Elastogalvanomagnetic Effect and Intervalley Scattering in *n*-Type Germanium

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(Received April 26, 1956)

Starting from a multivalley model of a cubic semiconductor, a calculation of the effect of elastic strain on certain galvanomagnetic effects is carried out. It is found that the effects are sensitive to the strength of the intervalley scattering. An experiment on germanium to which the calculation is applicable is described. It is concluded from a comparison of the theory and the experiment that the coupling constant which characterizes the coupling of the electrons to the intervalley phonons in the model of Herring is considerably smaller than the coupling constant to the acoustic phonons.

I. INTRODUCTION

EXPERIMENTS of Smith[1] have shown that, in a multivalley semiconductor, an elastic strain which destroys the symmetry equivalence of the valleys can produce large changes in the conductivity of the semiconductor. In such a strained crystal the minimum energies in the various valleys are not necessarily equal. Quantitative theories of the effects which result from the shifting of the energy valleys have been given by Herring[2] and by Adams.[3] The treatment of Herring shows that two principal mechanisms are involved in the large elastoresistance effect, namely, the transfer of electrons from valleys of higher energy to valleys of lower energy, and the alteration of the relaxation times for intervalley (IV) scattering.

The importance of the second of these mechanisms relative to the first depends on the strength of the IV scattering. Measurement of the elastoresistance coefficient alone provides no way of distinguishing these contributions. Extension of the above-mentioned theories to the Hall and magnetoresistance effects in elastically strained germanium shows that these elastogalvanomagnetic effects are quite sensitive to the amount of intervalley scattering which takes place. In this paper the calculation of certain elastogalvanomagnetic effects is described in some detail, and it is shown that the rate of IV scattering can be deduced from such coefficients. Some experimental data which show that the amount of IV scattering is small in germanium are presented.

II. PRELIMINARY DISCUSSION OF THE MODEL

In the detailed calculation of the effects we treat the phenomena due to elastic strain according to the model and methods of Herring,[2] while taking the point of view of Adams[3] toward the galvanomagnetic phenomena. In this section we describe the model and some of the results of Herring in our notation, and apply the ideas of statistical equilibrium to this model.

The energy minima considered are located on axes of threefold (111) or fourfold (001) symmetry in $\mathbf{k}$ space. The minima are enumerated by a superscript (i), which is often omitted in formulas which are applicable to any single valley. Let $\mathbf{a}^{(i)}$ be a unit vector directed from the origin in $\mathbf{k}$ space to the ith energy minimum. Then the surfaces of constant energy are ellipsoids of revolution about an axis along $\mathbf{a}$. The energy as a function of $\mathbf{k}$, measured with the energy of the minimum as zero and the location of the minimum as the origin for $\mathbf{k}$, is $E=(\hbar^2/2m)\mathbf{k}\cdot\boldsymbol{\alpha}\cdot\mathbf{k}$, where the second rank effective mass tensor $\boldsymbol{\alpha}=\alpha_1\mathbf{1}+(\alpha_3-\alpha_1)\mathbf{aa}$, $\mathbf{1}$ being the unit tensor.

In our model we neglect all effects of strain except those arising from shifting of the energy minima with respect to one another. This approximation depends on the assumption that all of the impurities are ionized, and there are no intrinsic carriers, so that application of a strain does not change the total number of electrons. If $\mathbf{u}$ is the strain tensor as defined by Herring ($\delta\mathbf{r}=\mathbf{u}\cdot\mathbf{r}$), symmetry requires that $\delta(E^{(i)}-\zeta)$, the shift of the energy minimum with respect to the Fermi level ζ, has the form

$$\delta(E^{(i)}-\zeta)=\Omega_u(\mathbf{a}^{(i)}\cdot\mathbf{u}\cdot\mathbf{a}^{(i)}-\tfrac{1}{3}\epsilon), \tag{1}$$

where Ω_u is a constant with the dimensions of energy and $\epsilon=$dilation$=$trace $\mathbf{u}$. If a cylindrical specimen of crystal with axial direction $\mathbf{e}$ is stretched axially by a force per unit area X, application of elastic theory shows that

$$\delta(E^{(i)}-\zeta)=SX\Omega_u[(\mathbf{a}^{(i)}\cdot\mathbf{e})^2-\tfrac{1}{3}], \tag{2}$$

where S is an elastic compliance, and is, in terms of the elastic constants,

$$S=1/(C_{11}-C_{12}) \quad \text{for (001) minima (Si)}, \tag{3}$$

$$S=1/C_{44} \quad \text{for (111) minima (Ge)}. \tag{4}$$

Hereafter we designate these two types of energy minima by Si and Ge, the fourth group semiconductors in which they occur.

Now we consider the two consequences of the energy shifts, which are mentioned above:

1. It is assumed that the carriers are nondegenerate, and also that $\delta E\ll kT$. Then the change in the number

[1] C. S. Smith, Phys. Rev. **94**, 42 (1954).
[2] C. Herring, Bell System Tech. J. **34**, 237 (1955).
[3] E. N. Adams, Chicago Midway Laboratories Technical Report CML-TN-P8 (unpublished).

of electrons in a valley is

$$\frac{\delta n^{(i)}}{n^{(i)}}=\frac{\delta n^{(i)}}{n_0}=-\frac{\delta(E^{(i)}-\zeta)}{kT}, \tag{5}$$

where $n^{(i)}=n_0$, independent of i, the number of electrons in a valley in the unstrained state.

2. In an IV scattering process, an electron makes a transition from a state in one valley to a state in another valley. Since the energy of the electron plus the energies of any phonons which participate in this process must be conserved, and, since the density of final states is proportional to the square root of the energy of the final state above the bottom of its valley, a shifting of the valley energies will affect the probability of an IV transition.

In the crystals considered, the vectors in $\mathbf{k}$ space which connect pairs of minima, and which represent the change in $\mathbf{k}$ in an IV scattering, may not all be identical or symmetrical with one another. There may be different kinds of IV transitions, which should be considered separately in the theory. In this treatment, however, we use the approximation introduced by Herring,[2] and employ the same functional description for all the IV transitions, and, in considering phonon induced transitions, attribute all the phonons involved to the same branch of the vibrational spectrum. Since symmetry requires that

$$\sum_i \frac{\partial\tau^{(i)}}{\partial E^{(j)}}=\sum_i \frac{\partial\tau^{(i)}}{\partial E^{(j)}}=0, \tag{6}$$

the dependence of relaxation time on the energy minima in the framework of this model can be described by

$$\frac{\partial\tau^{(i)}}{\partial E^{(j)}}=\beta(E)\left(\frac{1}{N}-\delta_{ij}\right), \tag{7}$$

where $\tau^{(i)}$ is the relaxation time in valley i, N is the number of valleys, and β is a function of E, the energy of the electron above the minimum of its valley. The change in $\tau^{(i)}$ due to a strain is

$$\delta\tau^{(i)}=\sum_j \beta(E)\left(\frac{1}{N}-\delta_{ij}\right)\delta E^{(j)}=-\beta\delta(E^{(i)}-\zeta). \tag{8}$$

In (7) and (8), δ_{ij} is the Kronecker delta.

Since the effect described by Eq. (8) arises from the IV scattering, it is apparent that the magnitude of β must depend on the strength of this scattering. We can obtain a quantitative measure of this dependence from considerations of detailed balancing. If $\tau^{(ij)}$ is the relaxation time for scattering from valley i to valley j, and τ_L is the relaxation time for intravalley scattering, then

$$\frac{1}{\tau^{(i)}}=\frac{1}{\tau_L}+\sum_{j\neq i}\frac{1}{\tau^{(ij)}}. \tag{9}$$

In equilibrium, the number of electrons scattered from valley i to valley j must be the same as the number scattered from valley j to valley i:

$$n^{(i)}\left\langle\frac{1}{\tau^{(ij)}}\right\rangle=n^{(j)}\left\langle\frac{1}{\tau^{(ji)}}\right\rangle. \tag{10}$$

Here the angular brackets $\langle\ \rangle$ mean the average over the Boltzmann distribution. In the case of averages which are to be taken in the unstrained state of the crystal, a subscript 0 is added to the bracket. By differentiating Eqs. (9) and (10) with respect to $E^{(i)}$ and using the definition (7), it is found that

$$\left\langle\frac{\beta}{\tau^2}\right\rangle_0=\frac{1}{2kT}\frac{N}{N-1}\left\langle\frac{1}{\tau_I}\right\rangle_0, \tag{11}$$

where τ_I is the total IV scattering time, defined by

$$\frac{1}{\tau_I}=\frac{N-1}{\tau_0}, \tag{12}$$

and τ_0 is the value of the $\tau^{(ji)}$ in the unstrained state. In Herring's model for phonon-induced IV scattering,

$$\frac{1}{\tau^{(i)}}=W_L\left(\frac{E}{k\theta}\right)^{\frac{1}{2}}\frac{T}{\theta}+W_I\frac{1}{\exp(\theta/T)-1}\left[\left(\frac{E}{k\theta}+1\right)^{\frac{1}{2}}+e^{\theta/T}\left(\frac{E}{k\theta}-1\right)^{\frac{1}{2}}\right], \tag{13}$$

in which the first term represents $(1/\tau_L)$ and the second term $(1/\tau_I)$.

III. METHOD OF CALCULATION

The solution of the Boltzmann equation, for an electron with ellipsoidal energy surfaces, in the presence of uniform electric and magnetic fields, has been given, to all orders of the magnetic field, in the form which we wish to use, by Adams.[3] In this form the conductivity for one electron is

$$\xi=\frac{2e^2}{3mkT}\boldsymbol{\alpha}\cdot\langle E\tau\mathbf{S}\rangle, \tag{14}$$

$$\mathbf{S}=\frac{1}{\mathbf{1}+\gamma\mathbf{b}\times\boldsymbol{\alpha}}=\frac{\mathbf{1}-\gamma\mathbf{b}\times\boldsymbol{\alpha}+\gamma^2\Delta\boldsymbol{\alpha}^{-1}\cdot\mathbf{bb}}{1+\gamma^2\Delta\alpha^{-1}{}_{\mathbf{bb}}}. \tag{15}$$

Here $\gamma=eB\tau/mc$, $\Delta=$determinant $\boldsymbol{\alpha}=\alpha_1{}^2\alpha_3$, $\mathbf{b}$ is a unit vector in the direction of the magnetic field, and the notation $\alpha^{-1}{}_{\mathbf{bb}}$ means $\mathbf{b}\cdot\boldsymbol{\alpha}^{-1}\cdot\mathbf{b}$. Setting

$$f=\tfrac{1}{3}\,\mathrm{Tr}\boldsymbol{\alpha}=\tfrac{1}{3}(2\alpha_1+\alpha_3), \tag{16}$$

and

$$\xi_0=\frac{2e^2}{3mkT}f\langle E\tau\rangle_0, \tag{17}$$

(14) becomes

$$\xi=\frac{\xi_0}{f}\frac{\boldsymbol{\alpha}\cdot\langle E\tau\mathbf{S}\rangle}{\langle E\tau\rangle_0}. \tag{18}$$

The total conductivity is, then

$$\boldsymbol{\sigma}=\sum n^{(i)}\boldsymbol{\xi}^{(i)}, \tag{19}$$

where $\sum$ means sum over all valleys. If $\mathbf{B}=0$, then $\mathbf{S}=1$ and

$$\boldsymbol{\xi}=(\xi_0/f)\boldsymbol{\alpha}. \tag{20}$$

In the unstrained state $n^{(i)}=n_0$, and

$$\boldsymbol{\sigma}=\sigma=Nn_0\xi_0. \tag{21}$$

The change in conductivity when the crystal is strained is

$$\delta\boldsymbol{\sigma}=\sum(\boldsymbol{\xi}^{(i)}\delta n^{(i)}+n^{(i)}\delta\boldsymbol{\xi}^{(i)}). \tag{22}$$

The change of $\boldsymbol{\xi}^{(i)}$ comes about through the change in $\tau^{(i)}$. It is easy to verify that $\delta(\tau\mathbf{S})=\mathbf{S}^2$. Substituting for $\delta n^{(i)}$ from Eq. (5) and for $\delta\tau^{(i)}$ from (8), we find

$$\delta\boldsymbol{\sigma}=-\frac{n_0\xi_0}{f\langle E\tau\rangle_0}\sum(\boldsymbol{\alpha}^{(i)}\cdot\langle E\tau\mathbf{S}^{(i)}\rangle_0 +\boldsymbol{\alpha}^{(i)}\cdot\langle E\beta\mathbf{S}^{(i)2}\rangle_0 kT)\frac{\delta(E^{(i)}-\zeta)}{kT}, \tag{23}$$

with $\delta(E^{(i)}-\zeta)$ related to the stress by (2).

IV. MAGNETORESISTANCE

The evaluation of the summations in (23) for a general direction of the magnetic field is quite lengthy, and of little interest because of the small number of constants required to describe our model. We, therefore, specialize our consideration of (23) to a particular direction of the magnetic field for each type of minima, namely, that direction for which $\alpha^{-1}{}_{\mathrm{bb}}$ is the same for all the valleys. This means

$$\mathbf{b}=(111)/\sqrt{3} \quad \text{for Si}, \tag{24}$$

$$\mathbf{b}=(001) \quad \text{for Ge}, \tag{25}$$

and, in both cases, $\Delta\alpha^{-1}{}_{\mathrm{bb}}=\alpha_1 h$, where

$$h=(2\alpha_3+\alpha_1)/3. \tag{26}$$

With this restriction on the magnetic field, the denominator of $\mathbf{S}^{(i)}$, Eq. (15), is independent of i, and factors out through the summation. The addition is much simplified, and we find, for the unstrained state,

$$\boldsymbol{\sigma}=Nn_0[fS_0\mathbf{1}-\alpha_1 hS_1\mathbf{1}\times\mathbf{b}+\Delta S_2\mathbf{bb}], \tag{27}$$

where

$$S_\mu=\frac{\xi_0}{f\langle E\tau\rangle_0}\left\langle\frac{E\tau\gamma^\mu}{1+\alpha_1 h\gamma^2}\right\rangle_0. \tag{28}$$

When $(\mathbf{S}^{(i)})^2$ is expanded and the results

$$\boldsymbol{\alpha}\cdot(\mathbf{b}\times\boldsymbol{\alpha})=\Delta(\mathbf{1}\times\boldsymbol{\alpha}^{-1})\cdot\mathbf{b}, \tag{29}$$

$$\boldsymbol{\alpha}\cdot(\mathbf{b}\times\boldsymbol{\alpha})\cdot(\mathbf{b}\times\boldsymbol{\alpha})=\Delta(\mathbf{bb}-\alpha^{-1}{}_{\mathrm{bb}}\boldsymbol{\alpha}), \tag{30}$$

are applied, it is found that the only summations are

$$\sum\boldsymbol{\alpha}^{(i)}\delta(E^{(i)}-\zeta)=JN\mathbf{A} \tag{31}$$

and

$$\sum\boldsymbol{\alpha}^{(i)-1}\delta(E^{(i)}-\zeta)=-(\alpha_1/\Delta)JN\mathbf{A}, \tag{32}$$

where

$$\mathbf{A}=\begin{vmatrix} e_x^2-\frac{1}{3} & 0 & 0\\ 0 & e_y^2-\frac{1}{3} & 0\\ 0 & 0 & e_z^2-\frac{1}{3}\end{vmatrix} \quad \text{(Si)}, \tag{33}$$

$$\mathbf{A}=\frac{2}{3}\begin{vmatrix} 0 & e_xe_y & e_xe_z\\ e_xe_y & 0 & e_ye_z\\ e_xe_z & e_ye_z & 0\end{vmatrix} \quad \text{(Ge)}, \tag{34}$$

and

$$J=(\alpha_3-\alpha_1)X\Omega_u S/3. \tag{35}$$

If we define

$$U\equiv\frac{\xi_0}{f\langle E\tau\rangle_0}\left\langle\frac{E\beta(1-\gamma^2\alpha_1 h)}{(1+\gamma^2\alpha_1 h)^2}\right\rangle_0, \tag{36}$$

$$W\equiv\frac{\xi_0}{f\langle E\tau\rangle_0}\left\langle\frac{E\beta\gamma}{(1+\gamma^2\alpha_1 h)^2}\right\rangle_0, \tag{37}$$

and use (31) and (32), the change in conductivity can be written

$$\delta\boldsymbol{\sigma}=JNn_0\left\{-U\mathbf{A}-2W\alpha_1(\mathbf{1}\times\mathbf{A})\cdot\mathbf{b} -\frac{1}{kT}[S_0\mathbf{A}+\alpha_1 S_1(\mathbf{1}\times\mathbf{A})\cdot\mathbf{b}]\right\}. \tag{38}$$

In practice, the resistivity tensors are more useful. The inversion of $\boldsymbol{\sigma}$ gives

$$\boldsymbol{\sigma}^{-1}=\frac{fS_0\mathbf{1}+\alpha_1 hS_1\mathbf{1}\times\mathbf{b}+\dfrac{(\alpha_1 hS_1)^2-f\Delta S_0S_2}{fS_0+\Delta S_2}\mathbf{bb}}{Nn_0[(fS_0)^2+(\alpha_1 hS_1)^2]}, \tag{39}$$

and $\delta(\boldsymbol{\sigma}^{-1})$ can be obtained by straightforward multiplication:

$$\delta(\boldsymbol{\sigma}^{-1})=-\boldsymbol{\sigma}^{-1}\cdot\delta\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}^{-1}. \tag{40}$$

The most practical arrangement from the experimental point of view is one in which the magnetic field is perpendicular to the axial force $\mathbf{e}$, and the current is parallel to $\mathbf{e}$. Measurement of the resistivity is accomplished, then, by measuring the component of the electric field parallel to $\mathbf{e}$. For detailed consideration we choose

$$\mathbf{e}=(1/\sqrt{2})(1\bar{1}0) \tag{41}$$

which satisfies the condition $\mathbf{b}\cdot\mathbf{e}=0$ for both cases. Figure 1 shows the arrangement of sample, fields, and

electrodes for making the elastomagnetoresistance measurement under the conditions being discussed.

Let $\mathcal{E}(B)$ be the electric field in direction $\mathbf{e}$ due to unit current in direction $\mathbf{e}$. Then, in the unstrained state

$$\mathcal{E}(B)=\mathbf{e}\cdot\boldsymbol{\sigma}^{-1}\cdot\mathbf{e}=\frac{1}{Nn_0fS_0(1+r^2)}, \tag{42}$$

where

$$r=a_1hS_1/fS_0. \tag{43}$$

The change in $\mathcal{E}$ when the strain is applied is

$$\delta\mathcal{E}(B)=\mathbf{e}\cdot\delta(\boldsymbol{\sigma}^{-1})\cdot\mathbf{e}$$
$$=-\frac{J\mathbf{e}\cdot\mathbf{A}\cdot\mathbf{e}}{fkT}\frac{1}{Nn_0fS_0}\frac{1}{1+r^2}\left(1+\frac{UkT}{S_0}\right). \tag{44}$$

With the geometry being considered, $\mathbf{e}\cdot\mathbf{A}\cdot\mathbf{e}=\frac{1}{3}$ for Ge and $\frac{1}{6}$ for Si. The ordinary elastoresistance is

$$\frac{\delta\mathcal{E}(0)}{\mathcal{E}(0)}=-\frac{J\mathbf{e}\cdot\mathbf{A}\cdot\mathbf{e}}{fkT}\left(1+\frac{U^0kT}{S_0^0}\right), \tag{45}$$

in which a superscript 0 is used to mean "evaluated with $B=0$." To characterize the elastomagnetoresistance, we calculate

$$G=\left[\frac{\delta\mathcal{E}(B)-\delta\mathcal{E}(0)}{\mathcal{E}(B)-\mathcal{E}(0)}\right]\Big/\frac{\delta\mathcal{E}(0)}{\mathcal{E}(0)}$$
$$=\left[\frac{S_0^0}{S_0}\frac{1+(UkT/S_0)}{1+(U^0kT/S_0^0)}-(1+r^2)\right]\Big/\left[\frac{S_0^0}{S_0}-(1+r^2)\right]. \tag{46}$$

V. HALL EFFECT

The effect of elastic strain on the low-field Hall effect follows by a similar calculation from (23). In fact, all the results of the last section up to (40) are correct to first-order terms in the magnetic field for arbitrary field direction, because the special direction of $\mathbf{b}$ was only used in the evaluation of $\alpha^{-1}{}_{\mathbf{bb}}$. In the measurement of the Hall effect it is also convenient to have $\mathbf{b}\cdot\mathbf{e}=0$, and the current parallel to $\mathbf{e}$. We define $\mathcal{E}_H$ as that part of the electric field component in the direction $\mathbf{b}\times\mathbf{e}$ which is proportional to the magnetic field. Then the result for the elasto-Hall effect can be expressed

$$F=\frac{\delta\mathcal{E}_H/\mathcal{E}_H}{\delta\mathcal{E}(0)/\mathcal{E}(0)}=\left[\frac{f}{h}\frac{1+(2W^0kT/S_1)}{1+(U^0kT/S_0)}-1\right]\frac{\mathbf{b}\cdot\mathbf{A}\cdot\mathbf{b}}{\mathbf{e}\cdot\mathbf{A}\cdot\mathbf{e}}. \tag{47}$$

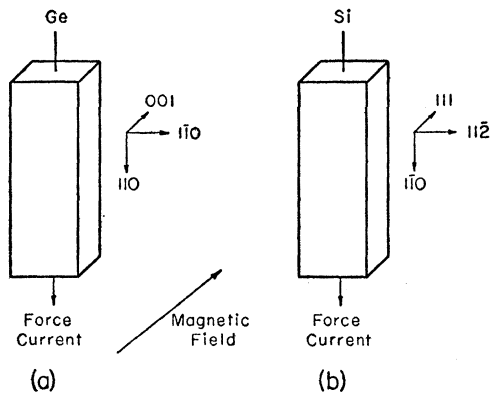

FIG. 1. Sample arrangements which satisfy the conditions for measurement of the EMR function described in the text.

VI. DISCUSSION OF RESULTS

In order to compare formulas (46) and (47) with experimental data, some evaluation of the integrals involved is necessary. A very simple, although somewhat unrealistic, approximation in which the integrations can be easily performed, and which gives some insight into the relationship between the functions F and G, and the IV scattering, is the approximation in which τ_I, τ, and β are regarded as constants over the energy distribution. Let $\nu=\tau/\tau_I$, the fractional amount of IV scattering, and set $N/(N-1)\approx1$. Then, from (11),

$$\beta kT=\tfrac{1}{2}\nu\tau \tag{48}$$

and

$$G=1-\frac{f^2}{f^2-\alpha_1h}\frac{\nu}{1+\frac{1}{2}\nu}, \tag{49}$$

$$F=\frac{f}{h}\frac{1+\nu}{1+\frac{1}{2}\nu}-1. \tag{50}$$

An important characteristic to be noted in these results is the sensitivity of the function G to the strength of the IV scattering. For a mass ratio of 20, which we take as representative of germanium, the factor $f^2/(f^2-\alpha_1h)$ in (49) has the value 4.65. In actual cases, in which dispersion of the relaxation times also contributes to the magnetoresistance, this sensitivity is reduced, but usually remains larger than that of the function F. It should also be noted that, although in the calculation of G all orders of the magnetic field were included, the field exactly cancels in Eq. (49).

VII. EXPERIMENT

Previous work[4,5] on n-type germanium has indicated that IV scattering is not an important factor in limiting the mobility of electrons below room temperature. In an attempt to obtain a more precise estimate of the amount of IV scattering, we have made some measurements of the elastomagnetoresistance (EMR) effect in this material. For this purpose, the EMR has certain advantages over the elasto-Hall effect, one being the greater sensitivity already referred to. In addition, in the Hall measurement, the region of the crystal in which the Hall voltage is determined, i.e., the cross section at the Hall electrodes, is not the same as that in which the potential drop which determines the resistance occurs. Thus, if there is any gradation in

[4] R. W. Keyes, Phys. Rev. **99**, 1655 (1955).
[5] R. W. Keyes, Phys. Rev. **100**, 1104 (1955).

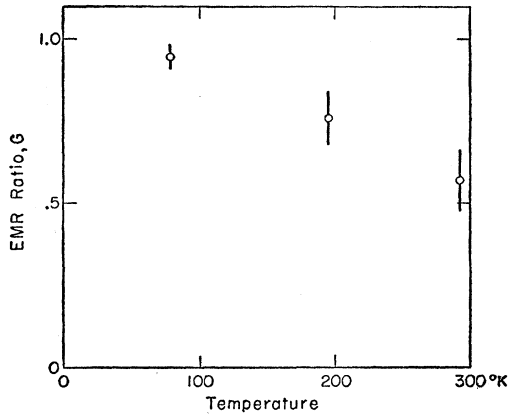

FIG. 2. Experimental values of the EMR ratio, G. The indicated errors represent only the scatter in the data.

sample properties or dimensions, or in the strain pattern, the ratio of EMR to elastoresistance will probably not be affected as much as the ratio of elasto-Hall effect to elastoresistance.

The germanium used for the experiment was n type, and contained 1.7×10^{15} excess electrons per cm^3. The mobility product at 77°K was 27 000 cm^2/volt sec. Contacts were applied to the sample with soft solder. The technique of Smith was followed closely, in both the mechanical features and the electrical instrumentation. Magnetic fields to 3000 gauss at 77°K and to 8000 gauss at 300°K were used. The EMR ratios defined by (46) were independent of field, to an accuracy of about 10%, within these ranges. The measurements were made at three convenient bath temperatures. The results are shown in Fig. 2. The uncertainty due to scattering of the data is also shown.

VIII. COMPARISON OF THEORY AND EXPERIMENT

The data of Fig. 2 indicate the presence of some IV scattering in germanium at room temperatures. In order to make the quantitative estimate of the strength of this scattering which is required, we have calculated by a numerical method the values of the integrals which occur in G in the limit of small magnetic fields for various values of W_I/W_L, the electron-lattice coupling parameters of Eq. (13). It is found that a factor H^2 can be cancelled from Eq. (46), and then, when the magnetic field is set equal to zero,

$$G=1-\frac{\dfrac{kT\langle E\beta\rangle_0}{\langle E\tau\rangle_0}\;\dfrac{3\dfrac{\langle E\beta\tau^2\rangle_0}{\langle E\beta\rangle_0}-\dfrac{\langle E\tau^3\rangle_0}{\langle E\tau\rangle_0}}{\langle E\tau\rangle_0}}{1+\dfrac{kT\langle E\beta\rangle_0}{\langle E\tau\rangle_0}\;\dfrac{\langle E\tau^3\rangle_0}{\langle E\tau\rangle_0}\;\dfrac{\alpha_1 h}{f^2}\;\dfrac{\langle E\tau^2\rangle_0^2}{\langle E\tau\rangle_0^2}}. \tag{51}$$

It has been suggested by Herman[6] that the energy minima lie at the zone edges in the n-type germanium. If this is so, θ should be near the Debye temperature, 362°K according to Keesom and Pearlman.[7] Our highest temperature point then corresponds to $\theta/T=1.25$, and our calculation of G as a function of W_I/W_L for this case is shown in Fig. 3. A comparison with Fig. 2 shows that W_I/W_L lies between 0.1 and 0.2. Measurements of the temperature dependence of electrical properties[5,8] are inconsistent with the higher part of this range, and we therefore feel that W_I/W_L must lie near 0.1 (if $\theta=362$°K). Herring and Vogt[9] have recently concluded that, if the energy minima are at the zone faces, the smallness of the IV scattering is not in disagreement with theoretical understanding of the electronic properties of germanium.

The possibility that the energy minima are not at the zone faces, and that the smallness of the IV scattering at 300°K is due to a high excitation temperature for the IV phonon has also been examined. According to work of Hsieh,[10] the highest energy phonon in germanium corresponds to a θ of about 500°K. We have, therefore, also calculated G as a function of W_I/W_L for $\theta/T=1.8$ and shown this result in Fig. 3. It can be seen by comparison with Fig. 2 that values of W_I/W_L of 0.25 to 0.5 are consistent with our data in this case.

The calculation of G for $\theta/T=1.8$ can also be used to compare our data point at 195°K with the theory under the assumption $\theta=362$°K. This comparison gives the result that W_I/W_L lies between 0.1 and 0.2, in agreement with our result for the 290°K data point and $\theta=362$°K.

Obviously, extension of measurements of this type to higher temperatures in order to confirm the applicability of the model and to fix more precisely W_I/W_L

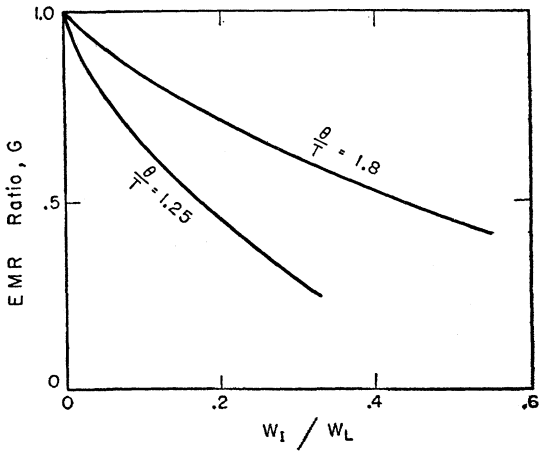

FIG. 3. Calculated values of G as a function of W_I/W_L, the ration of the coupling constants for intervalley and intravalley scattering in the model of Herring.

[6] F. Herman, Phys. Rev. **95**, 847 (1954).

[7] P. H. Keesom and N. Pearlman, Phys. Rev. **91**, 1347 (1953).
[8] F. Morin, Phys. Rev. **93**, 62 (1953).
[9] C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).
[10] Y. Hsieh, J. Chem. Phys. **22**, 306 (1954).

and θ is indicated. However, the use of less pure specimens to extend the exhaustion region of conductivity to higher temperature creates difficulties in the interpretation of the results because of the simultaneous introduction of impurity scattering.

A small amount of impurity scattering can have a fairly large effect on the value of G. When the $\langle E\beta\tau^2\rangle$ integral is evaluated with the relaxation time of Eq. (13) for a case in which there is only a small amount of intervalley scattering, the relaxation time and the integrand become very large near the zero of energy. If some other scattering mechanisms, in particular impurity scattering, reduces the value of τ at low energies, the value of the integral will be seriously reduced. In our germanium we estimate that at 300°K the value of the impurity mobility is 10^6 cm^2/volt sec, and that this much impurity scattering will decrease $\langle E\beta\tau^2\rangle$ by 25% if $W_I/W_L=0.1$. The effect is smaller for smaller values of W_I/W_L. Small amounts of impurity scattering may have a comparable effect[11] on the $\langle E\tau^3\rangle$ integral also, but the total effect of impurity scattering appears to be somewhat less than the other uncertainties at 300°K.

Impurity scattering has a greater effect on the scattering integrals at lower temperatures. However, it is seen from (51) that, as β approaches zero, G approaches 1, regardless of the law of intravalley scattering, and that our conclusion that the intervalley scattering is small does not depend on an accurate evaluation of the integrals. For the specific orientation considered the transfer of electrons from one valley to another produces the same change in the magnetoresistance as in the resistance, and a deviation of G from one means that the elastoresistance effect is not due entirely to the transfer of electrons. The exact quantitative interpretation of the value of G in terms of coupling constants for intervalley scattering does depend on the ratio of various scattering integrals. We expect such ratios to be of order of magnitude 1, and, because their precise values are sensitive to details of the scattering mechanism, an intrepetation of G based on the constant τ approximation (49) would not be entirely unsatisfactory.

[11] H. Brooks, *Advances in Electronics* (Academic Press, Inc., New York, 1955).

CONCLUSIONS

A calculation, based on Herring's model of the elastoresistance effect, shows that the elastogalvanomagnetic effects provide a sensitive measure of the amount of IV scattering in germanium. The amount of IV scattering increases with temperature, due to the increasing excitation of the IV phonon. Experiment shows that, at room temperature, 10 to 20% of the scattering is of the IV type. The scatter in the measurements of the EMR effect, and the uncertainties in other details of the scattering mechanism which may affect the scattering integrals, prevent us from determining accurately the values of θ and W_I/W_L, the parameters of Eq. (13). If the energy minima are at the zone faces, then θ is near 360°, and W_I/W_L is small, about 0.1. If the smallness of the IV scattering at room temperature is attributed to a high excitation temperature for the IV phonon, and θ is assumed to have the highest value consistent with the elastic properties of germanium, 530°K, then W_I/W_L may have a value as high as 0.5. In any case, it appears that the coupling constant for IV scattering is considerably less than that for intravalley lattice scattering.

ACKNOWLEDGMENTS

The author is indebted to A. I. Bennett for the preparation of the samples used in the experiment and to E. N. Adams for discussion of numerous points.