Measurements at lower temperatures and a more detailed discussion of these results and experimental techniques will be presented in the future.

We wish to thank Dr. P. W. Anderson for helpful discussions on the interpretation of these results and Dr. G. Feher for the loan of some equipment.

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dc Magnetoconductivity and Energy Band Structure in Semiconductors

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THEORIES of the behavior of semiconductors under the influence of electric and magnetic fields give directly the magnetoconductivity tensor (L), whereas the usual measurements give the magnetoresistivity tensor ($P=L^{-1}$). The comparison of theory and experiment is ordinarily made by mathematical inversion of L. This procedure often proves unwieldy and limits the usefulness of the theory, usually restricting it to low or high H regions.

The purpose of this letter is to report an experimental method uniquely suited to the direct evaluation of the components of L. Application of this method to n-type germanium and silicon reproduces the known energy band structure.

Our technique consists in rotating H in one particular plane of crystalline symmetry, but in different sample planes in several experiments; all components of P can then be measured using only the usual Hall and resistivity probes—thence, L can be computed *vs* the position of H in the crystal plane. For cubic materials, which are here considered, only a single rectangular sample, cut with all three directions on axes of cubic symmetry, is required. The application to materials of other structure and to other directions in cubic materials becomes obvious.

Let ϕ be the angle between H and a direction of cubic symmetry, H being rotated through a plane of cubic symmetry. Then Fig. 1 shows how one can obtain all four (the number is reduced due to the symmetry of the experiment) independent components of $P(\phi)$ by measurement of P_{11} and P_{21} for both ϕ and ψ. (P_{23} is the "planar hall" field first reported by Goldberg and Davis.[1]) We have thus far performed the experiment at 80°K for n-type germanium and silicon. The so-obtained $L(\phi)$ contains considerable information about the band structure.

We have developed the theory for this experiment in the mass tensor approximation: Using the distribution function of Blochinzev and Nordheim,[2] we obtain:

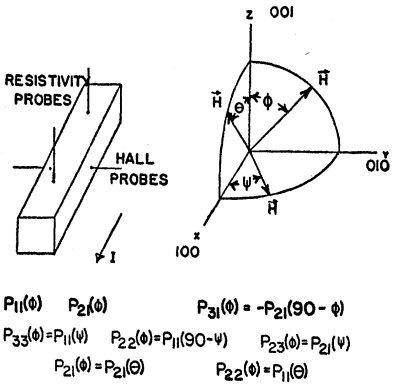

FIG. 1. Measurement of the magnetoresistivity tensor for cubic materials.

$$\mathbf{j}=-\frac{e^2}{4\pi^3}[\sigma^1\mathbf{F}+(e/c)\rho^1(M^{-1}\mathbf{F}\times\mathbf{H}) +(e/c)^2\lambda^1(M\mathbf{H}/|M|)(\mathbf{F}\cdot\mathbf{H})], \quad (1)$$

$$\sigma^1\equiv\int\frac{(\partial f_0/\partial E)\tau\mathbf{v}\mathbf{v}d\Omega}{1+[e\tau/c]^2(M\mathbf{H}\cdot\mathbf{H}/|M|)},$$

where ρ^1 and λ^1 are identical to σ^1 but for the replacement of τ by τ^2 and τ^3, respectively. Using the relation $\mathbf{v}=\hbar M^{-1}\mathbf{k}$, a theorem of Herring[3] concerning the equality of energy shell averages, and the original distribution function of Bronstein,[4] Eq. (1) becomes

$$\mathbf{j}=-\frac{e^2}{6\pi^3}[\sigma M^{-1}\mathbf{F}+(e/c)(1/|M|)\rho(M\mathbf{H}\times\mathbf{F}) +(e/c)^2(1/|M|)\lambda\mathbf{H}(\mathbf{H}\cdot\mathbf{F})], \quad (2)$$

$$\sigma=\int\frac{(\partial f_0/\partial E)\tau(E)Ed\Omega}{1+[e\tau/c]^2(M\mathbf{H}\cdot\mathbf{H}/|M|)},$$

with similar expressions for ρ and λ in τ^2 and τ^3. Thus,

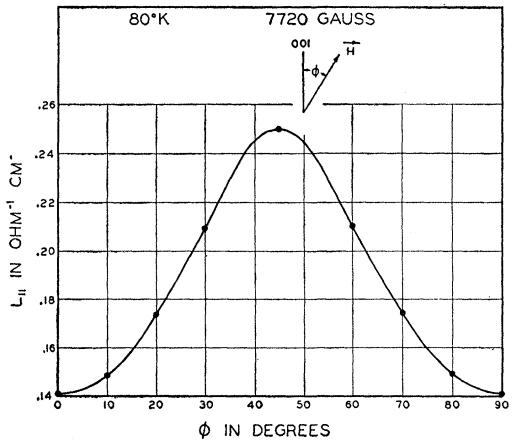

FIG. 2. Transverse magnetoconductivity for n-type germanium.

the transport integrals become scalars but are functions of **H**, making a convenient and simple calculation possible. These results and all the following are valid for all $|\mathbf{H}|$.

$L(\phi)$ has been calculated from (2) by using the mass tensors for the three simplest cubic solutions (see, e.g., Shibuya[5]): 100-type spheroids (case A), 111-type spheroids (case B), and 110-type spheroids (case C). In the principal axis system $E=\frac{1}{2}\hbar^2[(k_x^2+k_y^2)/b+k_z^2/a]$. We show here only the results for $L_{11}{}^A(\phi)$ and $L_{11}{}^B(\phi)$:

$$-\frac{6\pi^3}{e^2}L_{11}{}^A=\frac{1}{b}\int(\partial f_0/\partial E)\tau E d\Omega\left\{\left(\frac{b}{a}\right)\left(\frac{1}{1+(\omega\tau)^2}\right)+\frac{2+(\omega\tau)^2(1+a/b)}{1+(\omega\tau)^2(1+a/b)+(\omega\tau)^4[(a/b)+\frac{1}{4}(1-a/b)^2(\sin 2\phi)^2]}\right\}$$

$$-\frac{6\pi^3}{e^2}L_{11}{}^B=\tfrac{2}{3}\left(\frac{1}{b}\right)\left(2+\frac{b}{a}\right)\int(\partial f_0/\partial E)\tau E d\Omega\left\{\frac{2+(\omega\tau)^2\times\frac{2}{3}(2+a/b)}{1+(\omega\tau)^2\times\frac{2}{3}(2+a/b)+(\omega\tau)^4\times\frac{1}{9}[(2+a/b)^2-(1-a/b)^2(\sin 2\phi)^2]}\right\};$$

$L_{11}{}^C$ is of the form $C_1L_{11}{}^A+C_2L_{11}{}^B$, $\omega=(eH)/(ab)^{\frac{1}{2}}c$.

Figures 2 and 3 show the experimental $L_{11}(\phi)$ for n-type Ge and Si. Comparison of theory and experiment shows directly that the spheroidal assumption is the correct one and that case A must hold for Si and case B for Ge, in agreement with other work. Within experimental error, the angular dependence of the integrands exactly matches the experimental curves; thus τ probably does not vary strongly with energy.

Further application of the theory has resulted in the following relations for $K\equiv a/b$ for cases A and B, respectively:

$$\left[\frac{L_{11}{}^{H=0}-(L_{22}+L_{23})_{45^\circ}}{K(L_{11}{}^{H=0}-L_{11}{}^{45^\circ})-2L_{23}}\right]^A=\tfrac{1}{2}\left(\frac{K+(1/K)-2}{K(1+K)-2}\right),$$

$$\left[\frac{L_{22}-L_{11}}{L_{11}{}^{H=0}-L_{11}}\right]_{90^\circ}^B=\frac{9}{(2+K)(2+1/K)}$$

$$=\frac{\rho_0}{\rho_L}\left(\frac{\rho_T{}^2+(RH)^2-\rho_L\rho_T}{\rho_T{}^2+(RH)^2-\rho_0\rho_T}\right).$$

These expressions hold independently of any assumption on τ except that it be constant on a surface of constant energy. Comparison with experiment gives $K=15.5\pm0.5$ for Ge and $K=5.2\pm0.5$ for Si. The accuracy can be improved by working at higher $|\mathbf{H}|$ and lower temperature. Comparison of various components of $L(\phi)$ shows that $K>1$ in both cases. The latter relation indicates a simple method for finding K for case B from ρ_L, ρ_T, and ρ_0 (longitudinal, transverse, and zero-field resistivity) and R (Hall constant in the appropriate units). In addition, having found the effects of the band structure on the transport integrals, one is at liberty to make more exact investigations on the behavior of τ. Further and more complete details will be subsequently reported.

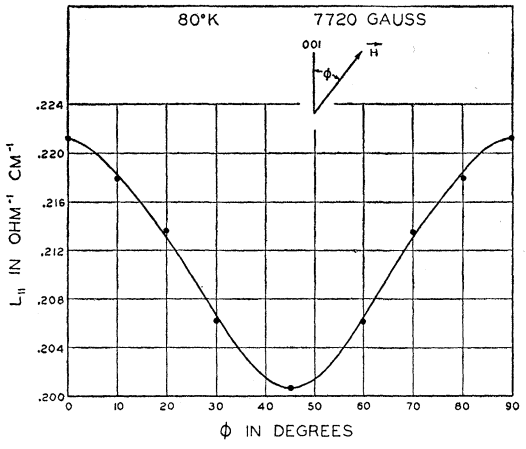

FIG. 3. Transverse magnetoconductivity for n-type silicon.

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Acceptors Quenched into Germanium

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RECENTLY Logan[1] and Hopkins and Clarke[2] have presented evidence which seems to be contradictory to my earlier experiments[3] and which appears to be mutually contradictory. The purpose of this Letter is to point out that the differences among us can be rationalized by noting the effects of dislocations on the annealing process which Logan has found.

Logan claims that my data on the number of acceptors quenched into Ge are larger by a factor of 5 than his. However, if my data are plotted along with Logan's, they are extremely close to the curve which Logan has drawn through his n-type data. My sample was 30 ohm-cm n-type, and therefore, it may be argued that the number of acceptors I observed should be compared with Logan's data taken on p-type Ge. If this be the basis of comparison, Logan's data and mine differ by a factor of at most 2.2. This factor includes the correction for the fact that my acceptor concentrations