Suppression of Pair Effects in Double π-Meson Photoproduction at Low Energies

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The matrix element for the photoproduction of a (π^+,π^-) pair by γ rays on protons is examined in the low-energy region. Using the Deser-Thirring-Goldberger definition of the nucleon-meson interaction in a symmetric pseudoscalar theory with pseudoscalar coupling, it is shown that the leading term in an expansion in $\boldsymbol{k}/M$ ($\boldsymbol{k}$ being the kinetic energy of the light particles and M the mass of the nucleon), is exactly the pair-term contribution in the perturbation result ($\sim eg^2$), but occurring with the much smaller g_s^2 coupling constant. With the actual value of the pion mass, terms proportional, for instance, to $(\mu/M)g_P^2$ then become predominant.

1. INTRODUCTION

RECENT experimental results[1] have assigned an upper limit of $(1.4\pm0.3)\%$ to the ratio of the yields of 47-Mev negative and positive pions produced by 500-Mev γ rays incident on hydrogen. If this value be compared with the lowest order perturbation prediction in pseudoscalar meson theory,[2] it is evident that the latter gives too large an estimate, owing to its general tendency to overestimate effects in the S state. Calculations using the Tamm-Dancoff method have already been carried out by Nelkin,[3] showing that S-state effects are considerably depressed. It seemed, however, that an examination of the pair effects by using the standard pseudoscalar relativistic theory with pseudoscalar coupling (*ps-ps*) in the low-energy region could be of some utility. It is the aim of this paper to look into this problem. The technique developed by Deser, Thirring, and Goldberger[4] provides the mathematical tools to achieve this program; accordingly, we carry out the calculations assuming the mass of the outgoing pions μ to be zero. In this limit, the leading term for the double production is the process in which the γ ray interacts with the outgoing meson current.[5]

This particular matrix element is computed in Sec. 2 by straightforward expansion in $\boldsymbol{k}_i/M$, the $\boldsymbol{k}_i$ being the kinetic energies of the light particles (π mesons, photon) and M the nucleon mass. In the low-energy region, the terms of order $\boldsymbol{k}_i/M$ can be neglected compared with the leading term of this expansion. The result is exactly that of the pair-term contribution, to the lowest order of perturbation theory, but occurring with the g_s^2 coupling constant, as determined by D.T.G. The latest value for this constant, deduced from the phase shifts given by Orear,[6] yields 0.14, thus indicating a nearly complete suppression of the pair effect.

In Sec. 3, with the actual value of μ, the relative orders in μ/M for processes in which the γ ray interacts with the nucleon current or its cloud, are listed, with an explicit example. A short discussion follows, in which it is made clear that the whole contribution to the photoproduction of pairs of π mesons comes from the relative μ/M terms so far neglected in the perturbation calculations. The low experimental ratio for the yields of negative to positive pions is thus not unexpected.

2. INTERACTION WITH THE OUTGOING MESON CURRENT

A particular matrix element for this process is represented diagrammatically in Fig. 1. These terms are characterized by the fact that it is always possible to split them up by the lines B_1 and B_2 into three parts:

(a) the interaction $\Theta_\mu(k_+,k')$ of the γ ray with one of the outgoing mesons;

(b) the propagation $\Delta'(k')$ of the dressed meson between its interactions with the γ ray and the nucleon;

(c) the interaction $T_{ij}(k',k_-,p)$ of the nucleon with the two mesons.

One has, for (a):

$$\Theta_\mu(k_+,k')=i(k_{+\mu}+k'_\mu)F(k_+^2/M^2,\ k'^2/M^2,\ (k_+-k')^2/M^2)$$
$$+i(k_{+\mu}-k'_\mu)F_1(k_+^2/M^2,\ k'^2/M^2,\ (k_+-k')^2/M^2).$$

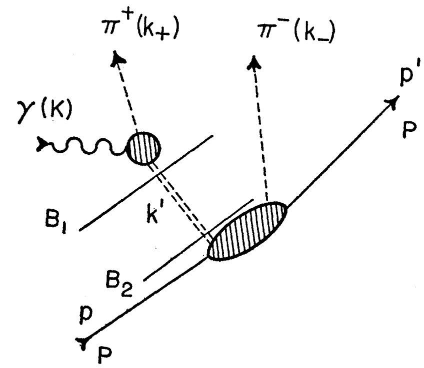

FIG. 1. A typical process for the interaction of the γ ray with the outgoing meson current.

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[1] M. L. Sands, *Proceedings of the Fifth Annual Rochester Conference on High-Energy Physics, 1955* (Interscience Publishers, New York, 1955), p. 48. See also W. K. H. Panofsky, *Proceedings of the Fifth Annual Rochester Conference on High-Energy Physics, 1955* (Interscience Publishers, New York, 1955), p. 50.

[2] R. D. Lawson, Phys. Rev. **92**, 1272 (1953).

[3] See Schweber, Bethe, and de Hoffmann, *Mesons and Fields* (Row, Peterson, and Company, Evanston, 1955), Vol. 2, Sec. 49 g.

[4] Deser, Thirring, and Goldberger, Phys. Rev. **94**, 711 (1954), to be referred to as D.T.G.

[5] R. D. Lawson, reference 2, showed this feature in the lowest order perturbation result $\sim eg^2$. It can be shown that it is also true at any order in a perturbation expansion. See also A. Klein, Phys. Rev. **95**, 1061 (1954), where a similar argument is developed in the case of the nuclear forces in the adiabatic limit.

[6] J. Orear, Phys. Rev. **100**, 288 (1955).

A development in powers of $K/M=(k_+-k')/M$ yields:

$$\Theta_\mu(k_+,k')=i(k_{+\mu}+k'_\mu)[F(k_+^2/M^2,k_+^2/M^2,0)+O(K/M)]. \quad (2.1)$$

From

$$\Delta'^{-1}(k_+)=(k_+^2+\mu^2)\left[1+\frac{(k_+^2+\mu^2)}{M^2}\Phi(k_+^2)\right],$$

one deduces the interaction

$$\Theta_\mu(k_+,k_+)=\frac{1}{i}2k_{+\mu}\left[1+2\frac{(k_+^2+\mu^2)}{M^2}\Phi(k_+^2)+\frac{(k_+^2+\mu^2)^2}{M^2}\Phi'\left(\frac{k_+^2}{M^2}\right)\right]$$
$$=(1/i)2k_{+\mu}[1+O(\mu^2/M^2)];$$
$$(\Phi'=\partial\Phi/\partial k_+^2). \quad (2.2)$$

By comparison between (2.1) and (2.2), it follows that

$$F(k_+^2/M^2,k_+^2/M^2,0)=-1+O(\mu^2/M^2). \quad (2.3)$$

For (b), one has simply:

$$\Delta'(k')=(k'^2+\mu^2)^{-1}\left[1+\frac{(k'^2+\mu^2)}{M^2}\Phi(k'^2)\right]^{-1}$$
$$=\Delta(k')[1+O(\mu^2/M^2)]. \quad (2.4)$$

Finally, for (c) one can write:

$$T_{ij}(k',k_-,p)=\frac{1}{M}\left[A_{ij}\left(\frac{k'}{M},\frac{k_-}{M},\frac{p}{M}\right)\right],$$

which, developed in powers of k'/M and k_-/M, leads to

$$T_{ij}(k',k_-,p)=\frac{1}{M}\left[A_{ij}\left(0,0,\frac{p}{M}\right)+\sum_n\frac{k_n}{M}\frac{\partial}{\partial k_n}A_{ij}\bigg|_{k'=k_-=0}+\cdots\right]$$
$$=\frac{1}{M}\left[A_{ij}\left(0,0,\frac{p}{M}\right)+r_{ij}O\left(\frac{k_n}{M}\right)\right]. \quad (2.5)$$

In (2.5), n runs over the indices prime and minus for the k's, whereas r_{ij} is the isotopic spin dependence of the k/M terms. By use of the D.T.G. results for the meson-nucleon scattering,[7] it is easily shown that

$$u_c^\dagger(p)A_{ij}(0,0,p/M)u_c(p')=[g_r^2\delta_{ij}(1+2M\dot{M})+r_{ij}'O(\mu/M)]u_c^\dagger u_c. \quad (2.6)$$

Here g_r^2 is a renormalized coupling constant defined from the unrenormalized g_u^2 by

$$g_r^2=Z_3(\partial\alpha'/\partial\alpha)^2g_u^2,$$

α' and α being the renormalized and the unrenormalized constant external fields, respectively, as defined by D.T.G. in their α formalism; furthermore, the u_c are the renormalized spinors.

Collecting now Eqs. (2.1)–(2.6), one can express the matrix element[8] for the process of Fig. 1 as

$$eg_r^2(2/i)k_{+\mu}[1+O(k/M)][(k_+-K)^2+\mu^2]^{-1}\times(1+O(\mu^2/M^2))(1/2M)[(1+2M\dot{M})+O(k/M)]$$
$$=eg_r^2ik_{+\mu}(k_+K)^{-1}(2M)^{-1}\times[(1+2M\dot{M})+O(k/M)]. \quad (2.7)$$

The first term in the bracket of the final result (2.7), i.e., the leading term in the expansion in k/M, is the same as the pair term contribution in the lowest order perturbation expansion, except for the factor $(1+2M\dot{M})$, which is not known. However, the product

$$g_s^2=g_r^2(1+2M\dot{M})$$

can be determined from the values of the α_1 and α_3 S-phase shifts, or directly from the data of the π-mesonic atoms. For instance, the last values of Orear's solution: $\alpha_3=-0.11\eta$; $\alpha_1=+0.16\eta$, lead to a $g_s^2=0.14$. This means nothing other than a complete suppression of the pair term, since terms of relative order μ/M, but occurring with g_r^2 acting in full force, are much more important for the actual value of μ than the g_s^2 core term.

3. INTERACTION WITH THE DRESSED NUCLEON CURRENT

We quote simply the relative order in μ/M of the different kinds of interaction of the γ ray with the nucleon or its cloud, compared with that with the outgoing meson current.

From the general form of this nucleonic interaction, given in a compact form by

$$S^{-1}\left(i\frac{\partial}{\partial A_\mu}\frac{1}{i}\frac{\partial}{\partial\alpha_i}\frac{1}{i}\frac{\partial}{\partial\alpha_j}S\right)S^{-1}, \quad (3.1)$$

where A_μ is a constant external electromagnetic field, the following ways of double photoproduction are likely to contribute:

(a) The γ ray is absorbed by the dressed nucleon which then propagates freely (i.e., it is possible, at least in one point, to split the diagram representation by cutting one single nucleon line) until the two mesons are emitted.

(b) The photoproduction of one meson takes place

[7] In this connection, see reference 4, Eqs. (2.29), (2.32), (3.2), (3.3), (3.4), and (5.4) which furnishes logically the result quoted in (2.6) of the present paper.

[8] It is clear that, in the limit $\mu=0$, terms of order μ/M are to be understood as being of order (kinetic energy)/(nucleon mass).

first, and then the dressed nucleon propagates freely until it emits the second meson.

(c) The whole effect occurs without free propagation of the nucleon.

Just above the threshold, the relative orders in μ/M compared with the outgoing meson current term are, respectively:

(a) $O(\mu/M)$, occurring with g_s^2.

(b) $O(\mu^2/M^2)$, occurring with $g_P g_r$.

(c) $O(\mu^2/M^2)$, occurring with g_r^2.[9]

As an illustration of the determination of these relative orders, consider briefly case (b). At threshold, with zero pion mass, it contributes a term proportional to $eg_r g_P(2M)^{-2}\gamma_\mu$, since the photoproduction gives a factor $eg_P(2M)^{-1}\gamma_\mu\gamma_5$ and the remaining interaction, a factor $g_r(i\gamma p+M)^{-1}\gamma_5$, in accordance with the cancellation of the dressed propagator and vertex effects.[10] Being strictly zero at threshold, this matrix element, between $u_c^\dagger(p+\Delta p)$ and $u_c(p)$, starts with a $\Delta p/M$ factor, with increasing Δp, and its contribution is then of order $eg_r g_P(4M^3)^{-1}\mu$ just above threshold. Compared to the leading term in (2.7), this behaves just with the relative order μ^2/M^2, but with the coefficient $g_r g_P$, as stated earlier.

Thus, in conclusion, it is likely that the whole contribution to the photoproduction of two pions comes entirely from the terms of relative order k/M in (2.7) as well from the effects (b) and (c). Calculations of the effect (b) have been made, using the semiphenomenological Chew method, with results in good agreement with experiment.[11] So far, the theoretical predictions were based essentially on the perturbation expansion, giving too large an estimate owing to the pair effects. On the other hand, in the light of the present analysis, the low experimental ratio of negative to positive pions is no longer unexpected.

It is a pleasure to thank Dr. S. Deser, Dr. P. Martin, and Dr. G. Källén for many helpful discussions.

[9] g_P is the Kroll-Rudermann constant, determined by the photoproduction of a single π meson. The relation between g_P and g_r is $g_P=(F+H)g_r$. For the definition of F and H, see reference 4, Eq. (4.3). Further, (a) has to be evaluated with care as, formally, it is infrared-divergent at threshold for $\mu=0$. In this connection, see A. Klein quoted in reference 5.

[10] Reference 4, Eq. (5.4).

[11] F. Zachariasen, Phys. Rev. **100**, 1809(A) (1955).

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High-Energy Interference Effect of Bremsstrahlung and Pair Production in Crystals*†

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With the use of monocrystalline targets for bremsstrahlung and pair production experiments, interference phenomena are expected to occur which will markedly change the γ-ray spectrum as well as the pair energy distribution, for certain angles giving enhancement of radiation and pairs. The effect increases with energy; it sets in at $\delta\lesssim n_0(2\pi/a)$, where δ is the minimum momentum transfer to the target atoms, a the lattice constant, and n_0 a number of the order 2 or 3. This corresponds to ~200-Mev primary energy for bremsstrahlung, ~1 Bev for pair production. The effect is confined to angles between the primary beam and a line of atoms, of order $\theta\lesssim(a_{\text{screen}}/\lambda\!\!\!^{-}_C)(mc^2/E)$. The temperature motion of the lattice reduces the interference effect to some extent. The Born approximation was used for the quantitative analysis of the problem.

I. INTRODUCTION

IN a recent Letter to the Editor,[1] reference was made to deviations of bremsstrahlung and pair production cross sections in crystalline targets from the Bethe-Heitler formulas, and estimates of the order of magnitude of the deviations as well as the "threshold value" of primary energy were given. We here present a detailed analysis of this phenomenon.

With a classical picture of an electron passing a row of regularly spaced atoms in a lattice, it can easily be seen that, for sufficiently high speed of the electron, coherence in the successive interactions might well occur. One could expect deviations from the one-atom bremsstrahlung formula if, in the rest system of the electron, the collision frequency $c\gamma/a$ approaches the frequency of the radiation $\sim mc^2/h$. Considerations of this kind indeed led Williams[2] to surmise a corresponding interference effect for the first time.

Quantum-mechanically, such an effect should not be considered as being due to the electrons represented by waves, as their wavelength would be short compared to the lattice constant at high energies. It will rather be

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[1] F. J. Dyson and H. Überall, Phys. Rev. **99**, 604 (1955).

[2] E. J. Williams, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **13**, 4 (1935).