Highly Excited Rotational States of Some Nuclei. Intensity of α Decay to Rotational Excited Levels

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The α spectra of Pu^{240} and U^{233} have been investigated. A more precise value $(8.5\pm1.5)\times10^{-4}$, for the intensity of the Pu^{240} α line corresponding to the transition to the 4+ level of the daughter nucleus has been obtained. New α lines corresponding to the levels with energies 164 kev (0.07±0.04%) and 237 kev (0.04 ±0.02%) have been found in the α spectrum of U^{233}. Thus, successive transitions to rotational levels starting from the ground level (spin 5/2) and up to the level with spin 15/2 have been observed.

Earlier results on the α decay of Am^{241} (on the population of the levels between 5/2 and 13/2) are used in the discussion. The α-line intensities are compared with available theoretical formulas.

INTRODUCTION

It is now well known that the lower levels of heavy nuclei far removed from closed nucleonic shells are of a rotational nature. For even-even nuclei the excitation energy of such levels is given by the formula

$$E_{\text{rot}}=(\hbar^2/2\mathcal{J})I(I+1); \tag{1}$$

for odd nuclei the excitation energy is given by

$$E_{\text{rot}}=(\hbar^2/2\mathcal{J})[I(I+1)-I_0(I_0+1)]. \tag{2}$$

In these formulas I is the nuclear spin for the considered excited state, I_0 is the nuclear spin for the ground state of the given rotational band, and $\mathcal{J}$ is the effective nuclear moment of inertia.

Rotational levels are very prominent in α disintegration. However, for a long time no more than three levels belonging to a single rotational band were observed. In 1955, five rotational levels were found for Am^{241}.[1]

It seems of interest to determine the possible number of levels in a given rotational band.

Attempts have been made recently to calculate the intensity of α decay to rotational levels. A formula for the intensity of α lines of odd elements has been proposed by Bohr, Fröman, and Mottelson[2]:

$$P=P_0(Z,E)\sum_{l=I-I_0}^{I+I_0} C_l|\langle I_0lI_00|I_0lII_0\rangle|^2, \tag{3a}$$

$$\log_{10}P_0(Z,E)=C(Z)-D(Z)/\sqrt{E}, \tag{3b}$$

where P is the probability of α decay to a given level, and I and I_0 have the same meaning as in (2); l is the angular momentum of the α particle and the $\langle I_0lI_00|I_0lII_0\rangle$ are the Clebsch-Gordan coefficients. Formula (3b) accounts for the dependence of the α-decay probability on the energy. C and D are parameters which depend weakly on Z; they are tabulated in reference 2. The coefficients C_l determine the probability of the emission of α particles with angular momentum l. They may be obtained from the α-decay intensities of the neighboring even-even nuclei.

Landau[3] has shown that for highly prolate nuclei which emit practically all α particles from a narrow region surrounding the major semiaxis, a simple formula for the intensity of α decay to rotational levels can be deduced. His formula for even-even nuclei has the following form:

$$P=C(2I+1)e^{-\alpha I(I+1)}, \tag{4}$$

where C and α do not depend on the energy levels. According to Landau this formula should also be valid for odd nuclei. Here the empirical constants C and α can be theoretically related to the dimensions and the eccentricity of the nucleus. The Clebsch-Gordan coefficients do not enter the formula.

Ter-Martirosian[4] has proposed a different formula for the α-decay intensity of odd nuclei:

$$P=C\sum_{l=I-I_0}^{I+I_0}|\langle I_0lI_00|I_0lII_0\rangle|^2 \times(2l+1)e^{-\alpha_1I(I+1)-\alpha_2l(l+1)}. \tag{5}$$

In this formula C is a normalization factor, α_1 and α_2 are empirical constants, and the other quantities have the same meaning as above.*

In contrast to (3), the dependence of the α-decay probability on energy is described in (5) by aid of the parameter α_1, and is not assumed to be known beforehand. Indeed, there are no reasons to assume that the probability of α decay to levels of the rotational band depends on the energy in the same fashion as does the decay probability to the ground states of various nuclei.

Formulas (4) and (5) do not pretend to describe

[1] Goldin, Novikova, and Tretyakov, Conference of the Academy of Sciences, U.S.S.R. on the Peaceful Applications of Atomic Energy, Session of the Division of Physico-Mathematical Sciences, 1955, p. 226 (translated by Consultants Bureau, New York, 1955; p. 167).

[2] Bohr, Fröman, and Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 29, No. 10 (1955).

[3] L. D. Landau, Report at Sixth Annual Conference on Nuclear Spectroscopy in Moscow, January, 1956 (Izvest. Akad. Nauk S.S.S.R.; Ser. Fiz., to be published).

[4] K. A. Ter-Martirosian (private communication).

* In more detail these formulas are discussed by Goldin, Peker, and Novikova (to be published in Uspekhy Fiz. Nauk USSR, July, 1956).

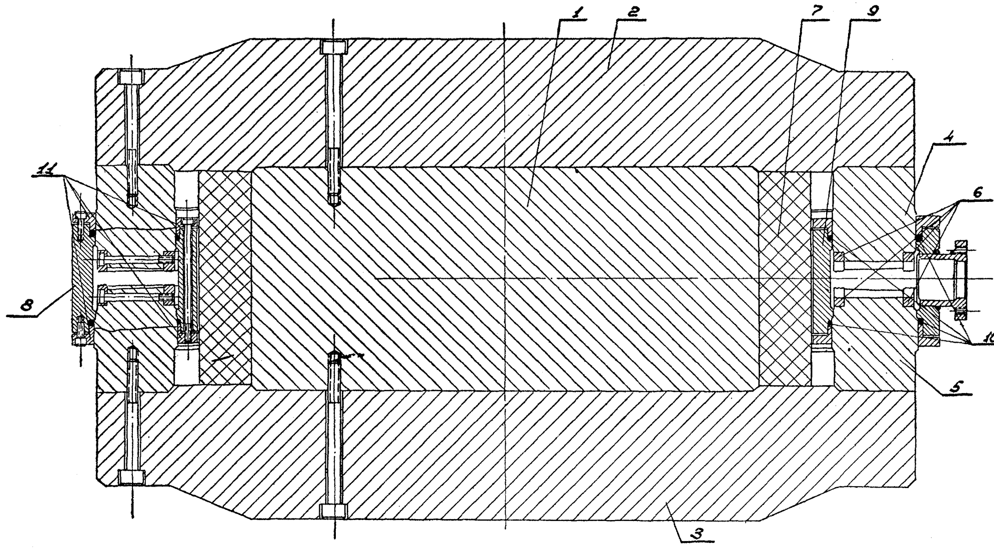

FIG. 1. Diagram of the α spectrometer. 1—core; 2, 3—covers; 4, 5—poles; 6—shimming rings; 7—coil; 8, 9—cylindrical walls of the vacuum chamber; 10—O-ring gaskets; 11—press rings.

accurately the emission of α particles with large angular momenta, as they were obtained on the basis of an approximate consideration in which only the first terms of the expansion in I and l were retained.

It should be borne in mind that even a semiquantitative agreement between theory and experiment is of interest. It is all the more of interest since the values of C and α in (4) or C, α_1, and α_2 in (5) can be theoretically deduced by considering the nuclear shape and its dimensions. Thus, the experimental values of these parameters may be used to study the geometry of nuclei.

From the foregoing it should be clear that interesting data should be obtained by measuring a possibly larger number of levels belonging to a given rotational band.

EXPERIMENTAL

The fine structure of the α spectra of Am^{241}, Pu^{240}, and U^{233} was studied.

The measurements were carried out by means of a precision magnetic α spectrometer with a resolution of 7.5 kev and $2\times10^{-4}/4\pi$ transmission. The spectrometer has been described in reference 1 and in greater detail in a forthcoming paper.[5] A diagram of the instrument is shown in Fig. 1.

The Am^{241} α spectrum was described in an earlier publication of ours[1] and was not remeasured after that. The Am^{241} α-decay scheme is represented in Fig. 2.

The intensity of the Pu^{240} α line corresponding to the transition to the 4+ level was remeasured. This line was first detected by Asaro and Perlman.[6] These authors, however, gave only an approximate value of the intensity—0.1%. The Pu^{240} α spectrum in the 5-Mev region is shown in Fig. 3. A group of α particles corresponding to the transition to the 4+ level is clearly discernible. The excitation energy is ~158 kev, the intensity (0.085±0.015)%. Because of the low probability of transition to the 4+ level, we were compelled to work with thick sources (50 μg/cm², 17% Pu^{240}); the exposure time was about 100 hours. Alpha decay of Pu^{240} to the first excited level has been described in the literature.[7,1] The Pu^{240} α-decay scheme and the level scheme of the daughter nucleus U^{236} are presented in Fig. 4. Attempts to detect the 6+ level have yet not been successful.

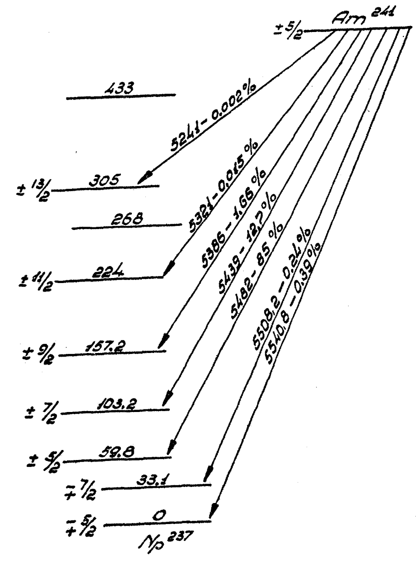

FIG. 2. Alpha-decay scheme of Am^{241} and the level scheme of Np^{237}. (Energy in kev.)

[5] L. L. Goldin and E. F. Tretyakov, Report at Sixth Annual Conference on Nuclear Spectroscopy in Moscow, January, 1956 (Izvest. Akad. Nauk S.S.S.R.; Ser. Fiz., to be published).

[6] I. Perlman and F. Asaro, *Annual Reviews of Nuclear Science* (Annual Reviews, Inc., Stanford, 1954), Vol. 4, p. 157.

[7] F. Asaro and I. Perlman, Phys. Rev. 88, 4, 828 (1953).

The main lines of the U^{233} α spectrum are well known. In addition to the strong α lines at 4815.7 kev (83.5%), 4773.2 kev (14.9%), and 4717.4 kev (1.6%), a weak line at 4489 kev (0.03%) was observed.[1] We have carefully investigated the region between the 4717.4- and 4489-kev lines. The source used in these experiments was obtained by electrolytical deposition of uranium on platinum from an alcohol solution of uranyl nitrate. As in the experiments with Pu^{240}, we were forced to work with a thick source and to widen the entrance slit of the instrument considerably (up to 4 mm). Several exposures were made, each of which lasted almost 2 weeks.

One of the spectra obtained by us is shown in Fig. 5. Alpha lines of 161-kev energy ($0.07\pm0.04\%$) and 234-kev energy ($0.04\pm0.02\%$) can be seen in the figure.

Combining the present results with those previously obtained,[1] we arrive at the decay scheme shown in Fig. 6. The spin of the Th^{229} ground state is 5/2. The energy levels are satisfactorily described by formula (2). Theoretically, the ratios of the energies of the levels are equal to 1:2.28:3.86:5.72:7.85. Experimentally the ratios 1:2.3:3.8:5.5:7.7 were found. Thus, there can be no doubt concerning the rotational nature of these levels. Identical parity and monotonically increasing spins 5/2, 7/2, 9/2, 11/2, 13/2, and 15/2 should be assigned to the excited states. Alpha particles corresponding to the transitions from the ground state of U^{233} (spin 5/2) to the level 15/2 carry off an angular momentum $l\geqslant 6$. Such extended rotational level systems have not been described in the literature heretofore.

DISCUSSION OF THE RESULTS

It is of interest to compare the α-line intensities for even-even nuclei with formula (4). Some of the nuclei described in the literature can be used for this purpose, but the uncertainty in the experimental intensity values do not permit one to do this quite reliably. More precise measurements of the Pu^{240} α-line intensities now make such a comparison possible. Choosing the value of α in (4) to be $\alpha=0.46$, we find that the α-line intensities are in the ratios $1:5e^{-6\times0.46}:9e^{-20\times0.46}=1:0.32:0.9\times10^{-3}$, whereas the experiment yields $1:0.32:1\times10^{-3}$. Considering the approximate nature of the theory, the agreement with the experiment is rather unexpected.

It should be borne in mind that even a departure of the measured intensities of the 4+ lines by factors of 2–3 from their theoretical values should not be considered as a serious failure of the theory.

Consider now the alpha disintegration of Am^{241} (Fig. 2). The wide rotational band begins from the 59.8-kev level. The successive α-line intensity ratio equals to

$$1:0.15:1.9\times10^{-2}:1.8\times10^{-4}:2.5\times10^{-6}.$$

Alpha decay to the ground level of the rotational band can be satisfactorily described by formula (3b); i.e., according to the Bohr-Fröman-Mottelson terminology,[2] the transitions are favored. The spin and parity of the states do not change in such transitions.[2] As the spin of the ground state of the rotational band is equal to 5/2,[8] three groups of α particles always participate in the transitions, carrying away three different even angular momenta: $l=0$, 2, or 4 for the first level (59.8 kev), $l=2$, 4, or 6 for the second and third levels, and $l=4$, 6, or 8 for the fourth and fifth levels. We shall now compare the experimental data with formulas (3).

The coefficients C_l can be taken from α-decay data for even-even nuclei. A value of 0.7 has been proposed for C_2, and a value 0.01 for C_4 in the case of Am^{241} was suggested.[2] The magnitudes of C_6 and C_8 are not known.

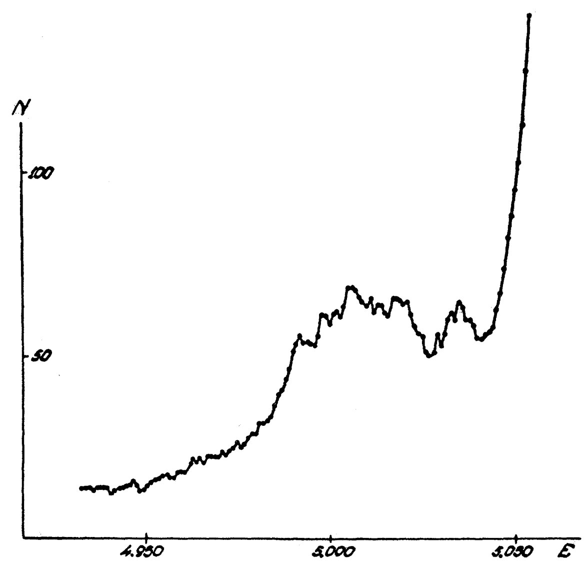

FIG. 3. Pu^{240} alpha spectrum in the 5-Mev region. (The points represent average values of N.)

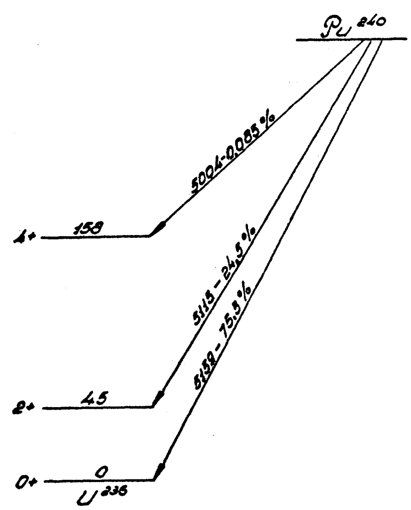

FIG. 4. Alpha-decay scheme of Pu^{240} and the level scheme of U^{236}. (Energy in kev.)

[8] F. Asaro and I. Perlman, Phys. Rev. 93, 1423 (1954).

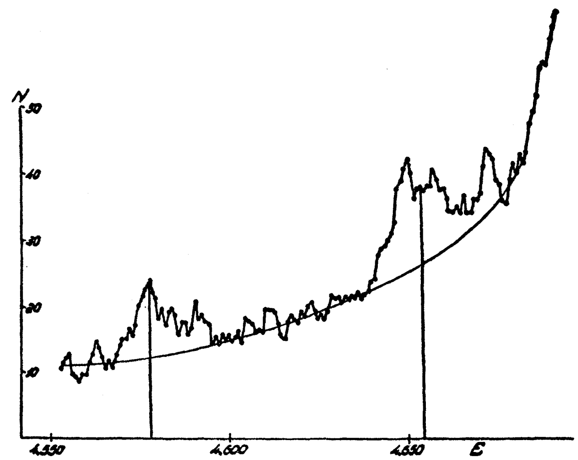

FIG. 5. U^{233} alpha spectrum in the 4.6-Mev region. (The points represent average values of N.)

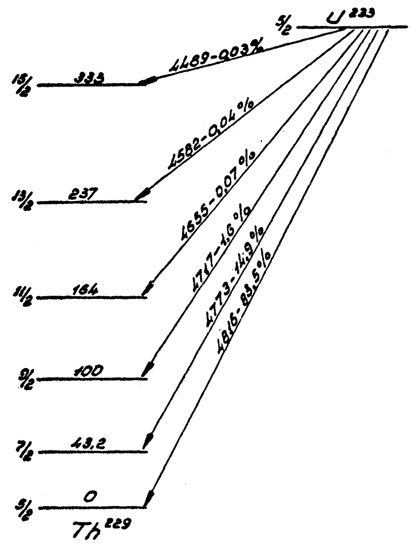

FIG. 6. Alpha-decay scheme of U^{233} and the level scheme of Th^{229}. (Energy in kev.)

We may assume them to be zero as the contribution from the higher momenta is not large.

On these assumptions, formula (3) yields the following intensities for a sequence of α lines:

$$1:0.11:1.6\times10^{-2}:1.5\times10^{-4}:1.3\times10^{-5}.$$

These are in fair agreement with experiment. Formula (5) gives even better agreement with experiment. Putting $\alpha_1=0.050$ and $\alpha_2=0.364$, we find for the α-line intensities the ratio

$$1:0.12:2.1\times10^{-2}:2.1\times10^{-4}:2.9\times10^{-5}.$$

Formula (4) yields:

$$1:0.15:1.2\times10^{-2}:4.4\times10^{-4}:0.9\times10^{-5}.$$

In the latter case the agreement with experiment is less satisfactory; this shortcoming, however, is compensated by the simplicity of the formula, which contains only a single parameter which must be determined experimentally.

Summing up, one may assert that the α-line intensities of Am^{241} exhibit regularities which can be described by several different formulas. The experiment does not permit one to choose between these formulas. Such agreement, however, is not always observed.

Consider the two lower levels of Np^{237} (Fig. 2). The first excited level at 33.1-kev is usually considered as a rotational satellite of the ground state.[9] The parity of both of these states is opposite to that of the ground state of Am^{241}. Therefore, only α particles with uneven momenta participate in the disintegration. Alpha particles carrying away an angular momentum $l=1$ should be predominant in the transitions to the ground state as well as in transitions to the 33.1-kev level. Retaining only one term corresponding to $l=1$ in the formulas (3) and (5), one gets for the decay probability to these levels the relation:

$$\frac{P(33.1)}{P(0)}<\left|\left\langle\frac{5}{2}\,1\,\frac{5}{2}\,0\right|\frac{5}{2}\,1\,\frac{7}{2}\,\frac{5}{2}\right\rangle\Big|^2\Big/\left|\left\langle\frac{5}{2}\,1\,\frac{5}{2}\,0\right|\frac{5}{2}\,1\,\frac{5}{2}\,\frac{5}{2}\right\rangle\Big|^2=0.3.$$

The experimental value is 0.61. There is thus a considerable departure from formulas (3) and (5). Formula (4), which does not involve the Clebsch-Gordan coefficients, is of course not in contradiction with the observed intensity values. Definite conclusions, however, can be drawn only if it should be reliably shown that the 33-kev level is in fact a rotational satellite of the ground level.

Consider now U^{233}. The intensities of successive α lines are given by the ratios

$$1:0.18:1.9\times10^{-2}:8\times10^{-4}:5\times10^{-4}:3.5\times10^{-4}.$$

The rapid and regular variation of the intensity observed for the first four lines is then replaced by only a slight variation. The intensities of the first lines can easily be described by any of the proposed theoretical formulas. A comparison of the α-line intensities of U^{233} and Am^{241} is sufficient to show this. However, not one of these formulas is able to describe the "plateau" which is characteristic for the last lines. Possibly, the departures from the theoretical formulas inevitable for high momenta are particularly important in U^{233}.

[9] Jaffe, Passel, Browne, and Perlman, Phys. Rev. **97**, 142 (1955).

CONCLUSION

The experimental data available at present are sufficient to permit one to attempt a theoretical treatment of the intensities of α lines belonging to a single rotational band. A problem of equal importance is to obtain the theoretical relation between the empirical constants in the formulas and the geometrical parameters of the nucleus.

A careful study of the α spectra of Pu^{240}, Am^{241}, and U^{233} has made it possible for the first time to carry out a comparison of the existing formulas with experiment.

Formula (4), proposed by L. D. Landau, satisfactorily describes the intensity of α decay to various levels, at least for even-even nuclei. For odd nuclei the picture is not clear.

ACKNOWLEDGMENTS

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Nuclear Saturation and Two-Body Forces: Self-Consistent Solutions and the Effects of the Exclusion Principle*

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The equilibrium properties of uniform nuclear matter have been studied in previous papers by using a self-consistency method to determine two-body reaction matrices and the average effective one-particle potential which they generate. In this paper the self-consistency method is simply illustrated by using some explicit examples. An investigation is also made of the previously neglected effects of the exclusion principle on transitions to intermediate states. A variational expression is used for the reaction matrix, utilizing as a trial function the wave function which is exact if the exclusion effect is neglected. Two-body potentials are used which simulate the actual forces, i.e., square wells with a repulsive core of 0.35 $\hbar/\mu c$, range of 1.15 $\hbar/\mu c$, and depth 98.3 Mev, acting on s-states only. It is found that for effective mass values of 0.5 M and 0.6 M, the reaction matrix is appreciably altered, particularly for low values of relative momentum. The requirements of self-consistency, however, almost entirely compensate for the change in the reaction matrices so that at normal density in the effective mass approximation the final result for the average binding energy is *precisely* the same as that obtained when exclusion effects are neglected. The reasons for this simple result are discussed.

Further approximations are discussed which are suitable for more exact computations and which allow inclusion of the exclusion effects, the departures from the effective mass approximation, and the effects of "propagation off the energy shell." Approximation methods for a finite nucleus are discussed and a simplified Hartree-Fock method using "pseudo-potentials" (in the sense of Fermi) is described.

I. INTRODUCTION

IN previous papers an approximation method for treating quantum mechanical systems of many particles has been developed and applied to the determination of the equilibrium properties of nuclear matter, such as the binding energy, equilibrium density, surface energy, etc.[1–5] We shall here develop in some detail the procedure used to obtain approximate solutions to a nuclear system of infinite extent by the method of self-consistency. In addition, it will be shown, by an explicit calculation of the average nuclear binding energy using a specific two-body nuclear interaction potential, how an approximation previously made in neglecting the effects of the exclusion principle in the intermediate states may now be removed. It is found that the average properties of the nuclear matter are altered only very slightly by the exclusion effects, although there exist appreciable alterations in the details of the reaction matrix and hence in some details of the velocity-dependent potential.

We also show how the effects of propagation off the energy shell in excited states can be included approximately in a simplified form of the theory suitable for computation. The effects of the finite size of the nuclear matter have already been studied in previous papers.[2,3] Some comments are made on an alternative method for investigating these effects.

II. FORMULATION OF THE SELF-CONSISTENCY PROBLEM

In earlier work[3–5] it was shown that an excellent approximation to the binding energy of extended nuclear matter can be obtained by solving the following

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* Work performed under the auspices of the U. S. Atomic Energy Commission.

† On leave of absence from the Naval Research Laboratory, Washington, D. C.

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2 K. A. Brueckner, Phys. Rev. **96**, 508 (1954).
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