# Annihilation of Antinucleons

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In this note, the expected $\pi$- and $K$-meson multiplicities resulting from the annihilation of a nucleon pair at rest have been calculated on the basis of two versions of the statistical theory: the Fermi and the Pomeranchuk-Landau models. The Fermi model yields smaller pion multiplicities and larger probabilities for $K$-meson pairs than does the Pomeranchuk-Landau model.

## INTRODUCTION

It is now well established experimentally that multiple production of mesons takes place in nuclear collisions of sufficiently high energy.[1,2] In view of the doubtful validity of the perturbation method for calculating the cross sections for these events (which correspond to higher order processes), various phenomenological (statistical) theories[3] have been put forward. The central idea of these theories is to exploit the large strength of the interaction in order to formulate a model which should be valid asymptotically in the limit of very strong interaction. However, usually the basic "ansatz" can be formulated in different ways. Calculations based on these various theories for nucleon-nucleon and pion-nucleon collisions have been given elsewhere.[4]

However, for one interesting case, the uncertainties of the various multiple-production theories are considerably reduced: this is the annihilation of a nucleon pair. Perturbation calculations have been made of this annihilation process by Helstrom,[5] by Ashkin, Auerbach, and Marshak,[6] and by McConnell.[7] It has been shown that the mesonic annihilation into a pair of spin-zero mesons (scalar or pseudoscalar) has a $1/v$ dependence and the average lifetime of a slow antiproton (or antineutron) is probably longer than the time that it would take the antiproton to be slowed down and be captured in an orbit about the nucleus. The annihilation would then proceed extremely rapidly.

The most important point to note is that the annihilation takes place practically at rest and hence there is *no* Lorentz contraction of the fundamental volume in the center-of-mass system. Hence, there is no anistropy in this system and this reduces the uncertainty in the predictions of the various theories.

As the first model for the calculation, we take the statistical version of Fermi's theory. The importance of imposing rigid momentum conservation has been pointed our by Lepore and Stuart[8] and, in greater detail, by Milburn.[9] In the following section we shall outline the calculation, following Milburn. It is shown subsequently to what extent the calculations are changed by taking account of the interaction of the outgoing mesons (Pomeranchuk-Landau model).

## STATISTICAL THEORY

The transition probability per unit time is given by

$$\omega = 2\pi |\mathcal{H}_{fi}|^2 (dN/dW), \tag{1}$$

where $\mathcal{H}_{fi}$ is the matrix element of the interaction and $dN/dW$ is the density of states. The statistical assumption is that

$$\mathcal{H}_{fi} = AG^{\frac{1}{2}}(\Omega/V)^{\frac{1}{2}}, \tag{2}$$

where $\Omega$ is the interaction volume, $V$ is an arbitrarily chosen normalization volume (which drops out in the final result), $G$ takes account of the symmetry character of the final state, and $A$ is a constant independent of the particular final state (provided it is consistent with all the conservation laws) and serves only to determine the over-all transition rate. This last may hence be omitted in the calculation of branching ratios for the various processes. The vanishing of the matrix element if the conservation laws are not obeyed can be taken care of, by considering in $dN/dW$ only the region of the phase space accessible without violation of these laws.

Pauli's method of replacing conservation laws by discontinuous factors in the phase space factors has been used by Lepore and Stuart to deduce the density of accessible states in multiple meson production. The general formulas do not lend themselves to easy manipulation, and it is preferable to consider certain simplified cases and estimate the errors introduced. We shall see

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[1] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **91**, 758 (1953); **95**, 1026 (1954).

[2] Sands, Bloch, Tuesdale, and Walker quoted in H. Bethe and F. de Hoffman, *Mesons and Fields* (Row, Peterson and Company, Evanston, 1955), Vol. II.

[3] E. Fermi, Progr. Theoret. Phys. Japan **5**, 570 (1950); Phys. Rev. **81**, 683 (1953); **93**, 1434 (1954); *Elementary Particles* (Yale University Press, New Haven, 1951). W. Heisenberg, Nature **164**, 65 (1949); Z. Physik **126**, 569 (1949); **133**, 65 (1952). H. J. Bhabha, Proc. Roy Soc. (London) **A219**, 293 (1953). L. Landau, Izvest. Akad. Nauk U.S.S.R. **17**, 51 (1953). I. Pomeranchuk, Doklady Akad. Nauk U.S.S.R. **78**, 88 (1951).

[4] See references quoted in H. Bethe and F. de Hoffman, *Mesons and Fields* (Row, Peterson and Company, Evanston, 1955) Vol. II, and R. H. Milburn, Revs. Modern Phys. **27**, 1 (1955).

[5] C. W. Helstrom, Phys. Rev. **78**, 88(A) (1950).

[6] Ashkin, Auerbach, and Marshak, Phys. Rev. **79**, 266 (1950).

[7] J. McConnell, Proc. Roy. Irish Soc. **A50**, 189 (1945); **A51**, 173 (1947); **A56**, 45 (1954).

[8] J. V. Lepore and R. N. Stuart, Phys. Rev. **94**, 1724 (1954).

[9] R. H. Milburn, reference 4.

TABLE I. Symmetry factors to take account of the complete symmetry of the final state and of the conservation of isotopic spin. The symmetry factor $G(n_\pi,n_K;I)=\mathfrak{N}(n_\pi,n_K;I)/n_\pi!n_K!$, where $\mathfrak{N}(n_\pi,n_K;I)$ is the number of representations for $n_\pi$ pions with $T=1$ and $n_K$ $K$ mesons with $T=\frac{1}{2}$ coupled to total isotopic spin $I$. The entries are $\mathfrak{N}(n_\pi,n_K;I)$.

$\mathfrak{N}(n,0;I)$

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $I=0$ | 0 | 1 | 1 | 3 | 6 | 15 | 36 | 91 | 232 | 603 |
| $I=1$ | 1 | 1 | 3 | 6 | 15 | 36 | 91 | 232 | 603 | 1585 |

$\mathfrak{N}(n,2;I)$

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $I=0$ | 1 | 1 | 2 | 4 | 9 | 21 |
| $I=1$ | 1 | 2 | 4 | 9 | 21 | 51 |

that the predicted multiplicities are quite insensitive to the simplifications.

We have considered the possible production of heavy mesons ($\tau$, $K$ etc), collectively called $K$ mesons, with a mass $m_K\sim 965$ electron masses. We assume that $K$ mesons possess spin 0. It has been fairly well established by experiment that the production of these unstable particles is subject to an additional conservation rule, the conservation of "strangeness."[10] This has the effect, for the case under discussion, of allowing production of $K$ mesons only in pairs (either a $K^+-\bar{K}^0$ pair, a $K^+-K^-$ or a $K^0-\bar{K}^0$ pair). This conservation law has been imposed in computing the probable multiplicities.

The simplified cases have been chosen so as to sandwich the actual case. The three cases considered are

(a) All mesons are relativistic;

(b) All mesons are nonrelativistic;

(c) The $K$ mesons are nonrelativistic but the pions are relativistic. It is convenient to express all masses in units of the nucleon mass and all energies in units of the rest energy of the nucleon. The appropriate densities of the states are

$$\frac{dN}{dW}=\frac{1}{W}\left(\frac{\pi W^3}{2}\right)^{n-1}\frac{(4n-4)!}{(2n-1)!(2n-2)!(3n-4)!}, \tag{3a}$$

$$\frac{dN}{dW}=\frac{(8\pi^3)^{n-1}(m_K{}^{n_K}m_\pi{}^{n_\pi})^{\frac{3}{2}}}{(n_Km_K+n_\pi m_\pi)^{\frac{3}{2}}}\times\frac{(W-n_Km_K-n_\pi m_\pi)^{3n/2-5/2}}{\Gamma(\frac{3}{2}(n_K+n_\pi-1))}, \tag{3b}$$

$$\frac{dN}{dW}=\frac{(2\pi m_K)^{\frac{3}{2}(n_K-1)}(8\pi)^{n_\pi}}{(n_Km_K)^{\frac{3}{2}}}\times\frac{(W-n_Km_K-n_\pi m_\pi)^{3(n_\pi+\frac{1}{2}n_K)-5/2}}{\Gamma(\frac{3}{2}(n_K+n_\pi-1))}, \tag{3c}$$

where $n=n_\pi+n_K$, and $W=2.00$, $m_K=0.520$, $m_\pi=0.148$. With this choice of units, the relative probabilities are

$$P(n_\pi,n_K)=G(0.945\Omega/\Omega_0)^{n-1}\frac{d}{dW}N(n_\pi,n_K), \tag{4}$$

where[11] $\Omega_0=\frac{4}{3}\pi(\hbar/m_\pi c)^3$, $\hbar/m_\pi c=1.4\times10^{-13}$ cm is the fundamental "volume" in which the annihilation takes place. It is to be noted that in the present case $\Omega$ *does not* involve any Lorentz contraction factor. We shall disregard the factor $(0.945\Omega/\Omega_0)^{n-1}$ for the calculations in the present section.

The symmetry factor $G$ takes care of automatically restricting the final state to be symmetric[12] for the interchange of identical bosons and is constructed so as to conserve the isotopic spin. It is given by:

$$G=\frac{\mathfrak{N}(n_\pi,n_K;I)}{n_\pi![(\frac{1}{2}n_K)!]^2}, \tag{5}$$

and is tabulated in Table I.

Using these expressions, the average and root-mean-square multiplicities of pions and of the total number of mesons have been computed. Since the $K$ mesons are produced in pairs, if at all, it is more significant to give the percentage probability $P_K$ for a $K$-meson pair to be present. The results are summarized in Table II.

It is worthy of note that the average multiplicities for the three cases considered do not differ much and give a total number of mesons of 2–3. In approximately half of the cases, a $K$-meson pair is also formed. The approximate agreement between the average and root-mean-square values shows that the fluctuations in these numbers is rather small.

## EFFECT OF A FINAL-STATE INTERACTION

In the above we have tacitly assumed that the branching ratios were given essentially by the phase space factors. Heisenberg[3] has considered the production of mesons in nuclear collisions as a shock-wave problem, described by a nonlinear equation. The theory, while not directly applicable to the annihilation of a nucleon pair at rest, contains a point worthy of notice and this is the "propagation" of the nonlinear "shock wave." This would have the effect of dissipating the energy originally concentrated in the high wave numbers in the shock front to lower wave numbers. It would, hence, in the present problem, suggest a preference for states with higher multiplicities.

Pomeranchuk[3] has pointed out that a strong final-

[10] M. Gell-Mann, Proceedings of the International Conference on Elementary Particles, Pisa, 1955, Nuovo cimento (to be published); R. E. Marshak, University of Rochester report NYO-7138 (unpublished).

[11] This choice of $\Omega_0$ is supported by the large ("geometrical") cross sections for annihilation recently observed at Berkeley [O. Chamberlain, Bull. Am. Phys. Soc. Ser. II, 1, 9 (1956)].

[12] We have incorporated isotopic spin invariance in the present calculation without explicitly treating the effects of invariance under charge conjugation. Charge conjugation invariance yields selection rules for nucleon pair annihilation which tend to increase the pion multiplicity [see T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956) and C. J. Goebel, Bull. Am. Phys. Soc. Ser. II, **1**, 52 (1956)].

state interaction of the mesons can alter the predictions of the statistical theory. The effect is somewhat similar to Heisenberg's nonlinear shock-wave propagation and may be outlined as follows: the choice of $\Omega$ in Eq. (2) to be independent of the multiplicity is incorrect for a strongly interacting system of particles and should be replaced by

$$\Omega' = n\Omega_1, \tag{6}$$

where $n$ is the number of interacting particles and $\Omega_1$ is the effective volume over which the interaction of a single particle (assumed to be a short-range force) is effective. The extra factor $n^n$ in the phase space factors which results can easily be seen to induce a preference for the higher-multiplicity states.

Pomeranchuk (and following him Landau[3]) assumes that the meson-meson interaction has a range of the order of a Compton wavelength of the pion. There are grounds[13,14] to believe that there does exist a strong meson-meson interaction of this range. To make a quantitative estimate of the effect of such a strong interaction, it is necessary to calculate the expected multiplicities for definite values of the ranges of interaction. We have made calculations for three different cases:

(1) Only pions interact strongly, with a range $\sim\hbar/\mu c$;

(2) Only pions interact strongly, with a range $\sim 0.75(\hbar/\mu c)$;

(3) Both pions and $K$ mesons interact strongly, with a range $\sim 0.75(\hbar/\mu c)$. The resulting multiplicities are given in Table III on the basis of approximation ($c$).

It is appropriate to mention a few points concerning the various approximations adopted. Among the three cases considered, Eq. (3c) is expected to be the most appropriate, since the numbers calculated show that the pions would, in general, be relativistic while the $K$ mesons would always be nonrelativistic.

An examination of the numbers in Table II and III reveals the following: On the Fermi model (i.e., the pions do not interact in the final state), the pion multiplicities are low—of the order of 2 pions; $K$-meson production occurs only in about 25% of the cases. If one assumes that the pions interact strongly in the final state, the pion multiplicities are somewhat enhanced. If the $K$ mesons are not strongly interacting in the final state (cases 1 and 2), the $K$-meson production is further reduced.[15] However, since $P_K$ is low even in the Fermi model, the test between the two theories is afforded by the somewhat larger number of pions predicted by the Pomeranchuk-Landau Model for the annihilation process.

In conclusion, it is to be pointed out that throughout this discussion, we have omitted any consideration of angular momentum conservation. So far it has not been found possible to incorporate this conservation law without greatly complicating the statistical model. The effect of considering the angular momenta of the outgoing particles would be to require that the symmetry factor $G(n_\pi,n_K;I)$ be further multiplied by the number of distinct angular momentum eigenfunctions corresponding to a specified total angular momentum. Since the requirement of total symmetry of the final state for interchange of identical particles has already been taken care of, the effect of considering the angular momenta of the outgoing particles is, qualitatively, to induce a preference for higher multiplicities. To make a quantitative estimate however, one would have to incorporate additional details into the statistical model.

TABLE II. Average multiplicities of mesons resulting from the annihilation of an antiproton[a] with a proton ($p\bar{p}$) and with a neutron ($n\bar{p}$), respectively. The entries ($a$), ($b$), ($c$) correspond to the three different approximations adopted in computing these numbers [see Eq. (3a), (b), (c)]: ($a$) all mesons are relativistic; ($b$) all mesons are nonrelativistic; and ($c$) $K$ mesons are nonrelativistic but pions are relativistic.

| | ($a$) | | ($b$) | | ($c$) | |
|---|---|---|---|---|---|---|
| Quantity | $p\bar{p}$ | $n\bar{p}$ | $p\bar{p}$ | $n\bar{p}$ | $p\bar{p}$ | $n\bar{p}$ |
| $\langle n_\pi \rangle$ | 0.92 | 0.99 | 0.99 | 1.05 | 1.85 | 2.01 |
| $\langle n_\pi^2 \rangle^{\frac{1}{2}}$ | 1.43 | 1.52 | 1.60 | 1.69 | 2.43 | 2.74 |
| $\langle n_\pi + n_K \rangle$ | 2.12 | 2.27 | 2.49 | 2.63 | 2.69 | 2.86 |
| $\langle (n_\pi + n_K)^2 \rangle^{\frac{1}{2}}$ | 3.24 | 3.31 | 3.59 | 3.65 | 3.14 | 3.20 |
| $P_K$ | 79 | 78 | 86 | 86 | 37 | 35 |

[a] The results for antineutron annihilation, namely $n\bar{n}$ and $p\bar{n}$, are identical to the results for $p\bar{p}$ and $n\bar{p}$, respectively.

TABLE III. Average multiplicities of mesons resulting from annihilation of an antiproton, with a proton ($p\bar{p}$) and with a neutron $n\bar{p}$, respectively, when a final state interaction is considered. The entries 1, 2, 3 correspond to three different assumptions concerning the interaction: (1) only pions interact strongly with a range $\sim\hbar/\mu c$; (2) only pions interact strongly with a range $\sim 0.75\hbar/\mu c$; (3) both pions and $K$ mesons interact strongly with a range $\sim 0.75\hbar/\mu c$. In all three cases the pions are considered to be relativistic and $K$ mesons nonrelativistic.

| | Case 1 | | Case 2 | | Case 3 | |
|---|---|---|---|---|---|---|
| Quantity | $p\bar{p}$ | $n\bar{p}$ | $p\bar{p}$ | $n\bar{p}$ | $p\bar{p}$ | $n\bar{p}$ |
| $\langle n_\pi \rangle$ | 3.44 | 3.52 | 2.70 | 2.81 | 2.33 | 2.45 |
| $\langle n_\pi^2 \rangle^{\frac{1}{2}}$ | 3.58 | 3.60 | 2.98 | 3.06 | 2.74 | 2.83 |
| $\langle n_\pi + n_K \rangle$ | 3.64 | 3.67 | 3.19 | 3.26 | 3.16 | 3.23 |
| $\langle (n_\pi + n_K)^2 \rangle^{\frac{1}{2}}$ | 3.69 | 3.71 | 3.30 | 3.35 | 3.36 | 3.39 |
| $P_K$ | 6 | 4 | 17 | 15 | 31 | 28 |

## ACKNOWLEDGMENTS

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[13] R. E. Marshak, Phys. Rev. **88**, 1208 (1952).

[14] Marc Ross, Phys. Rev. **95**, 1687 (1954).

[15] The fraction of $K$ mesons in high-energy showers ($\sim 10^{12}$ ev) seems to be low as indicated by the roughly constant value $\sim 0.5$ for the ratio of $\pi^0$ mesons to charged shower particles up to the highest energies. A strong final state interaction of the type we consider would constitute an explanation. Moreover, it is quite possible that the strong interaction may obtain only in certain definite states of the interacting particles. The enhancement of the corresponding matrix elements and partial cross sections could then distort the predictions of the statistical theory.