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Photoproduction of Pi-Meson Pairs

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The cross section for photoproduction of pion pairs from hydrogen has been calculated from the cut-off static theory, assuming the energy is sufficiently low that one of the mesons is produced in an S state, the other in a P state. It is shown that the cross section for this process can be expressed exactly in terms of the P-wave scattering phase shifts, provided the S-wave meson-nucleon interactions and the meson-meson interaction can be neglected. The theoretical predictions obtained here are in general agreement with preliminary experimental results.

IT has recently been shown that a good understanding of the main experimental features of meson physics can be obtained from a simple phenomenological model.[1,2] This model assumes that the only important interaction is the absorption and emission of single mesons by a nucleon, and that only low-energy intermediate states are important for describing the behavior of low-energy processes. This cutoff Yukawa theory has been applied by Chew, with considerable success, to the scattering of P-wave mesons and the photoproduction of single mesons. The scattering of S-wave mesons, for which this model is inadequate, is relatively weak.

It is of course possible that the agreement with experiment might be fortuitous, and since the linearity of the interaction is the most arbitrary assumption in the theory, the most direct further test can be obtained by a study of states containing two mesons, in addition to a nucleon. Thus similar tests of the cutoff Yukawa theory can be obtained by a study of either inelastic meson-nucleon scattering, or the simultaneous photoproduction of two π mesons from a single nucleon. The photoproduction of a pair of mesons is especially useful for this purpose, because, as will be shown, some detailed predictions can be made without resorting to approximate calculations, provided the meson-meson interaction and the S-wave meson-nucleon interactions are negligible.

Since it is assumed that the only interaction is a P-wave interaction between a meson and a nucleon, the cross section for producing two S-wave mesons will depend only on recoil effects, and therefore is expected to be very small. The cross section for emitting both in P states is difficult to estimate; we suppose that the bombarding energy is low enough that the contribution of this process to the cross section is much smaller than the cross section for producing the S-wave and one P-wave meson. We shall calculate the matrix element for production of an S-wave and a P-wave meson, neglecting recoil effects.

We use the methods of Wick, Low, and Chew,[3,4] which have been applied with great success to the problems of scattering and photoproduction of a single meson. We wish to calculate the matrix element

$$M_{\mathrm{pq}}=\left(\Psi_{\mathrm{pq}},\ -\int \mathbf{j}(\mathbf{x})\cdot\mathbf{A}_{\mathrm{K}}(\mathbf{x})d^3x\Psi_0\right),\tag{1}$$

where Ψ_0 is the wave function of the physical nucleon, Ψ_{pq} is the wave function for a state in which two mesons, with momenta $\mathbf{p}$ and $\mathbf{q}$ (and ingoing scattered waves), are present in addition to the nucleon, while $\mathbf{j}(\mathbf{x})$ and $\mathbf{A}_{\mathrm{K}}(\mathbf{x})$ are the current and the electromagnetic potential for a photon with momentum $\mathbf{K}$. The isotopic spin indices have been suppressed. The units are such that $\hbar=c=1=\mu$ (the meson mass). We may write

$$\Psi_{\mathrm{pq}}=a_{\mathrm{q}}{}^*\Psi_{\mathrm{p}}-(H-\omega_q-\omega_p+i\epsilon)^{-1}V_{\mathrm{q}}\Psi_{\mathrm{p}},\tag{2}$$

where V_{q} is defined by

$$[H,a_{\mathrm{q}}{}^*]=\omega_q a_{\mathrm{q}}{}^*+V_{\mathrm{q}}.$$

Then (1) becomes:

$$M_{\mathrm{pq}}=\left(\Psi_{\mathrm{p}},\left[\int \mathbf{j}\cdot\mathbf{A}d^3x,\ a_{\mathrm{q}}\right]\Psi_0\right)+\left(\Psi_{\mathrm{p}},\ V_{\mathrm{q}}{}^*(H-\omega_q-\omega_p-i\epsilon)^{-1}\int \mathbf{j}\cdot\mathbf{A}d^3x\Psi_0\right)-\left(\Psi_{\mathrm{p}},\ \int \mathbf{j}\cdot\mathbf{A}d^3x a_{\mathrm{q}}\Psi_0\right).\tag{3}$$

If we specialize to the case that one of the mesons, the meson with momentum $\mathbf{q}$, is in an S state, then $V_{\mathrm{q}}=0$ and $a_{\mathrm{q}}\Psi_0=0$, so only the first term of (3) remains. Since the final state is of course symmetric, we must later symmetrize M_{pq}.

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[3] G. C. Wick, Revs. Modern Phys. **27**, 339 (1955).

[4] F. Low, Phys. Rev. **97**, 1392 (1955); G. F. Chew and F. Low, Phys. Rev. **101**, 1570, 1579 (1956).

To evaluate the commutator of $a_{\mathbf{q}}$ with the current, we note that we get contributions from two types of terms; first, there is the ordinary meson current,

$$\left[\int \mathbf{j}_m(\mathbf{x})\cdot\boldsymbol{\varepsilon}(2K)^{-\frac{1}{2}}e^{i\mathbf{K}\cdot\mathbf{x}}d^3x,\ a_{\mathbf{q}\alpha}\right]$$
$$=2ie\mathbf{q}\cdot\boldsymbol{\varepsilon}[8K\omega_q\omega(\mathbf{q}-\mathbf{K})]^{-\frac{1}{2}}\epsilon_{3\alpha\gamma}[a_{\mathbf{q}-\mathbf{K},\gamma}+a_{\mathbf{K}-\mathbf{q},\gamma}^*].\quad(4)$$

The isotopic spin index is here indicated explicitly, with an implied sum over the repeated index. The matrix element of (4) is reduced to a simple form by using the identities,

$$(\Psi_{\mathbf{p}},\ a_{\mathbf{q}-\mathbf{K},\gamma}\Psi_0)=-[\omega_p+\omega(\mathbf{q}-\mathbf{K})]^{-1}(\Psi_{\mathbf{p}},\ V_{\mathbf{q}-\mathbf{K},\gamma}{}^*\Psi_0),\quad(5)$$
$$(\Psi_{\mathbf{p}},\ a_{\mathbf{K}-\mathbf{q},\gamma}{}^*\Psi_0)=-[\omega(\mathbf{q}-\mathbf{K})-\omega_p]^{-1}(\Psi_{\mathbf{p}},\ V_{\mathbf{K}-\mathbf{q},\gamma}\Psi_0).$$

Secondly, we get a contribution from the interaction current term which is required to provide continuity of current:

$$\left[\int \mathbf{j}_{\mathrm{int}}(\mathbf{x})\cdot\boldsymbol{\varepsilon}(2K)^{-\frac{1}{2}}e^{i\mathbf{K}\cdot\mathbf{x}}d^3x,\ a_{\mathbf{q}\alpha}\right]=\mathbf{J}_{\mathbf{q}\alpha}\cdot\boldsymbol{\varepsilon}(4K\omega_q)^{-\frac{1}{2}}.\quad(6)$$

We can express the matrix element of (6) in the same form as (5) by noting that in the Yukawa (linear) theory

$$V_{\mathbf{p}\alpha}=if\boldsymbol{\sigma}\cdot\mathbf{p}(2\omega_p)^{-\frac{1}{2}}\tau_\alpha v(p^2)=p_i(2\omega_p)^{-\frac{1}{2}}U_{i\alpha},\quad(7)$$

and

$$\mathbf{J}_{\mathbf{q}\alpha}=ef\epsilon_{3\alpha\gamma}\tau_\gamma\boldsymbol{\sigma}.\quad(8)$$

Therefore,

$$\mathbf{J}_{\mathbf{q}\alpha}=-ie\epsilon_{3\alpha\gamma}\mathbf{U}_\gamma,\quad(9)$$
$$\mathbf{U}_\gamma{}^*=-\mathbf{U}_\gamma.$$

Equations (9) are valid in any theory in which only the gradient of the meson field variable, $(\nabla\phi)$, enters the interaction Hamiltonian (provided the source size is small compared with the wavelength); however, the interaction may involve an arbitrary function $f(\nabla\phi,\boldsymbol{\sigma},\boldsymbol{\tau})$.

Combining Eqs. (4) to (9), we find

$$M_{\mathbf{p}\beta,\,\mathbf{q}\alpha}=-\frac{ie\epsilon_{3\alpha\gamma}}{(4K\omega_q)^{\frac{1}{2}}}\left[\epsilon_i+\frac{2\mathbf{q}\cdot\boldsymbol{\varepsilon}(K_i-q_i)}{K\omega_q-\mathbf{K}\cdot\mathbf{q}}\right]\times(\Psi_{\mathbf{p}\beta},U_{i\alpha}\Psi_0).\quad(10)$$

In order to obtain a consistent neglect of higher partial wave effects, we may separate out the part in which one meson is an S state; this gives

$$M_{\mathbf{p}\beta,\,\mathbf{q}\alpha}=-ie\epsilon_{3\alpha\gamma}(4K\omega_q)^{-\frac{1}{2}}F(q)\epsilon_i(\Psi_{\mathbf{p}\beta},U_{i\gamma}\Psi_0).\quad(11)$$

where

$$F(q)=1-\left[\frac{\omega_q}{2K}-\frac{1}{4Kq}\log\frac{\omega_q+q}{\omega_q-q}\right]\approx1-\frac{2}{3K}(\omega_q-1).\quad(12)$$

It is evident that the meson current term gives a small part of the total effect. Finally, we remember, that Wick, Chew, and Low[3,4] have shown the remaining matrix element in (11) to give the matrix element for scattering of a P-wave meson with energy ω_p,

$$R_{\mathbf{p}\beta,\,\mathbf{q}\alpha}=(\Psi_{\mathbf{p}\beta},V_{\mathbf{q}\alpha}\Psi_0).\quad(13)$$

We write

$$R_{\mathbf{p}\beta,\,\mathbf{q}\alpha}=(4\omega_p\omega_q)^{-\frac{1}{2}}p_iq_j(\phi_0,T_{ij,\,\beta\alpha}(p)\phi_0),\quad(14)$$

where the ϕ_0 are bare-nucleon states; then

$$M_{\mathbf{p}\beta,\,\mathbf{q}\alpha}=-i(8\omega_q\omega_pK)^{-\frac{1}{2}}e\epsilon_{3\alpha\gamma}F(q)\times p_i\epsilon_j(\phi_0,T_{ij,\,\beta\gamma}(p)\phi_0).\quad(15)$$

Equation (15) is our final result—it has the form of an explicit relation between the matrix element for meson pair production and the P-wave scattering phase shifts.

If we retain only the scattering phase shift δ_{33}, we have, writing $\delta_{33}=\delta$,

$$T_{ij,\,\beta\gamma}(p)=-12\pi(\delta_{\beta\gamma}-\tfrac{1}{3}\tau_\beta\tau_\gamma)(\delta_{ij}-\tfrac{1}{3}\sigma_i\sigma_j)\times p^{-3}\sin\delta(p)e^{i\delta(p)}.\quad(16)$$

We then find directly, for the cross section for production of a (π^+,π^-) pair

$$\begin{aligned}d\sigma^{+-}=\;&12\alpha(4\pi)^{-2}K^{-1}k_+k_-d\omega_\pm d\Omega_+d\Omega_-\\&\times\{k_+{}^2\Sigma_+{}^2F_-{}^2(\tfrac{1}{2}\sin^2\theta_++\tfrac{1}{3})\\&+\tfrac{1}{9}k_-{}^2\Sigma_-{}^2F_+{}^2(\tfrac{1}{2}\sin^2\theta_-+\tfrac{1}{3})\\&-\tfrac{1}{3}k_+k_-\Sigma_+\Sigma_-F_+F_-\cos(\delta_+-\delta_-)\\&\times[(5/3)\cos\phi-\cos\theta_+\cos\theta_-]\},\end{aligned}\quad(17)$$

where we have written $k_{+,-}$ instead of p, q. Here $F_\pm=F(k_\pm)$, and $\Sigma_\pm=(k_\pm)^{-3}\sin\delta_\pm$; also, θ_+ and θ_- are the angles of the π^+ and π^- with respect to the photon, and ϕ is the angle between the two mesons. Similarly, we find for the cross section for production of a (π^+,π^0) pair

$$d\sigma^{+0}=\frac{8\alpha k_+k_0d\omega_+}{3(4\pi)^2K}d\Omega_0d\Omega_+k_0{}^2\Sigma_0{}^2F_+{}^2(\tfrac{1}{2}\sin^2\theta_0+\tfrac{1}{3}).\quad(18)$$

Equations (17) and (18) would be particularly useful when the P-wave meson has an energy close to the resonance energy for meson-nucleon scattering in the $J=\frac{3}{2}$, $T=\frac{3}{2}$ state. Note that the familiar ratios, 9:2:1, which occur in meson scattering, appear here as the ratios of the partial cross sections for producing a π^+ in a P state and a π^- in an S state, a π^0 in a P state and a π^+ in an S state, and a π^- in a P state and a π^+ in an S state. We therefore find $\sigma^{+-}=5\sigma^{+0}$. In the (π^+,π^-) differential cross section, there is a pronounced preference for emission of the positive meson at right angles to the photon beam, and with most of the available energy. This differs markedly from the result of Lawson[5]

[5] R. D. Lawson, Phys. Rev. 92, 1272 (1953).

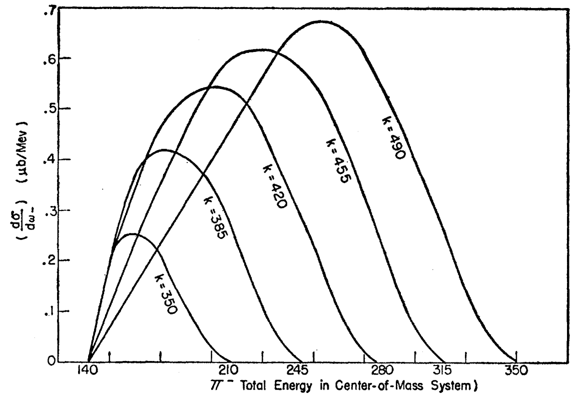

FIG. 1. Theoretical curves of $d\sigma/d\omega_-$ *versus* ω_- for various photon energies K, in the center-of-mass system.

who made a lowest order perturbation calculation using the relativistic theory.

Near threshold the P-wave scattering in other states may not be completely negligible compared with the scattering in the $(\frac{3}{2},\frac{3}{2})$ state. The smaller phase shifts are not given very accurately by the experimental data, so in this region we approximate them by the Born approximation to the cut-off Yukawa theory, which gives

$$\begin{aligned} T_{ij,\alpha\beta}(p) = {} & 4\pi f^2\omega_p^{-1}[\sigma_i\sigma_j\tau_\alpha\tau_\beta - \sigma_j\sigma_i\tau_\beta\tau_\alpha] \\ & -12\pi(\delta_{ij}-\tfrac{1}{3}\sigma_i\sigma_j)(\delta_{\alpha\beta}-\tfrac{1}{3}\tau_\alpha\tau_\beta) \\ & \times[p^{-3}\sin\delta(p)e^{i\delta(p)}-4f^2/3\omega_p], \end{aligned} \quad (19)$$

where f^2, the meson-nucleon coupling constant, is roughly 0.08. Using (19), we calculate

$$\begin{aligned} d\sigma^{+-} = {} & 12\alpha(4\pi)^{-2}K^{-1}k_+k_-d\omega_\pm d\Omega_+ d\Omega_- \\ & \times\{k_+^2F_-^2[\Sigma_+^2(\tfrac{1}{2}\sin^2\theta_+ + \tfrac{1}{3}) \\ & +\Sigma_+\xi_+\cos\delta_+(\tfrac{1}{3}-\tfrac{1}{2}\sin^2\theta_+)+(\xi_+^2/12)] \\ & +\tfrac{1}{9}k_-^2F_+^2[\Sigma_-^2(\tfrac{1}{2}\sin^2\theta_- + \tfrac{1}{3}) \\ & +\Sigma_-\xi_-\cos\delta_-((7/3)-(11/2)\sin^2\theta_-) \\ & +\xi_-^2(5\sin^2\theta_- + (49/12))] \\ & +\tfrac{1}{3}k_+k_-F_+F_-[\Sigma_+\Sigma_-\cos(\delta_+-\delta_-) \\ & \times(\cos\theta_+\cos\theta_- - (5/3)\cos\phi) \\ & +\Sigma_+\xi_-\cos\delta_+((19/6)\cos\phi-(11/2)\cos\theta_+\cos\theta_-) \\ & +\Sigma_-\xi_+\cos\delta_-(\tfrac{1}{2}\cos\theta_+\cos\theta_- - \tfrac{1}{6}\cos\phi) \\ & +\xi_+\xi_-(\tfrac{1}{2}\cos\theta_+\cos\theta_- - (4/3)\cos\phi)]\}, \end{aligned} \quad (20)$$

where $\xi_\pm = 4f^2/3\omega_\pm$. The perturbation theory result is obtained from (20) by replacing $\Sigma_\pm$ by $\xi_\pm$, and $\delta_\pm$ by zero.

Some of the recoil corrections may be taken into account in the following way. First we may replace the scattering matrix in the center-of-mass system of all three outgoing particles by the scattering matrix in the c.m. system of the nucleon and just one meson. We multiply $\Sigma_\pm$ by $[1+(\omega_\pm/M)]$, and to correct the statistical factors in the expression for $d\sigma$, we replace K^{-1} by $\nu^{-1}[1+(\nu/M)]^{-1}$, where ν is the frequency of the photon in the c.m. system, and we replace $d\omega_\pm$ by

$$d\omega_+ d\omega_-/d[\omega_+ + \omega_- + (\mathbf{k}_+ + \mathbf{k}_-)^2/2M].$$

In addition, there are recoil corrections to the meson theory, which we do not discuss here.

In Fig. 1 is shown $d\sigma^{+-}/d\omega_-$, plotted against ω_- and K. These cross sections were calculated assuming only $\delta_{33}\neq 0$, that is, from Eq. (17). Recoil corrections are not included. The phase shift δ_{33} was taken from a fit to the experimental meson-nucleon scattering by Feld.[6]

The present experimental results[7] show the peaking of $d\sigma^{+-}/d\omega_-$ at low ω_-. No angular distributions are as yet available. The best quantitative information is shown in Fig. 2 together with the theoretical predictions. The measured quantity is

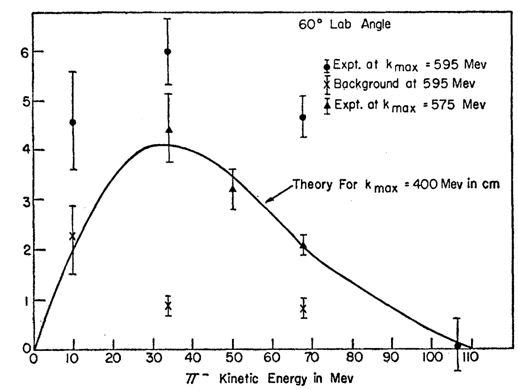

FIG. 2. Experimental measurements of the quantity $\int(dK/K)\times(d\sigma/d\omega_- d\Omega_-)$ as a function of ω_- for a laboratory angle $\theta_-=60°$, at laboratory photon energies of 575 and 595 Mev, in units of μb/sterad Mev.

$$\int_{1+\omega_-}^{K_{\max}}\frac{dK}{K}\left[\frac{d\sigma^{+-}}{d\omega_- d\Omega_-}\right]_{\theta_-=60°}.$$

There appears to be a general qualitative agreement with the theoretical predictions.

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[6] B. T. Feld (private communication).

[7] R. M. Friedman and K. M. Crowe, Phys. Rev. **100**, 1799 (1955); M. L. Sands (private communication).