with σ_f' taken to be constant, there results

$$\lambda_0 = N\sigma_f' v_{\rm thr}\eta_{\rm thr}^{-\frac{1}{2}} \frac{\int_{\eta_{\rm thr}}^{\infty} \eta^{\frac{1}{2}}(\eta-\eta_{\rm thr})^{\frac{1}{2}} \exp(-\eta^2)d\eta}{\int^{\infty} \eta^{\frac{1}{2}} \exp(-\eta^2)d\eta}.$$

The denominator is readily integrated to yield $\frac{1}{2}\Gamma(\frac{3}{4})$. However, the integral in the numerator must be approximated. Putting $\xi=\eta-\eta_{\rm thr}$, the numerator becomes

$$\int_{\eta_{\rm thr}}^{\infty} \eta^{\frac{1}{2}}(\eta-\eta_{\rm thr})^{\frac{1}{2}} \exp(-\eta^2)d\eta$$
$$=\exp(-\eta_{\rm thr}^2)\int_0^{\infty} (\xi+\eta_{\rm thr})^{\frac{1}{2}}\xi^{\frac{1}{2}} \exp(-2\eta_{\rm thr}\xi-\xi^2)d\xi,$$
$$\approx \eta_{\rm thr}^{-1}\exp(-\eta_{\rm thr}^2)\Gamma(\tfrac{3}{4})/2\sqrt{2},$$

where it may be noted that the major contribution to the integral occurs for values of η in the range

$$\eta_{\rm thr} \leq \eta \lesssim \eta_{\rm thr}+1/2\eta_{\rm thr},$$

and the approximations

$$\exp(-\xi^2)\approx 1, \quad (\xi+\eta_{\rm thr})^{\frac{1}{2}} \approx \eta_{\rm thr}^{\frac{1}{2}},$$

were used. This is completely adequate for the present purpose.

Using the values of the integrals indicated, and recalling that $\lambda_a = N\mathfrak{z}\pi r_0^2 c$, there results for the increase in positronium formation I in the present approximation

$$I=\frac{\lambda_0}{\lambda_0+\lambda_a}$$
$$=\frac{1.3\times10^6\eta_{\rm thr}^{-\frac{3}{2}}\exp(-\eta_{\rm thr}^2)}{1.3\times10^6\eta_{\rm thr}^{-\frac{3}{2}}\exp(-\eta_{\rm thr}^2)+(\mathfrak{z}/\sigma_f')(E_{\rm thr}/{\rm ry})^{-\frac{1}{2}}}$$

with σ_f' in units of πa_0^2. Figure 5 shows this result plotted *vs* the dimensionless field parameter $\epsilon=\eta_{\rm thr}^{-1}$ for a number of different values of $(\sigma_f'/\mathfrak{z})(E_{\rm thr}/{\rm ry})^{\frac{1}{2}}$.

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Density Effect on the Ionization in Gases by Electrons*

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The density effect on the ionization produced by electrons has been observed by comparing their specific ionization in He and H_2 gases at one and ten atmospheres pressure. The measurements were made by sending primary electrons of energy variable from 2–35 Mev through an ionization chamber into a Faraday cup. No density effect is observed or expected at one atmosphere pressure at these energies, but at ten atmospheres pressure, the results indicate the onset of the density effect at about 10 Mev in H_2 and 20 Mev in He. At 35 Mev, the percentage reduction in ionization from that expected without density effect amounts to (8.0 ± 1) in H_2 and (3.5 ± 1.3) in He. These results are in near agreement with the calculations of Sternheimer.

I. INTRODUCTION

FOR some time, it has been known[1,2] that the mass stopping power of any material for charged particles should be dependent on the density of the material. This dependence arises when the particle velocity approaches c and the relativistic extension of the particle's transverse field is affected by the polarization of the surrounding material. This effect has been demonstrated as influencing the energy loss of particles in solids,[3] the ionization produced in gases,[4,5] the grain density in nuclear emulsions,[6] and the light output of scintillators.[7]

In gases at atmospheric pressure, the density effect begins to occur when the kinetic energy of the charged particle is about 100 times its rest energy. In order to demonstrate the effect at lower particle energy, it is necessary to increase the gas pressure. The theory of the effect[8,9] indicates that the correction to the rate of energy loss depends on the gas pressure P and particle momentum p only as a function of the product $(pP^{\frac{1}{2}})$.

The measurements described in this paper were made with an ionization chamber at 10 atmospheres pressure using electrons of up to 35-Mev energy. The observed

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[1] W. F. G. Swann, J. Franklin Inst. **226**, 598 (1938).
[2] E. Fermi, Phys. Rev. **57**, 485 (1940).
[3] Goldwasser, Mills, and Hanson, Phys. Rev. **88**, 1137 (1952).
[4] Perry, Rathgeber, and Rouse, Proc. Phys. Soc. (London) **A66**, 541 (1953).
[5] Ghosh, Jones, and Wilson, Proc. Phys. Soc. (London) **A67**, 331 (1954).
[6] J. R. Fleming and J. J. Lord, Phys. Rev. **92**, 511 (1953).
[7] Theodore Bowen, Phys. Rev. **96**, 754 (1954).
[8] R. M. Sternheimer, Phys. Rev. **88**, 851 (1952).
[9] R. M. Sternheimer, Phys. Rev. **91**, 256 (1953).

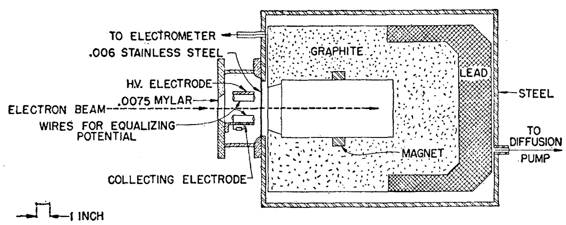

FIG. 1. Schematic drawing of the Faraday-cup beam collector showing the high-pressure ion chamber in front of the cup.

density effects should be the same as would attain at atmospheric pressure with electrons of up to 100 Mev.

II. APPARATUS

The methods and apparatus used were essentially the same as described in an earlier paper on specific ionization by electrons.[10] The primary electrons were accelerated in the Stanford 35-Mev linear accelerator, then formed into an energy-analyzed beam of $\frac{1}{2}$-in. by 1-in. rectangular cross section. This beam was then passed through an ionization chamber into a Faraday cup. (Figure 1 shows the configuration employed.) The ratio of the collected ionic charge to the charge collected in the Faraday cup is then proportional to the specific ionization. The details concerning the primary energy calibration, the method of measuring the accumulated charges, the tests indicating the reliable performance of the Faraday cup, and the various corrections to the observed data are given in reference 10. The only important change in the apparatus was the use of thicker windows, capable of withstanding the higher pressure, in the ionization chamber. These windows (0.0075-in. stainless steel, 4 in. in diameter at the exit) resulted in appreciable scattering or absorption of the low-energy primary particles. An experimental evaluation of this effect was made by measuring at atmospheric pressure with the thick window chamber and comparing these results with the previous atmospheric pressure measurements made with the same ion chamber fitted with thin windows.[10]

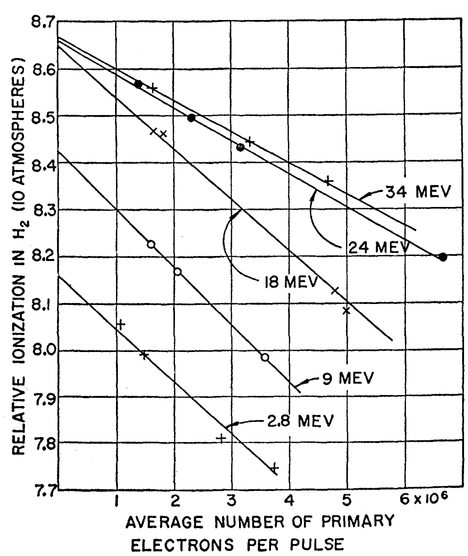

FIG. 2. Ratio of the collected ion charge to collected Faraday-cup charge (in arbitrary units) as a function of the average primary beam intensity.

[10] W. C. Barber, Phys. Rev. 97, 1071 (1955).

The recombination of ions in the gas was an important effect which limited the precision of the experiment. In order to minimize this effect, the ion density was kept as low as possible by employing gases of low electronic density (H_2 and He) and primary beams of large cross-sectional area and low intensity. A correction for the recombination was evaluated by observing the ratio of ion charge to Faraday cup charge as a function of primary beam intensity and extrapolating the resulting curves (straight lines) to zero beam intensity.

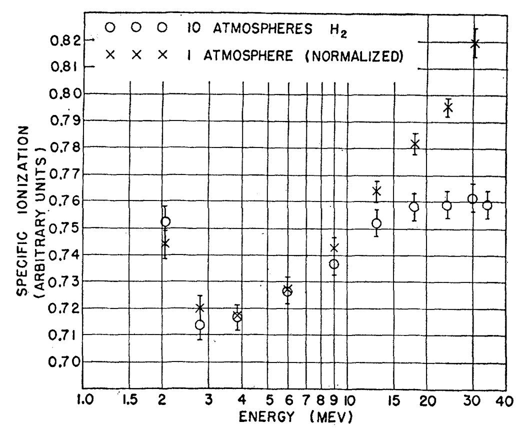

FIG. 3. Relative specific ionization in H_2 at one and ten atmospheres pressure as measured with the thick-window ion chamber shown in Fig. 1. The data were normalized to give agreement in the region from 2 to 6 Mev.

Figure 2 shows a few of these curves as measured under the best conditions. The extrapolations from the measured points of lowest intensity amount to 2 or 3% with possible errors of about 1%.

The necessity of avoiding gas "breakdown" inside the chamber limited the voltage which could be applied to 10 kv in the case of H_2 and 5 kv in the case of He. It was found that the long-term reproducibility of the measurement on H_2 was better than on He.

All measurements were made with the gases flowing through the ion chamber at about 1 cc/sec. The H_2 gas was electrolytic H_2 (supplied by the Stuart Oxygen Company) which was passed through a liquid N_2 trap to remove condensable vapors. The He gas had been mass spectrometer checked to be 99.99% pure. No further trapping or purification was used.

The gas pressure was measured with a barometer and a calibrated gauge. The experiments were all conducted at a uniform temperature of 23°C. The 10-

atmosphere data were taken at 145 psi (gauge) which, to 1% accuracy, gave the same number of atoms/cm^3 as at 0°C and 10 atmospheres absolute pressure.

III. RESULTS

Figures 3 and 4 show experimental data taken with H_2 and He by using the thick window chamber at both one and ten atmospheres pressure. The ordinate scales are arbitrary and the high- and low-pressure data have been normalized to correspond in the region from 2–6 Mev. The errors indicated on the figures are standard deviations calculated from the reproducibility of the measurements. Because the large beam cross section made the effect of the fringing fields of the ion chamber rather uncertain, and because the control on gas purity was not particularly good, absolute specific-ionization measurements were not attempted. However, the observed increase with pressure agreed with that theoretically calculated from the ratio of gas density and the change in secondary cut-off energy, T_0, to about 5%.

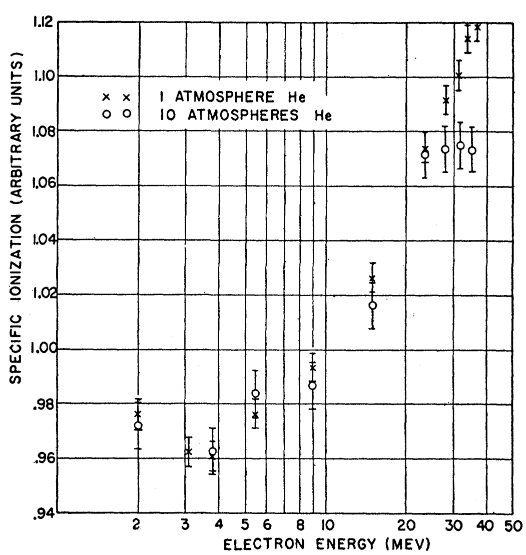

FIG. 4. Relative specific ionization in He at one and ten atmospheres pressure as measured with the thick-window ion chamber. The data were arbitrarily normalized to give the best average agreement in the region from 2 to 9 Mev.

Figures 5 and 6 show data used to evaluate the effect of the thick windows. The effect of the windows is chiefly due to single scattering whose cross section is nearly proportional to $1/E_0^2$. Therefore, the measured values of the thick-window ionization were multiplied by a factor $A(1-B/E_0^2)$, with A and B empirically chosen to give agreement with the thin-window data. With E_0 measured in Mev, the chosen value for B was 0.28. The value of A was near unity and was chosen to give agreement with the specific ionization measurements reported in reference 10.

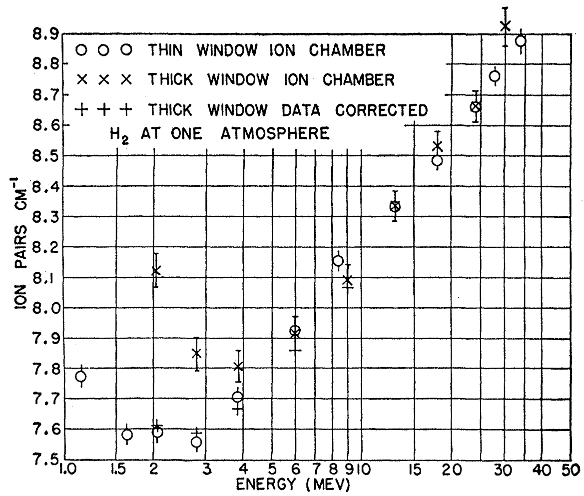

FIG. 5. Illustration of the effect of the thick windows on the specific ionization in H_2 observed with the apparatus shown in Fig. 1. The data were arbitrarily normalized at high energies. A window correction proportional to $1/E_0^2$ was applied to give the agreement indicated at low energies.

Figures 7 and 8 show a comparison of the specific ionization with theoretical curves of the expected rate of energy loss due to ionization and excitation. The theoretical curves show the expected density effect at 10 atmospheres pressure as given by Sternheimer.[8,9] The so-called "probable" energy loss

$$\frac{1}{\rho}\left(\frac{dE}{dx}\right)_{\text{probable}}=\frac{2\pi ne^4}{mv^2\rho}\left[\ln\frac{2mv^2T_0}{I^2(1-\beta^2)}-\beta^2-\delta\right], \quad (1)$$

which is Eq. (43) of reference 9, has been used for comparison with the ionization chamber measurements

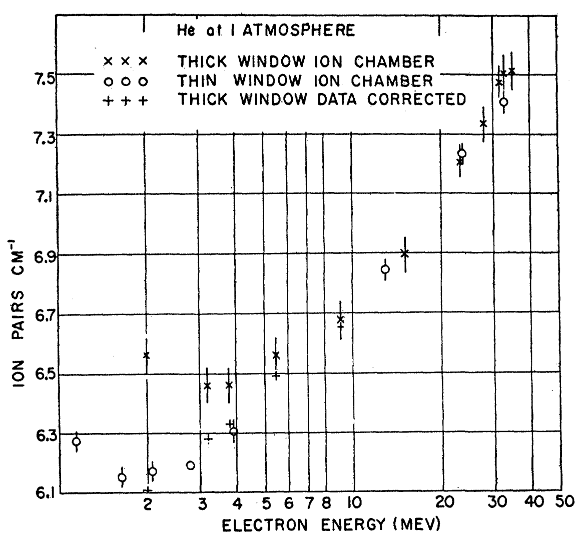

FIG. 6. Comparison of the measurements on He with thick and thin windows on the ion chamber. The same window correction was used as in the case of H_2 (Fig. 5).

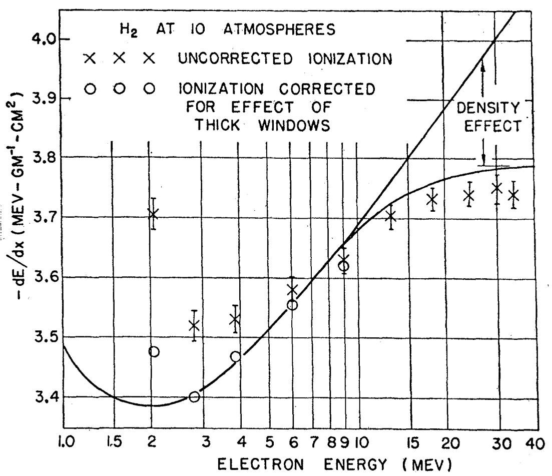

FIG. 7. Comparison of the specific ionization measurements in ten atmospheres of H_2 (crosses and circles) with the theoretically expected energy loss in H_2 (solid curve). The lower branch of the solid curve takes into account the expected density effect at ten atmospheres pressure while the upper branch is the expected energy loss without density effect. The points were arbitrarily normalized to fit the solid curve in the region from 2.8 to 6 Mev.

because the size and gas content of the chamber effectively impose a cut-off energy, T_0, on the secondary electrons which are fully detected. Equation (1) represents the rate of energy loss not including the losses in collisions where energies greater than T_0 are given to the secondary electron. It is valid for all particles, provided T_0 is sufficiently less than the primary energy. The δ represents the density effect and depends on the dielectric properties of the medium. It depends on primary momentum and pressure only through the product $(pP^{\frac{1}{2}})$. In evaluating expression (1), I was taken as 18 ev for H_2 and 27 ev for He. The values of T_0 were calculated to be 56 kev for the chamber filled with 10 atmospheres of H_2 and 51 kev with He.

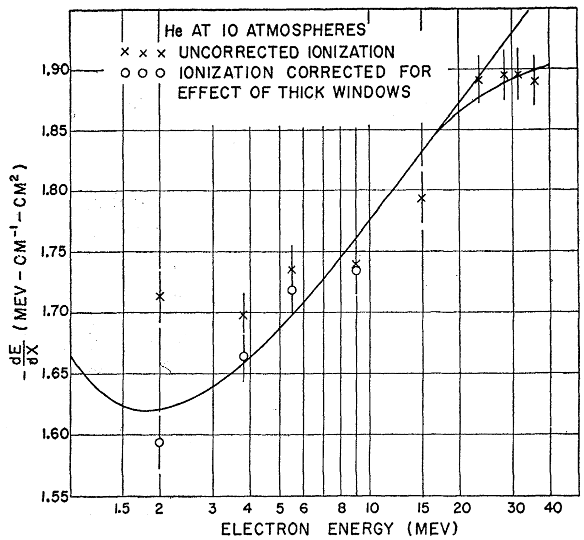

FIG. 8. Comparison (analogous to Fig. 7) of the measurements in ten atmospheres of He with the theoretically expected energy loss in He.

IV. CONCLUSIONS

The direct comparison of the one- and ten-atmosphere data (Figs. 3 and 4) shows clear evidence for a leveling off in the relativistic rise in ionization as is predicted by the density effect. This result is independent of possible errors in the window correction discussed above. The density effect is observed at lower energies in H_2 than in He, a result which is expected as a consequence of the tighter binding of the electrons in He. Comparisons of the data with the theory of energy loss (Figs. 7 and 8) show agreement within experimental error in the case of He and nearly so in the case of H_2. At 35 Mev the calculations of Sternheimer predict a reduction in energy loss due to the density effect of 6.5% and 3% in H_2 and He, respectively. The corresponding experimentally observed reductions are $(8\pm1)\%$ in H_2 and $(3.5\pm1.3)\%$ in He.