Theory of Cyclotron Resonance Absorption in Many-Valley Semiconductors*

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A general theory of cyclotron absorption, appropriate for energy-dependent scattering processes, is developed for electron diamagnetic resonance in germanium and silicon. An examination of the absorption arising from intravalley lattice scattering for nondegenerate electrons in germanium based on the restriction of an isotropic, energy-independent mean free path is made over temperatures ranging from liquid nitrogen to liquid helium for the resonance associated with the directional combination of magnetic and electric fields **H** [001], **E** [1$\bar{1}$0]. Comparison with the findings associated with the assumption of a constant mean free time reveals that precise absolute power absorption measurements would be required to distinguish between these two mechanisms.

THE original interpretation of cyclotron resonance absorption was made in terms of a constant scattering time τ.[1,2] The purpose here is to give a more general theory of cyclotron resonance absorption that has a wider range of applicability. This is done by removing the restriction of a constant τ and allowing for an energy dependence of the scattering time.

If surfaces of constant energy are also surfaces of constant τ, the integrals appearing in the conductivity tensor for the rf case may be written down directly from those in the dc case[3] by replacing $1/\tau(\mathcal{E})=\nu(\mathcal{E})$ by $\nu(\mathcal{E})+j\omega$, where ω is the applied angular rf frequency, yielding

$$\mathfrak{J}_n{}^i=\int_0^\infty \frac{(\nu+j\omega)^{2-n}\mathcal{E}^{\frac{3}{2}}(\partial f_0/\partial\mathcal{E})d\mathcal{E}}{(\nu+j\omega)^2+\gamma_i{}^2} \tag{1}$$

where $n=1,2$, or 3 and γ_i depends on the magnetic field and the band structure of the material involved.

At the bottom of the conduction band of germanium the surfaces of constant energy are ellipsoids of revolution with their major axes located along the $\langle 111\rangle$ axes in the Brillouin zone.[1,2] The eccentricity of the ellipsoids is denoted by $K=m_l/m_t$, the ratio of the longitudinal to the transverse mass. In the customary arrangement for observing cyclotron resonance, the rf electric field **E**, linearly polarized, is directed along a $\langle 110\rangle$ axis and the magnetic field **H** is rotated in a plane perpendicular to this axis. When $H_1=H_2=(H\sin\theta)/\sqrt{2}$ and $H_3=H\cos\theta$, where θ is the angle between **H** and the [001] axis (H is rotating in the $(1\bar{1}0)$ plane), the components of the conductivity tensor σ may be written[4]:

$$\begin{aligned}
\sigma_{11}&=\sigma_{22}=g[\Sigma_1+\tfrac{1}{2}yh^2H^2K^2\sin^2\theta\Sigma_3],\\
\sigma_{33}&=g[\Sigma_1+yh^2H^2K^2\cos^2\theta\Sigma_3],\\
\sigma_{12,21}&=g\{t(2\mathfrak{J}_1{}^c-\mathfrak{J}_1{}^a-\mathfrak{J}_1{}^b)+\tfrac{1}{2}yh^2H^2K^2\sin^2\theta\Sigma_3\\
&\qquad\mp hHK[\sqrt{2}t\sin\theta(\mathfrak{J}_2{}^a-\mathfrak{J}_2{}^b)+z\cos\theta\Sigma_2]\},\\
\sigma_{23,32}&=\sigma_{31,13}=g\{t(\mathfrak{J}_1{}^b-\mathfrak{J}_1{}^a)\\
&\qquad+(1/\sqrt{2})yh^2H^2K^2\sin\theta\cos\theta\Sigma_3\\
&\qquad\mp hHK[(1/\sqrt{2})\sin\theta(\mathfrak{J}_2{}^a+\mathfrak{J}_2{}^b+2y\mathfrak{J}_2{}^c)\\
&\qquad\qquad+t\cos\theta(\mathfrak{J}_2{}^a-\mathfrak{J}_2{}^b)]\}.
\end{aligned}\tag{2}$$

In these equations the first pair of subscripts refers to the upper sign and the second pair to the lower,

$$t=(K-1)/(2K+1),\quad y=3/(2K+1),$$
$$z=(K+2)/(2K+1),\quad g=-2^{\frac{3}{2}}e^2m_t/3y\pi^2\hbar^3m_l^{\frac{1}{2}},$$
$$h=e/m_l,\quad \Sigma_n=\mathfrak{J}_n{}^a+\mathfrak{J}_n{}^b+2\mathfrak{J}_n{}^c,$$
$$\gamma_a{}^2=\tfrac{1}{3}h^2KH^2[(K+2)+(K-1)(\sin^2\theta+\sqrt{2}\sin2\theta)],$$
$$\gamma_b{}^2=\tfrac{1}{3}h^2KH^2[(K+2)+(K-1)(\sin^2\theta-\sqrt{2}\sin2\theta)],$$

and

$$\gamma_c{}^2=\tfrac{1}{3}h^2KH^2[(K+2)-(K-1)\sin^2\theta].$$

For the electric field in the $[1\bar{1}0]$ axis $E_1=-E_2=E/\sqrt{2}$, $E_3=0$ and the absorbed power $P=\mathrm{Re}\,\mathbf{E}\cdot\mathbf{J}=\mathrm{Re}\,\mathbf{E}\cdot\sigma\cdot\mathbf{E}$ is given by

$$\begin{aligned}
P&=\mathrm{Re}\tfrac{1}{2}(\sigma_{11}+\sigma_{22}-\sigma_{12}-\sigma_{21})E^2\\
&=\mathrm{Re}\,g[(\mathfrak{J}_1{}^a+\mathfrak{J}_1{}^b)(1+t)+2\mathfrak{J}_1{}^c(1-t)].
\end{aligned}\tag{3}$$

Restricting the magnetic field to the [001] direction reduces all the $\gamma_i{}^2$ to $\frac{1}{3}h^2H^2K(K+2)\equiv\gamma$, so $\mathfrak{J}_n{}^a=\mathfrak{J}_n{}^b=\mathfrak{J}_n{}^c\equiv\mathfrak{J}_n$ and the absorbed power $P_{001}=4gE^2\,\mathrm{Re}\mathfrak{J}_1$. If the collision frequency ν varies with the energy as $\epsilon^{\frac{1}{2}}/l$, where l is an isotropic energy-independent generalized mean free path (an approximation for the case of intravalley lattice scattering in the acoustic mode[5]) and if Boltzmann statistics apply, it is shown in the

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[1] Lax, Zeiger, and Dexter, Physica **20**, 818 (1954).

[2] Dresselhaus, Kip, and Kittel, Phys. Rev. **98**, 368 (1955).

[3] The integrals for the dc case derive from the Boltzmann transport equation as shown in B. Abeles and S. Meiboom [Phys. Rev. **95**, 31 (1954)] but without restricting the energy dependence of τ. Extension to the ac case involves putting G of Eqs. (2) and (3) in M. Shibuya [Phys. Rev. **95**, 1385 (1954)] equal to $G_0e^{j\omega t}$.

[4] These may be deduced from Abeles and Meiboom (reference 3) by making suitable generalizations.

[5] M. Shibuya, reference 3.

appendix that

$$P_{001}/P_0=\tfrac{1}{2}[\alpha(w_1)+\alpha(w_2)], \qquad (4)$$

where $\alpha(\xi)=1-\xi^2-\xi^4\exp(\xi^2)\,\mathrm{Ei}(-\xi^2)$, $w_{1,2}=l(kT)^{-\frac{1}{2}}\times(\omega\pm\gamma)$, and $P_0=\sigma_0E^2$, P_0 being the absorbed power for $H=0$ and $\omega=0$. The critical condition which admits resonance cannot be put into as simple a form as in the constant τ theory where $\omega\tau>1$ is required.[1] However, this condition may be transformed to a relation containing mobility[2] $\mu>\mu_c=e/\omega m^*$. If $\omega/2\pi=8900$ Mc/sec and $m^*=0.12\,m_0$, $\mu_c\approx2.6\times10^5$ cm^2/volt sec. The mobility $\mu=\sigma_0R_\infty$ may be related to the generalized mean free path l by using the relations of Abeles and Meiboom[3]:

$$l=(kT)^{\frac{1}{2}}\left(\frac{9\sqrt{\pi}}{4}\right)\left(\frac{m_l}{e}\right)\left(\frac{1}{2K+1}\right)\mu. \qquad (5)$$

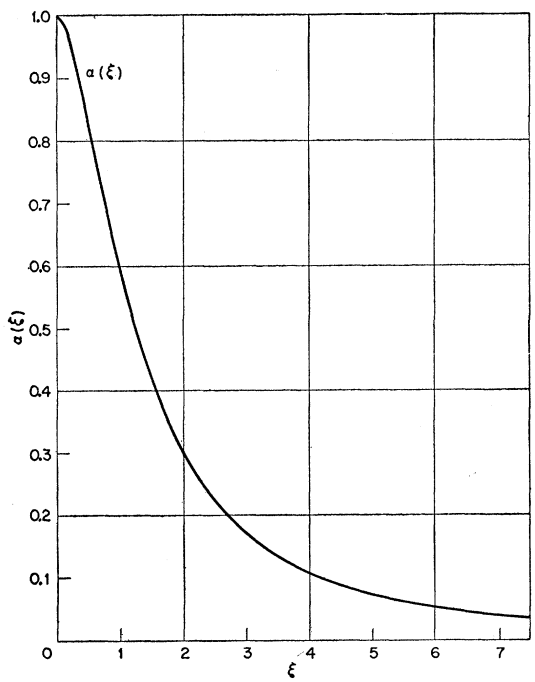

FIG. 1. Behavior of the function $\alpha(\xi)=1-\xi^2-\xi^4\exp(\xi^2)\,\mathrm{Ei}(-\xi^2)$. Appearance of resonance may be understood since $\alpha(w_2)\rightarrow1$ while $\alpha(w_1)$ remains essentially constant as $w_2\rightarrow0$, w_1 remains large.

The highly transcendental character of the relation (4) makes graphical presentation an appropriate means for studying the situation in more detail. A plot of $\alpha(\xi)$ is given in Fig. 1. Existence of a resonance point can be seen by considering the arguments in Eq. (4). Resonance will occur at $\omega=\gamma$ since here w_2 becomes zero and $\alpha(w_2)$ reaches a maximum. If this were the only term involved, the resonance peak would be symmetrical. The asymmetry present enters through the term $\alpha(w_1)$ and is more pronounced as $w_1(2\omega)$ decreases. When $w_1(2\omega)$ is large, say $\geqslant2.5$, the maximum will be sharp and only slightly displaced from the resonant field.

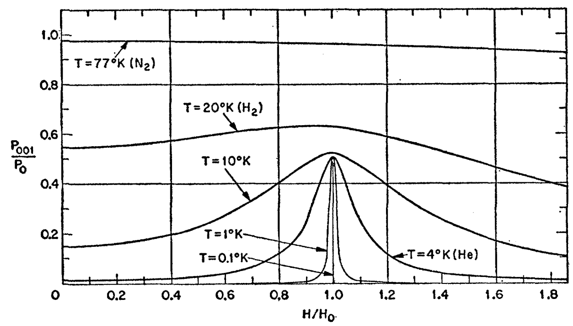

FIG. 2. The relative power absorption for **H** [001] and **E** [$1\bar{1}0$] characteristic of n-type germanium as a function of the normalized magnetic field, assuming an isotropic energy-independent mean free path. Temperatures are as shown and the mobility is assumed to be $1.9\times10^7T^{-\frac{3}{2}}$ cm^2/volt sec. For the case considered, $\omega/2\pi=8900$ Mc/sec and $H_0=440$ oersteds.

The absorbed power over a range of temperature is plotted in Fig. 2 against the normalized magnetic field H/H_0 for an idealized sample of n-type germanium with a mobility[6] of $1.9\times10^7T^{-\frac{3}{2}}$ cm^2/volt sec. The mass constants[2] were taken as $m_l=1.6m_0$ and $K=19$, yielding $w_{1,2}=1.8\times10^{-9}T^{-\frac{3}{2}}(\omega\pm\gamma)$. At 20°K, $w_1(2\omega)=2.26$ and $\mu=2.1\times10^5$ cm^2/volt sec placing this value very near the critical point for resonance as Fig. 2 indicates.

For comparison, the absorbed power is plotted in Fig. 3 for various values of an energy-independent τ, chosen to be equivalent to the temperatures in Fig. 2 using the relation $\mu=e\tau/m^*$. It can be seen that there are differences in the line shape and, in the nonresonant region, differences in the absolute magnitudes at the maxima as well. In principle, therefore, absorption curves arising from these two types of scattering could be differentiated particularly in the nonresonant region. However, the difficulty of obtaining absolute measurements seems to rule out such differentiation. Until absolute measurements can be made with the required precision, the procedure of arbitrarily letting the scatter-

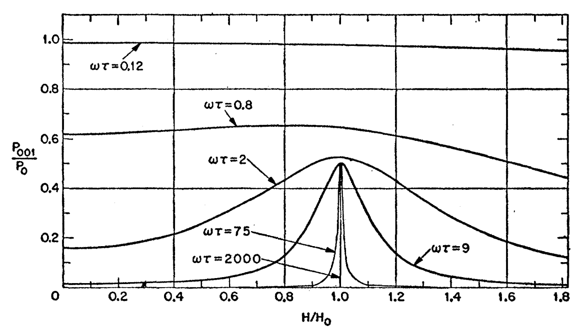

FIG. 3. The relative power absorption characteristic of n-type germanium, assuming an isotropic energy-independent mean free time as a function of the normalized magnetic field **H** [001] with **E** [$1\bar{1}0$]. The values of $\omega\tau$ shown are related to the temperatures of Fig. 2 through the relation $\mu=e\tau/m^*$. Other conditions are as in Fig. 2.

[6] W. Shockley, *Electrons and Holes in Semiconductors* (D. Van Nostrand Company, Inc., New York, 1950), p. 287.

ing time τ vary with the temperature in such a way as to fit the line shapes seems to be as practical an approach as any other.

APPENDIX

For the magnetic field in the [001] direction, the absorbed power is:

$$P_{001}=4gE^2\,\mathrm{Re}\int_0^\infty \frac{(\nu+j\omega)\,\mathcal{E}^{\frac{3}{2}}(\partial f_0/\partial\mathcal{E})d\mathcal{E}}{(\nu+j\omega)^2+\gamma^2}. \quad \text{(A-1)}$$

If one assumes that Boltzmann statistics apply and that the scattering frequency may be represented by $\nu=l^{-1}\mathcal{E}^{\frac{1}{2}}$, the integral of Eq. (A-1) becomes:

$$\mathrm{Re}\mathfrak{J}_1=\mathrm{Re}\frac{\exp(\mathcal{E}_F/kT)}{kT}\int_0^\infty \times\frac{(l^{-1}\mathcal{E}^2+j\omega\mathcal{E}^{\frac{3}{2}})\exp(-\mathcal{E}/kT)d\mathcal{E}}{l^{-2}\mathcal{E}-\omega^2+\gamma^2+j(2\omega\mathcal{E}^{\frac{1}{2}}l^{-1})}. \quad \text{(A-2)}$$

After taking the real part and collecting terms, $\mathrm{Re}\mathfrak{J}_1$ becomes

$$\mathrm{Re}\mathfrak{J}_1=Al\int_0^\infty \times\frac{x^2[x+(l^2/kT)(\omega^2+\gamma^2)]e^{-x}dx}{[x+(l^2/kT)(\omega^2+\gamma^2)]^2-(2l^2\omega\gamma/kT)^2}, \quad \text{(A-3)}$$

with the substitutions $x=\mathcal{E}/kT$ and $A=-(kT)\times\exp(\mathcal{E}_F/kT)$. Splitting into partial fractions yields:

$$\mathrm{Re}\mathfrak{J}_1=-\frac{Al}{2}\int_0^\infty \frac{x^2e^{-x}dx}{x+(l^2/kT)(\omega+\gamma)^2} +\frac{Al}{2}\int_0^\infty \frac{x^2e^{-x}dx}{x+(l^2/kT)(\omega-\gamma)^2}. \quad \text{(A-4)}$$

Defining $w_1^2=(l^2/kT)(\omega+\gamma)^2$ and $w_2^2=(l^2/kT)(\omega-\gamma)^2$, we have

$$\mathrm{Re}\mathfrak{J}_1=-\frac{Al}{2}\left[2\int_0^\infty xe^{-x}-(w_1^2+w_2^2)\int_0^\infty e^{-x}dx +w_1^4\int_0^\infty\frac{e^{-x}dx}{x+w_1^2}+w_2^4\int_0^\infty\frac{e^{-x}dx}{x+w_2^2}\right]. \quad \text{(A-5)}$$

With the substitution $t=x+w_k^2$ in the last two terms of Eq. (A-5) these terms become $-w_k^4\exp(w_k^2)\times\mathrm{Ei}(-w_k^2)$, where[7] $\mathrm{Ei}(-\xi)$ is the exponential integral

$$-\mathrm{Ei}(-\xi)=\int_\xi^\infty \frac{e^{-t}}{t}dt.$$

Thus

$$\mathrm{Re}\mathfrak{J}_1=\tfrac{1}{2}Al[\alpha(w_1)+\alpha(w_2)], \quad \text{(A-6)}$$

where $\alpha(\xi)=1-\xi^2-\xi^4\exp(\xi^2)\,\mathrm{Ei}(-\xi^2)$. Thus the absorbed power becomes

$$P_{001}=4g\times\tfrac{1}{2}AlE^2[\alpha(w_1)+\alpha(w_2)], \quad \text{(A-7)}$$

or in terms of $P_0=\sigma_0E^2$, the absorbed power for $H=0$, and $\omega=0$:

$$P_{001}/P_0=\tfrac{1}{2}[\alpha(w_1)+\alpha(w_2)]. \quad \text{(A-8)}$$

Clearly the existence and position of the resonance requires study of Eq. (A-5). Solution of the derivative of this with respect to γ set equal to zero involves a highly transcendental equation and rather than graphical evaluation of the extremal properties, it is just as well to plot (A-5) or its equivalent (A-8). If only the resonant term, $\alpha(w_2)$, is considered, it can be shown analytically that a maximum or resonance point occurs when $w_2=0$.

[7] See, for example, E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1945), fourth edition, p. 1.