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Ratios of Transition Probabilities between Rotational States in Odd-A Nuclei*

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A γ-coincidence method has been developed for measuring relative electric excitation cross sections of the first two excited levels in odd-A nuclei. This method has been applied to eight nuclei in the region $63 \leq Z \leq 75$. In general the results agree very well with the strong-coupling collective-model theory, indicating that in any one nucleus Q is constant to within about 20%. The deviations, though small, are outside the limits of experimental error for a number of nuclei.

INTRODUCTION

IN the collective model theory of nuclear structure the electric quadrupole moments of nuclei, static and dynamic, are given in terms of two parameters, one of which is customarily taken as the ground state spin and the other as the quantity defined by Bohr and Mottelson as intrinsic quadrupole moment.[1] It is well known that in many nuclei the spins of the low-lying levels as well as the relative level spacings are predicted correctly and accurately by this theory. Once the spins and positions of the energy levels in any nucleus are known, the theory gives the static and dynamic quadrupole moments in an unambiguous manner. The values of these moments are found to differ considerably from the values predicted from the level spacing. Discrepancies in Q^2 as large as factors of six are not uncommon.[2,3] However, in some odd-A nuclei electric excitation cross sections have been measured for both the first excited state and the second excited state; and the ratio of the $B(E2)$'s thus obtained is in fair agreement with the collective model value.[4] The present work is an attempt to measure this ratio of $B(E2)$'s systematically and accurately in the region $Z=63$ to $Z=75$ where the rotational level scheme is well developed. The improved accuracy of the method is obtained by measuring the ratio of electric excitation cross sections directly and simultaneously rather than by measuring two cross sections separately, in which case the ratio contains an accumulation of two independent errors.

EXPERIMENTAL METHOD

The level scheme in an odd-A rotational nucleus is shown in Fig. 1, where K is a half-integer (in all the nuclei investigated here $K \neq \frac{1}{2}$). The $K+1$ level decays by means of photons and conversion electrons of both $E2$ and $M1$ types. The $K+2$ level decays: (a) by the cascade transition ($K+2 \rightarrow K+1 \rightarrow K$), where ($K+2 \rightarrow K+1$) is again an $E2$-$M1$ mixture, and (b) by a direct ($K+2 \rightarrow K$) transition which is pure $E2$. The electric excitation of both levels depends, however, almost entirely on the $E2$ moments. In the present work, as in most experiments to date, the excitation cross section is determined from the number of de-excitation processes observed. For the $K+2$ state, evaluation of the cross section requires knowledge of both cascade and crossover transitions. However, in many nuclei the cascade is the predominant mode of decay, and will generally be so whenever the $M1/E2$ ratio for the ($K+2 \rightarrow K+1$) transition is considerably greater than unity. In these nuclei the ratio of the number of cascade transitions ($K+2 \rightarrow K+1 \rightarrow K$) to the number of direct transitions ($K+1 \rightarrow K$) when bombarded by charged particles of some given energy, is directly related to the ratio $B(E2, K \rightarrow K+2)/B(E2, K \rightarrow K+1)$. The simultaneous measurement of this "cascades-to-singles" ratio is the essential feature of this method. The influence of the crossover transition is considered as a comparatively small correction and this method of evaluating $E2$ moments will be the more accurate the bigger the $M1$ percentage in the $M1+E2$ transition ($K+2 \rightarrow K+1$).

The ratio "cascades to singles" is measured by viewing the target with two γ counters, one counting ($K+2 \rightarrow K+1$) photons and the other ($K+1 \rightarrow K$) photons. If the latter counts N_s sec^{-1} and the number of coincidences in the two counters is N_c sec^{-1}, then the ratio "cascades-to-singles" is given by $(N_c/N_s)(1/\eta)$, where η is the over-all counting efficiency for ($K+2 \rightarrow K+1$) γ rays.

Under the conditions of these experiments, the electric excitation is essentially of the pure $E2$ type. The cross section for this process for any given energy level is given by[5]:

$$\sigma(E) = \frac{2m(E-E_0)}{Z_2^2 e^2 \hbar^2} B(E2) f\{\eta, \xi(E)\}, \tag{1}$$

where E is the incident particle energy in the center-of-mass system, E_0 is the excitation energy of the nucleus, and the other notation is taken from reference 5.

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[1] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 27, No. 16 (1953).
[2] H. Mark and G. T. Paulissen, Phys. Rev. 100, 813 (1955).
[3] K. W. Ford, Phys. Rev. 95, 1250 (1954); A. W. Sunyar, Phys. Rev. 98, 653 (1955).
[4] A. Bohr, *Rotational States of Atomic Nuclei* (Ejnai Munksgaards Forlag, Copenhagen, 1954).
[5] K. Alder and A. Winther, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 29, No. 19 (1955).

The yield per incident particle of excited nuclei in a target of finite thickness ΔE is given by

$$Y=\int_{E-\Delta E}^{E}\sigma(E')\frac{n}{\kappa}dE', \qquad (2)$$

where n is the number of target atoms per cm^3 and κ is the proton energy loss per cm, dE/dx.

The ratio of $B(E2)$'s for the two excited levels can be expressed in the following way:

$$\frac{B(E2, K\rightarrow K+2)}{B(E2, K\rightarrow K+1)}=r\left\{\frac{N_c}{N_s}\frac{1}{\eta\epsilon_1}\right\}\frac{1+(\epsilon_1/\epsilon_2)\nu}{1-[(N_c/N_s)(1/\eta\epsilon_1)]}; \qquad (3)$$

here:

$$r=\frac{\displaystyle\int_{E-\Delta E}^{E}f\{\xi^{K+1}(E')\}\frac{E'-E_{K\rightarrow K+1}}{\kappa}dE'}{\displaystyle\int_{E-\Delta E}^{E}f\{\xi^{K+2}(E')\}\frac{E'-E_{K\rightarrow K+2}}{\kappa}dE'},$$

where E is the proton energy, ξ^{K+1} and ξ^{K+2} refer to the transitions $(K\rightarrow K+1)$, $(K\rightarrow K+2)$, respectively, ΔE is the target thickness, ν is the branching ratio: number of $K+2\rightarrow K$ photons/number of $K+2\rightarrow K+1$ photons, ϵ is related to the conversion coefficient α by

$$\epsilon=\frac{\text{No. of photons}}{\text{No. of photons}+\text{No. of electrons}}=\frac{1}{1+\alpha},$$

ϵ_1, ϵ_2 relating to the $(K+2\rightarrow K+1)$, $(K+2\rightarrow K)$ transitions respectively. The factor $1/\{1-[(N_c/N_s)(1/\eta\epsilon_1)]\}$ derives from the fact that the $(K+1\rightarrow K)$ photons arise both in direct transitions from the $K+1$ level and from cascade transitions from the $K+2$ level.

COINCIDENCE MEASUREMENTS, N_c/N_s

Protons of about 3 Mev were produced in the Massachusetts Institute of Technology-Rockefeller electrostatic generator. At the target the beam spot was less than 1/64 in. in diameter. The γ counters were 2 in. $\times$2 in. "Harshaw" NaI(Tl) canned crystals optically bonded to DuMont 6292 photomultiplier tubes. The coincidence counting assembly is shown in Fig. 2. The crystal faces have been brought as close as possible to the target in order to get high coincidence yields, and to minimize the effects of anisotropic distribution of the γ rays (in most cases the relevant distributions were unknown). With the geometry of Fig. 2 it can be shown

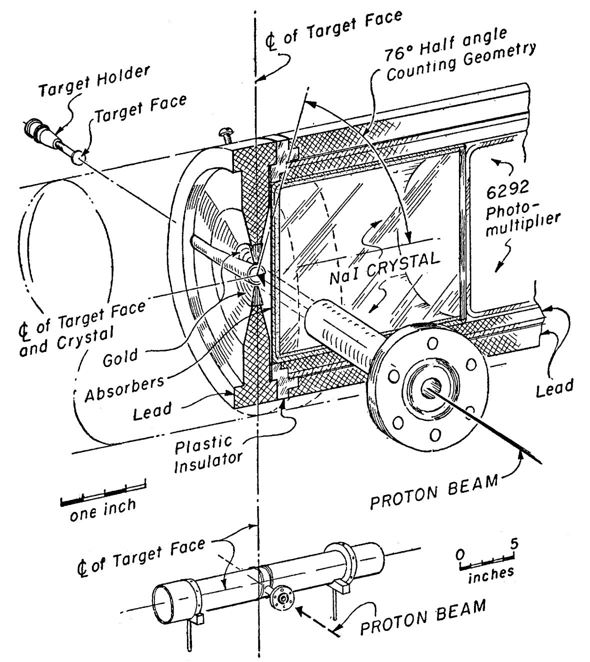

FIG. 2. Counting assembly cutaway diagram showing solid angle and lead shielding. The lower inset shows the complete assembly.

that the uncertainty thus introduced cannot exceed 2 percent. In order to avoid coincidence counts from Compton-back-scattering events, the lead and gold double-cone shield, detailed in the upper part of Fig. 2, was introduced.

Graded and "staggered" absorbers were placed in front of each crystal to absorb atomic x-rays. The "staggering" was done by building up the absorbers from a large number of thin concentric circular foils whose radii were chosen so that a γ ray penetrated the same amount of material in reaching the crystal regardless of its incident angle from the target. The uniformity of absorption thus obtained helps minimize the effects of the anisotropic distribution of the γ rays.

The coincidence circuit was of the "fast-slow" type with a single-channel pulse-height analyzer for each counter. The fast resolving time was 0.2 μsec.

The number of random coincidences was determined directly by comparing the total coincidence rate at widely differing proton currents. In the normal runs the random rate was always less than 0.1 of the real coincidence rate.

The experiment was performed in the following way: In one analyzer the window was opened wide to cover the range from below the escape peak of the $K+1\rightarrow K$ radiation to above the photopeak of the $K+2\rightarrow K+1$ radiation. In the second analyzer the window was comparatively narrow, and by changing the base line, both the singles spectrum and the coincidence spectrum were recorded simultaneously. The spectra (corrected for random coincidences) are shown in Figs. 4 to 7.

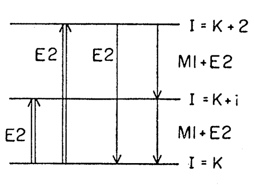

FIG. 1. Level scheme for a typical odd-A rotational nucleus, showing electric quadrupole excitation of the first two energy levels and the subsequent decay. The crossover from the second level to the ground state is pure $E2$ while the cascade transitions are mixed $M1$ and $E2$.

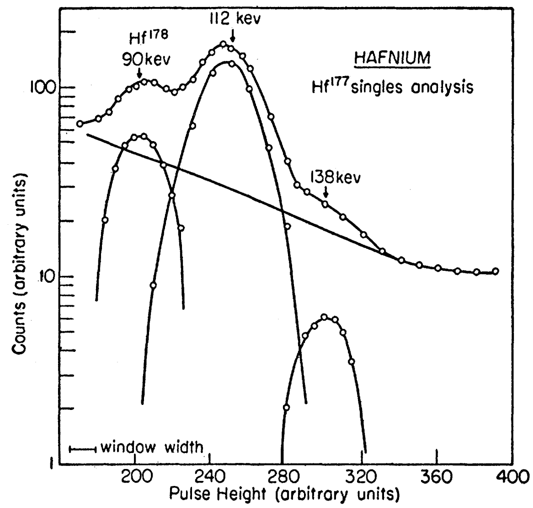

FIG. 3. Gaussian analysis of the singles pulse-height distribution from Hf^{177}. The 138-kev peak is clearly visible when the parabola corresponding to the 112-kev γ ray has been subtracted.

In the single-count spectrum one can usually discern a small peak due to $(K+2\rightarrow K+1)$ γ rays, on the high-energy side of the main peak produced by $(K+1\rightarrow K)$ γ rays. The general background arises mainly from proton bremsstrahlung. By graphical Gaussian analysis one can determine the height of the main peak to within 2 or 3%. Figure 3 illustrates the general method applied to the Hf^{177} singles spectrum, where the $(K+2\rightarrow K+1)$ γ ray is most difficult to resolve.

In the coincidence spectrum the two peaks are always clearly visible. Apart from corrections yet to be discussed one would expect the two peaks to be of equal area, i.e., to differ in height by the inverse ratio of their respective widths. The resolution of the counters as a function of energy has been determined experimentally and the expected ratio between peak heights could be evaluated. For an accurate determination one should, however, take into account the following processes:

(a) Every photopeak is accompanied by an escape-peak of energy 28 kev less than the full γ energy.

(b) The γ rays absorbed in the Pb and Au absorbers between the two crystals produce characteristic x-rays which in some cases lie within the coincidence window. The increase in counting rate introduced in this way can be estimated with fair accuracy. In most cases it is very small; only in Ho and Eu is the correction appreciable, $\sim$7%.

If (a) and (b) are taken into account, the ratio of peak heights, for the two coincidence peaks, can be calculated accurately. In all cases the measured ratio is different from the calculated ratio by not more than 5%. This is the expected experimental accuracy and is a reassuring internal-consistency check, particularly with respect to background subtraction and random correction.

The quantity N_c/N_s is deduced from the peak heights, after subtracting the background and correcting for the processes (a) and (b) discussed above.

EFFICIENCY MEASUREMENT, η

As has been explained in the first section, this experiment depends critically on the absolute counting of the γ's from the *first* $(K+2\rightarrow K+1)$ transition in the cascade. In the nuclei investigated, these radiations range in energy from 105 to 180 kev. The counter efficiencies have been determined directly for energies of 125 kev and 175 kev and interpolated for the energies required.

A Co^{57} source emitting γ rays of 119 and 131 kev served as an effective "125-kev" source. For the 175-kev calibration an As^{71} source was produced from a natural Ge target using 3.1-Mev protons in the reaction $Ge^{70}(p,\gamma)As^{71}$. The diameter of both sources was about the same as that of the beam spot. The sources were calibrated by "4π geometry" counting in a 2-in.-diameter well-type crystal. Small corrections had to be made for absorption in the aluminum wall, and for escape through the crystal. For the As^{71} source, a correction was also made for the loss of counts due to coincidence events between the 175-kev γ ray and an annihilation quantum, which shift the pulse to a higher energy. The calibrated sources were mounted in the target position, attached to the target proper (to include self-absorption in the target material) and counted in the usual manner.

BRANCHING RATIO MEASUREMENT, ν

The measurement of the crossover-to-cascade branching ratio is the least accurate part of the present work. In principle, a coincidence system such as ours can be used to evaluate branching ratios, and some measurements have been reported using such methods.[2,6,7] This procedure, however, is somewhat uncertain because the conversion coefficient for the $(K+1\rightarrow K)$ radiation must be known. In our work no great accuracy in the value of ν is required and therefore we preferred to measure it in the less accurate but more direct way of comparing the $(K+2\rightarrow K)$ and $(K+2\rightarrow K+1)$ peaks in the singles spectrum. For Ta we used the value for ν as reported by Stelson and McGowan,[8] who had measured it using essentially this latter procedure. For other nuclei we used our own data, with this Ta value[8] as a calibration point for the $(K+2\rightarrow K)$ γ ray.

CONVERSION COEFFICIENTS

The $M1$ to $E2$ mixing ratio, δ^2, for the $(K+2\rightarrow K+1)$ radiation was obtained by comparing the experimental

[6] G. M. Temmer and N. P. Heydenburg, Bull. Am. Phys. Soc. Ser. II, 1, 43 (1956).
[7] N. P. Heydenburg and G. M. Temmer, Phys. Rev. 100, 150 (1955).
[8] P. H. Stelson and F. K. McGowan, Phys. Rev. 99, 112 (1955).

branching ratio ν with the value ν' obtained for pure $E2$ transitions according to the collective model. This procedure presupposed the validity of the collective-model calculations, which is in effect the problem under investigation in this work. Nevertheless, it seems to us justified for the following reasons:

(a) The final results of this work confirm the validity of the collective model, so that the procedure is in this case consistent.

(b) It has been mentioned above that in cases of interest δ^2 is large. In such cases ϵ_1 [of Eq. (3)] is relatively insensitive to δ^2:

$$\frac{d\epsilon_1}{\epsilon_1}=\frac{\epsilon_1 d\delta^2}{(1+\delta^2)^2}(\alpha_E-\alpha_M),$$

where α_M, α_E are the $M1$ and $E2$ conversion coefficients respectively.

The calculations of conversion coefficients by Rose *et al.*[9] and Gellman *et al.*[10] are based on the assumption of point nuclei. For very high Z this assumption may not be valid, especially for $M1$ transitions. Wapstra and Nijgh[11] report the measurement of the conversion coefficient for a transition in Tl and according to that paper the $M1$ coefficient is only about 0.6 of the value given by Rose. The general trends of the dependence of this correction factor on Z and on energy are also discussed in that paper and it appears that for the transitions considered here the correction factor would be closer to unity. We have investigated this problem directly in the following manner: the ratio of coincidence counts to singles counts from the $(K+2\rightarrow K+1)$ transition could be determined directly from our data; this ratio is equal to $\eta_3/1+\alpha_3$, where α_3 is the conversion coefficient and η_3 the counter efficiency for the

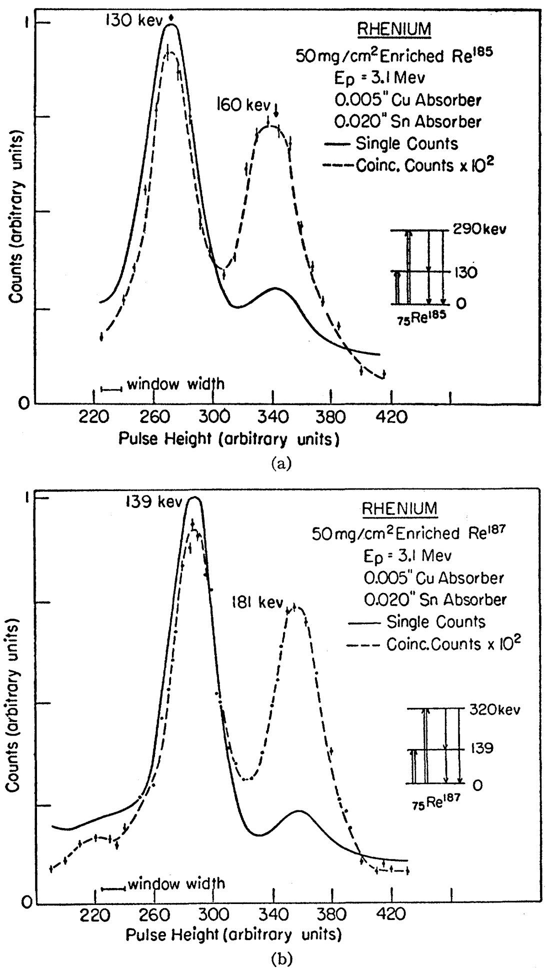

FIG. 5. Re[185] and Re[187], coincidences and singles pulse-height distribution.

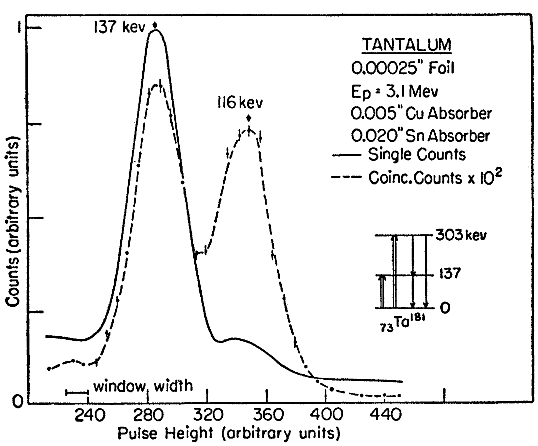

FIG. 4. Ta[181], coincidences and singles pulse-height distribution. The level scheme is shown at the right.

$(K+1\rightarrow K)$ radiation. Since we know η_3 fairly accurately, we have in effect a measurement of α_3. In order to get the value of the $M1$ coefficient we adopted the values of Huus *et al.*[12] for the $M1/E2$ branching ratio. Most of these transitions [in the $(K+2\rightarrow K+1)$ transitions] are predominantly $M1$ and therefore no accurate value of the mixing ratio is required. Our results are consistent with correction factors p (to the values given by Rose) of $0.8\leq p\leq 1$. We expect p to be the same for the $(K+2\rightarrow K+1)$ transitions as for the $(K+1\rightarrow K)$ transitions and for our final results we have adopted the value $p=0.8$. In order to show the dependence on p the ratios of $B(E2)$ are also given as calculated with the extreme (and improbable) values $p=1$ and $p=0.6$.

Tantalum-181

The low-lying levels of Ta[181] have been examined very thoroughly[2,7,8,12–14] and our own work on Ta

[9] Rose, Goertzel, Spinrad, Harr, and Strong, Phys. Rev. **83**, 79 (1951).

[10] Gellman, Griffith, and Stanley, Phys. Rev. **85**, 944 (1952).

[11] A. H. Wapstra and G. J. Nijgh, Nuclear Phys. **1**, 245 (1956).

[12] Huus, Bjerregaard, and Elbek, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **30**, No. 17 (1956).

[13] H. Mark and G. T. Paulissen, Phys. Rev. **99**, 1654 (A) (1955).

[14] F. K. McGowan and P. H. Stelson, Phys. Rev. **99**, 127 (1955).

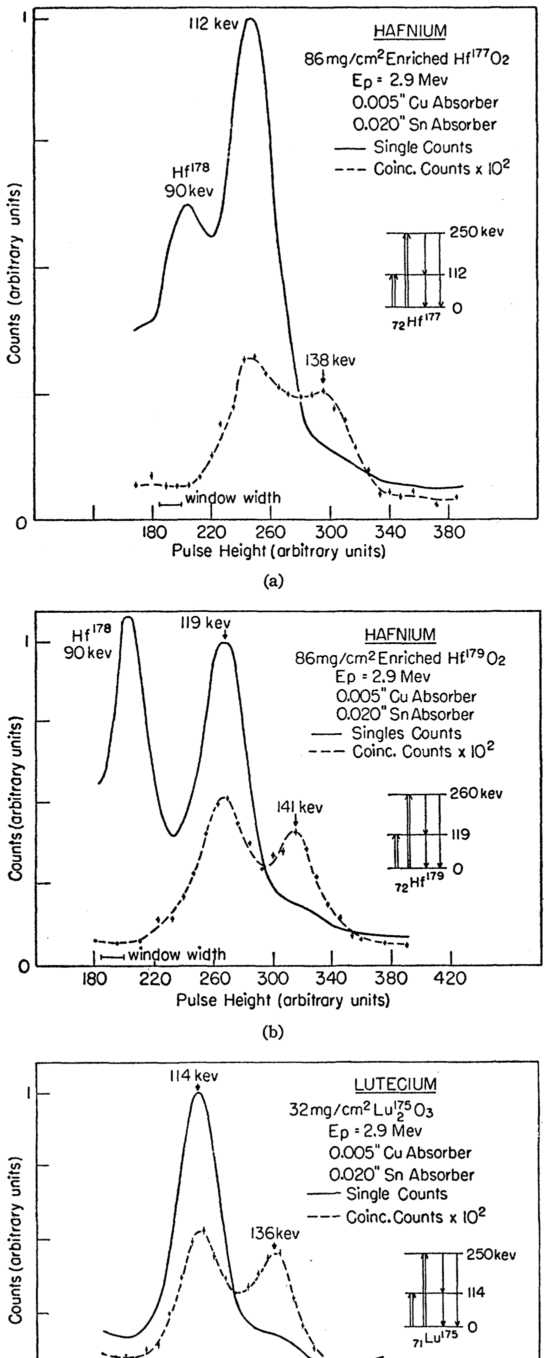

FIG. 6. Hf^{177}, Hf^{179}, and Lu^{175}, coincidences and singles pulse-height distribution.

could be used to serve as a check for the experimental procedures adopted.

The target was prepared from Ta foil 0.00025 in. thick mounted on thin Ta wire. In order to measure the energy loss of protons in the foil directly a second Ta target, thick to protons, was placed 2 cm behind the foil and counters were arranged to accept only γ's emitted from this thick target. The yield of γ's per proton for various proton energies was measured with the foil both in and out and the results were compared. A cylinder of Ta foil of the same diameter as the two targets was inserted coaxial with the beam and between the two targets to intercept Rutherford-scattered protons. The thickness of the 0.00025 target was found to be 650-kev at 3.1-Mev proton energy.

In the calculation of r [Eq. (3)], $\kappa=dE/dX$ was assumed to be constant over the range of proton energies involved. This assumption is based on material presented in the review article by Allison and Warshaw[15] on proton absorption in Au.

The Ta coincidences and singles spectra are shown in Fig. 4. The ratio of the two $B(E2)$'s was found to be 0.22 ± 0.04, in good agreement with previous work.[8,16,17]

The mixing ratio δ^2 of $M1/E2$ in the $(K+2\rightarrow K+1)$ radiation has been determined in Ta, in an angular correlation measurement.[14] From this one gets for ϵ_1 the value 0.485 (for $p=1$). If one calculates ϵ_1 from the collective model $E2$ radiation intensities[18] with the value of Stelson and McGowan for the branching ratio ν, $\nu=0.575$, one gets for ϵ_1: $\epsilon_1=0.475$, in close agreement with the experimental value. This derivation of ϵ_1 thus seems well justified.

Rhenium-185, Rhenium-187

Re targets were prepared by pressing powdered isotopically enriched Re metal on an aluminum screw covered with 0.001-in. tin foil. The thickness was 50 mg/cm². The targets for the nuclei discussed subsequently were prepared in a similar manner.

It is not possible by this method to obtain targets much thinner than 4 mg/cm². Targets which are thick to protons, however, are unsatisfactory for accurate cross section determinations because the dependence of κ [in Eqs. (2) and (3)] on energy is not known with sufficient accuracy.[14] A "subtraction method" was therefore employed: the targets were bombarded with protons of 3.1 Mev and 2.6 Mev. The differences of the counts per proton of the two energies give the thin-target yields (see Fig. 5).

In practice the thick-target yields were measured carefully and the thin-target values were obtained by applying a correction factor, A, based on the measurement at different proton energies. A should be very nearly the same for Re^{185} and Re^{187} since the excitation energies and therefore the ξ are rather close. The ratio

[15] S. K. Allison and S. D. Warshaw, Revs. Modern Phys. **25**, 779 (1953).
[16] Eisinger, Cook, and Class, Phys. Rev. **94**, 735 (1954).
[17] W. I. Goldburg and R. M. Williamson, Phys. Rev. **95**, 767 (1954).
[18] Alaga, Alder, Bohr, and Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 9 (1955).

of A's for the two isotopes may be calculated to high accuracy. In working out the data in any one of the isotopes, one can therefore make use of the A obtained for both and in this way the accuracy of this correction procedure is increased.

Hafnium-177, Hafnium-179, Lutecium-175

The isotopically enriched HfO_2 targets were 86 mg/cm^2 thick, while the Lu_2O_3 (natural Lu) target was 32 mg/cm^2 thick. They were bombarded with protons of energy 2.9 Mev and 2.5 Mev for the thin-target yield determination. The A factor was derived in these cases from pooled information from all three targets.

In the Hf^{177} and Hf^{179} single-counts spectra, Figs. 6(a) and 6(b), peaks are observed which are ascribed to even-isotope impurities.[7] With careful Gaussian analysis they can be isolated from the main odd-isotope peaks, and do not affect the accuracy of the measurements appreciably.

In Hf^{177} the branching ratio ν is so large that the measurement of the ratio of $B(E2)$'s has comparatively low accuracy.

Europium-153, Holmium-165

Targets of Eu^{153} and Ho^{165} were prepared from oxides of the natural elements. Ho is monoisotopic. Eu has two stable isotopes but only Eu^{153} has low-lying levels of the normal rotational type.[6,7] The lowest level of Eu^{151} has recently been reported to be at essentially the same energy as the second level in Eu^{153}.[19] The results reported here take into account the ratio of yields of these two γ rays. The targets were 30 mg/cm^2 thick and were bombarded with protons of energy 2.9 Mev and 2.5 Mev.

The main peaks in the singles spectrum, Figs. 7(a) and 7(b), are markedly asymmetrical; we ascribe this to the degradation of the γ's in the Pb—Au back-scattering shield and subsequent re-emission of characteristic x-rays.

This degradation is also responsible for the fact that the two coincidence peaks are of almost the same height. This may best be understood if one considers the degradation as a broadening of the $(K+1\rightarrow K)$ peak, and therefore instead of having a $(K+1\rightarrow K)$ peak which is narrower than the $(K+2\rightarrow K+1)$ peak and higher, one gets in these spectra $(K+1\rightarrow K)$ peaks which are as broad (and therefore also as high) as the $(K+2\rightarrow K+1)$ peaks.

The contribution of these degraded γ's must of course be known for the determination of the ratio of the $B(E2)$'s. Although it may be estimated, the error is sufficiently great to affect the over-all accuracy of the measurement.

RESULTS

The results are summarized in Table I. We have also included in the table a value recently obtained by

[19] G. M. Temmer (private communication).

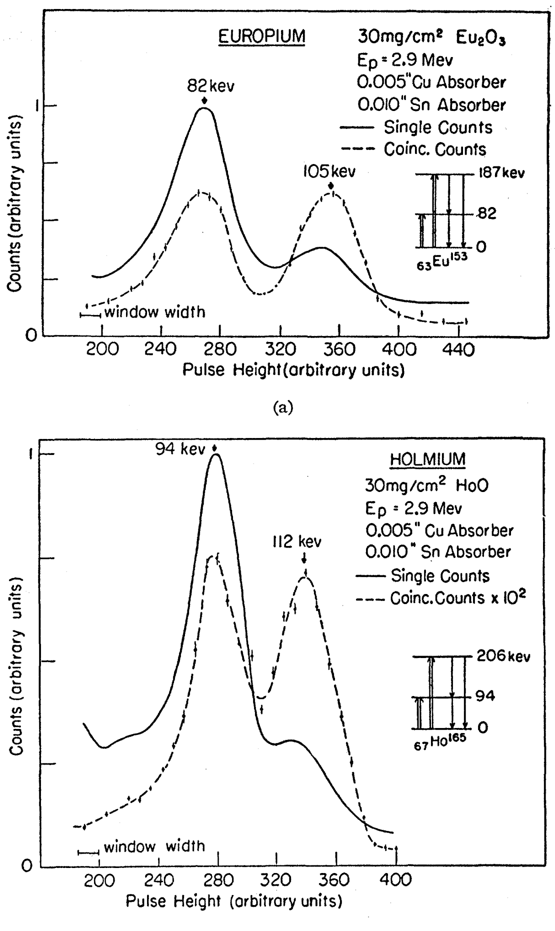

FIG. 7. Eu^{153} and Ho^{165}, coincidences and singles pulse-height distribution.

Sharp[20] by measuring relative cross sections for inelastic proton scattering from Tb. The results indicate that the intrinsic quadrupole moments Q_0 in any one nucleus for two different $E2$ transitions are remarkably constant. The agreement is certainly better than the agreement between Q_0's calculated from electric excitation cross sections, and from the level spacings. In particular it is interesting to note that the mean ratio $B(E2, K\rightarrow K+2)/B(E2, K\rightarrow K+1)$ for all the nuclei investigated is 0.267 ± 0.05 whereas the corresponding collective-model value is 0.315.

This seems to indicate that the general theory is correct and applicable to a class of nuclei and that the main discrepancy lies in the relation between Q_0 and the moment of inertia [reference 1: Eqs. (V, 7), (VI, 2), and (II, 6a)]. No regularity could be detected in the discrepancies between experiment and theory in the ratios of $B(E2)$'s.

Hf^{177} is unique in that it has an unusually large branching ratio ν. Compared to the crossover transition,

[20] Richard D. Sharp (private communication).

TABLE I. Ratios of $B(E2)$'s calculated from the experimental results using different values for the $M1$ conversion coefficient. The quantity p denotes the ratio between the value used and the value given by Rose *et al.* The numbers given in the last column are considered accurate to 20%, including the uncertainty in the conversion coefficient. Relative values for different nuclei are accurate to ~12%.

Nuclide	Ground state spin	$\left(\frac{N_c}{N_s}\right)\left(\frac{1}{\eta}\right)=\frac{\text{cascades}}{\text{singles }(K+1\rightarrow K)}$	$\nu=\frac{\text{crossover}}{\text{cascade}}$	$\frac{B(E2,K\rightarrow K+2)}{B(E2,K\rightarrow K+1)}$ Observed with $p=1$	Observed with $p=0.6$	Observed with $p=0.8$	Calculated
Tb	3/2	···	···	···	···	0.36[a]	0.555
Re^{187}	5/2	0.0760±0.005	0.22 ±0.05	0.31	0.25	0.28	
Re^{185}	5/2	0.0845±0.006	0.23 ±0.05	0.40	0.30	0.35	0.350
Eu^{153}	5/2	0.0465±0.003	1.3 ±0.02	0.24	0.21	0.285	
Ta^{181}	7/2	0.0467±0.003	0.574±0.05	0.24	0.20	0.22	
Hf^{177}	7/2	0.0342±0.003	4.3 ±0.6	0.34	0.30	0.32	
Lu^{175}	7/2	0.0484±0.003	0.90 ±0.15	0.24	0.22	0.23	0.257
Ho^{165}	7/2	0.0564±0.004	0.14 ±0.015	0.22	0.17	0.195	
Hf^{179}	9/2	0.048 ±0.004	0.50 ±0.07	0.25	0.19	0.22	0.204

[a] This value was obtained from inelastic proton scattering measurements [Richard D. Sharp (private communication)].

the cascade transition is (1±0.13) times the value predicted for a pure $E2$ transition. We may therefore conclude that the magnetic moment for this transition is exceedingly small, or in the description of the collective model: $g_\Omega \sim g_R$. The $(K+1\rightarrow K)$ transition in Hf^{177} is also reported[12] to be essentially pure $E2$ (see Table II).

We believe the relative value

$$\left[\frac{B(E2,K\rightarrow K+2)}{B(E2,K\rightarrow K+1)}\right]_{\mathrm{Re}^{187}}:\left[\frac{B(E2,K\rightarrow K+2)}{B(E2,K\rightarrow K+1)}\right]_{\mathrm{R}^{1185}}=0.80$$

to be accurate to ±0.08. This ratio, for which the collective-model value is unity, is the most accurate measurement we have of the modification of the mean rotational structure.

In Table II we summarize the information on the branching ratio $M1/E2$. The values for the $K+1\rightarrow K$ transition are taken from reference 12. The numbers for the two transitions are listed as $\delta^2/[K(K+2)]$ and $\delta^2/[(K+1)(K+3)]$ respectively, because according to the collective model these two numbers should be equal [being proportional to $(g_\Omega - g_R)^2/Q_0^2$]. Since the measurements are rather inaccurate, the differences between the two values may well be within the limits of experimental uncertainty for all nuclei except Lu and Eu^{153}. For these two there seems to be a real and significant discrepancy.

TABLE II. $M1/E2$ ratios. The first column is taken from Huus *et al.*[12] and the second column gives the values derived from our measurements. The values are normalized in such a way that the collective model theory would give equal numbers in both columns. The values in the last column have been computed under the assumption that Q_0 is constant. The uncertainty thus introduced is not taken into account in the errors quoted in the last column.

Nuclide	$K+1\rightarrow K$ $\frac{\delta^2}{K(K+2)}$	$K+2\rightarrow K+1$ $\frac{\delta^2}{(K+1)(K+3)}$
Re^{187}	0.8	1.3 ±0.3
Re^{185}		1.0 ±0.25
Eu^{153}	0.035	0.2 ±0.05
Ta^{181}	0.35	0.22±0.02[a]
Hf^{177}	0.001	0 +0.005
Lu^{175}	1.1	0.15±0.025
Ho^{165}	0.6	1.2 ±0.2
Hf^{179}		0.15±0.025

[a] This value is derived from the value of reference 8 for ν. From angular correlation measurements (reference 13), one gets the value 0.14.

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