambda$	$-3.7+13.7\xi\lambda$	$12.0-1.88\xi\lambda$	$3.58-2.2\xi\lambda$	$-0.4\xi\lambda$	$-1.7\xi\lambda$	$-2.78+4.2\xi\lambda$
	2.000	$3.82-0.41\xi\lambda$	$-0.35+3.31\xi\lambda$	$4.17+2.28\xi\lambda$	$1.10-0.03\xi\lambda$	$-0.52\xi\lambda$	$-0.41\xi\lambda$	$-0.69+0.62\xi\lambda$
0.1915	1.000	$31.7-12.0\lambda$	$-15.8+6.0\lambda$	$23.8-9.0\lambda$	$7.92-3.00\lambda$	0	0	$-7.92+3.00\lambda$
	1.064	$26.1-8.5\lambda$	$-12.7+4.9\lambda$	$19.9-5.8\lambda$	$6.51-2.00\lambda$	0.24λ	-0.30λ	$-6.15+2.14\lambda$
	1.414	$10.7-1.70\lambda$	$-4.6+1.75\lambda$	$8.6-0.52\lambda$	$2.60-0.29\lambda$	0.002λ	-0.16λ	$-1.92+0.45\lambda$
	2.000	$3.47-0.24\lambda$	$-1.48+0.43\lambda$	$2.81+0.07\lambda$	$0.80-0.02\lambda$	-0.03λ	-0.04λ	$-0.45+0.06\lambda$
0.4028	1.000	$15.5-10.6\lambda$	$-7.7+5.3\lambda$	$11.6-7.9\lambda$	$3.86-2.65\lambda$	0	0	$-3.86+2.65\lambda$
	1.064	$13.4-7.8\lambda$	$-7.0+4.6\lambda$	$9.8-5.3\lambda$	$3.19-1.78\lambda$	0.22λ	-0.25λ	$-2.96+1.86\lambda$
	1.414	$6.3-1.9\lambda$	$-3.8+1.9\lambda$	$4.2-0.7\lambda$	$1.27-0.27\lambda$	0.05λ	-0.12λ	$-0.84+0.36\lambda$
	2.000	$2.14-0.35\lambda$	$-1.46+0.44\lambda$	$1.28-0.05\lambda$	$0.362-0.018\lambda$	0.000λ	-0.023λ	$-0.167+0.042\lambda$
0.6882	1.000	$4.62-4.70\lambda$	$-2.31+2.35\lambda$	$3.46-3.52\lambda$	$1.15-1.17\lambda$	0	0	$-1.15+1.17\lambda$
	1.064	$4.29-3.66\lambda$	$-2.42+2.23\lambda$	$2.98-2.41\lambda$	$0.96-0.80\lambda$	0.10λ	-0.10λ	$-0.86+0.81\lambda$
	1.414	$2.29-1.06\lambda$	$-1.66+0.99\lambda$	$1.29-0.41\lambda$	$0.37-0.12\lambda$	0.042λ	-0.044λ	$-0.21+0.136\lambda$
	2.000	$0.71-0.188\lambda$	$-0.59+0.204\lambda$	$0.34-0.049\lambda$	$0.088-0.008\lambda$	0.007λ	-0.006λ	$-0.032+0.012\lambda$

angular distribution of γ rays to within the order considered here. A similar effect has been noted in connection with the contribution of the term containing $c_0^{(1)}$ in Eq. (2.2) which turned out to be of a higher order.

IV. ESTIMATES AND VALUES OF INTEGRALS

Calculations for the three different types of observations discussed in Sec. III have been performed for the following values of $\xi=\eta_f-\eta_i$:

$\xi=0$ (no excitation);
$\xi=0.1915$ (5-Mev protons on Pt^{194}, 330-kev level);
$\xi=0.4028$ (3.3-Mev protons on Cd^{114}, 555-kev level);
$\xi=0.6882$ (2.4-Mev protons on Cd^{114}, 555-kev level).

The values of orbit eccentricity ϵ and the corresponding values of δ given by Eq. (7.8) used in the computations of the single and double integrals in Eqs. (9.1) to (9.5) are given in Table I. Computation of all integrals was performed in the form given in Eqs. (9.1) to (9.5), using Simpson's rule with w as the independent variable. The results for $\mathcal{T}_0$, $\mathcal{T}_c$, and $\mathcal{T}_s$, defined in Eqs. (9.6) to (9.8), are given in Table II for representative values of ϵ.

The quantities available from the coincidence type experiment discussed in the previous section are given in Eqs. (14.2)$\cdots$(14.5), (15). In order to make their comparison with theory possible, the values were calculated making use of the numbers in Table II. The results are listed in Table III for a few of the values of ϵ in Table II.[9]

The limited-type coincidence experiment also discussed in the previous section, in which the eccentricity, but not the plane of the incident orbit is observed in coincidence with the γ rays, yields the quantities $D+\delta D$, $N_2+\delta N_2$, and $N_4+\delta N_4$ of Eqs. (10.1), $\cdots$(10.9) which are listed in Table IV for typical values of ϵ, so as to facilitate comparison with experiment.[9]

If only the angular correlation of the γ rays with respect to the incident beam direction is measured, one obtains only the quantities

$$\int_1^\infty \epsilon d\epsilon (D+\delta D,\ N_2+\delta N_2,\ N_4+\delta N_4).$$

TABLE IV. Values of the coefficients of the angular dependent terms available in a limited coincidence experiment which defines only the eccentricity of the orbit of the incident particle in coincidence with the γ ray.

ξ	ϵ	$100(D+\delta D)$	$100(N_2+\delta N_2)$	$100(N_4+\delta N_4)$
0.0000	1.000	$44.4-108\xi\lambda$	$88.9-217\xi\lambda$	$-66.7+162\xi\lambda$
	1.064	$35.5-74\xi\lambda$	$58.3-116\xi\lambda$	$-25.8+30\xi\lambda$
	1.414	$12.9-11.4\xi\lambda$	$3.73-16.1\xi\lambda$	$5.74-12.4\xi\lambda$
	2.000	$3.82-0.41\xi\lambda$	$-2.95-5.92\xi\lambda$	$1.00+1.50\xi\lambda$
0.1915	1.000	$31.7-12.0\lambda$	$63.4-24.0\lambda$	$-47.5+18.0\lambda$
	1.064	$26.1-8.5\lambda$	$42.6-13.2\lambda$	$-18.5+3.5\lambda$
	1.414	$10.7-1.70\lambda$	$4.6-1.74\lambda$	$3.9-1.32\lambda$
	2.000	$3.47-0.241\lambda$	$-0.91-0.515\lambda$	$0.70+0.108\lambda$
0.4028	1.000	$15.5-10.6\lambda$	$30.9-21.2\lambda$	$-23.2+15.9\lambda$
	1.064	$13.4-7.8\lambda$	$21.7-12.0\lambda$	$-9.0+3.2\lambda$
	1.414	$6.33-1.94\lambda$	$3.78-1.57\lambda$	$1.58-1.07\lambda$
	2.000	$2.14-0.352\lambda$	$0.37-0.383\lambda$	$0.25+0.055\lambda$
0.6882	1.000	$4.62-4.7\lambda$	$9.23-9.39\lambda$	$-6.93+7.04\lambda$
	1.064	$4.29-3.7\lambda$	$6.84-5.53\lambda$	$-2.71+1.51\lambda$
	1.414	$2.29-1.06\lambda$	$1.66-0.74\lambda$	$0.33-0.41\lambda$
	2.000	$0.714-0.188\lambda$	$0.325-0.145\lambda$	$0.038+0.0112\lambda$

TABLE V. Values of the coefficients of the angular dependent terms available from observation of the γ rays alone.

ξ	$100\int_1^\infty (D+\delta D)\epsilon d\epsilon$	$100\int_1^\infty (N_2+\delta N_2)\epsilon d\epsilon$	$100\int_1^\infty (N_4+\delta N_4)\epsilon d\epsilon$
0.0000	$28.4-21\xi\lambda$	$-1.54-50\xi\lambda$	$-0.022-0.24\xi\lambda$
0.1915	$23.5-3.4\lambda$	$8.1-5.0\lambda$	$0.50-0.078\lambda$
0.4028	$12.7-3.6\lambda$	$7.9-4.2\lambda$	$-0.12-0.105\lambda$
0.6882	$3.97-1.79\lambda$	$3.20-1.84\lambda$	$-0.20-0.027\lambda$

[9] The values listed in Tables III, IV, V for $\xi=0$ are consistent with the vanishing of the reorientation effect for no excitation, as discussed in the previous section. The limit $\xi\to 0$ is implied in the terms listed as being proportional to ξ.

These values were obtained by numerical integration over ϵ and are listed in Table V.[9]

The computational error of the values listed in the tables is believed to be about 1 in the last figure listed. In Table V considerable cancellation has taken place in obtaining the values given in the last column. However, the quantity $N_4+\delta N_4$ is the most difficult of the three to obtain experimentally since it is most sensitive to the angular definition of the photon direction.

The values of $\int_1^\infty D\epsilon d\epsilon$, $\int_1^\infty N_2\epsilon d\epsilon$, and $\int_1^\infty N_4\epsilon d\epsilon$ listed for $\xi=0$ represent the exact values

$$\int_1^\infty D\epsilon d\epsilon=(\pi^2/16)-(1/3)\cong 0.284,$$

$$\int_1^\infty N_2\epsilon d\epsilon=(7\pi^2/16)-(13/3)\cong -0.0154,$$

$$\int_1^\infty N_4\epsilon d\epsilon=(17/9)-(49\pi^2/256)\cong -0.00022,$$

obtainable by direct analytic integration.

The quantity $\int_1^\infty (D+\delta D)\epsilon d\epsilon$ listed in Table V represents the angle-independent part of the correction due to the reorientation effect. As a result the quantity

$$\int_1^\infty (D+\delta D)\epsilon d\epsilon \Big/ \int_1^\infty D\epsilon d\epsilon,$$

represents the factor by which the presence of the reorientation effect increases the total cross section.

It is clear from examination of Tables III, IV, and V that the coincidence-type experiments offer opportunity for obtaining effects due to reorientation which are relatively larger than for the non-coincidence-type experiment. Specifically, one sees from Table III that a judicious choice of coincidence angles which selects the coefficients of $\sin 2\varphi$, $\sin 4\varphi$ ($\mathrm{Im}[\mathcal{T}_0{}^*\mathcal{T}_s]$, and $\mathrm{Im}[\mathcal{T}_s{}^*\mathcal{T}_c]$) may offer the possibility of measuring λ directly.

There seems to be an indication that the reorientation effect is relatively larger for higher excitation, but this will depend on the static quadrupole moment of the level in question. An additional inference which may be drawn from the tables is that the reorientation effect is larger for values of ϵ near 1. This is probably due to the fact that larger values of ϵ correspond to distant collisions for which the amplitude for excitation is small, and the relative importance of the reorientation effect can be expected to be reduced for large ϵ.

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