evaluate the average energy difference for ϵ at the point of inflection of the ground state energy shown in Fig. 8. Although this deformation does not represent an equilibrium state, at least the quadrupole parameter and deformation change at the same rate, i.e., Eq. (24) is satisfied. The energy difference for oscillations in $\epsilon(\Delta\Lambda=0)$ is now nearly equal to the value for rotations and γ oscillations.

From the above arguments it is therefore plausible, and in agreement with experimental evidence (Sec. IV) that the mass parameter has approximately the same value for the various modes of quadrupole oscillations.

VI. CONCLUSIONS

The main conclusion of this paper is that the nucleons outside filled shells appear, in spite of their relatively small numbers, to be of greater significance in contributing to collective properties than was sometimes believed. According to the oscillator model used here, the outer particles contribute fully half of the static quadrupole moment; thus core distortion is important here but not dominant. Even more strikingly, the outer particles contribute the major share of the collective motion so that the nucleons inside filled shells play only a minor role for the dynamic properties.

This behavior resembles the situation in the usual shell model where the nucleons in filled shells do not, in general, enter in the properties of the low-lying states. The evident similarity of the shell model and collective model in this important respect is very encouraging for the prospect of constructing a unified model of nuclear structure.

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Neutron Resonance Parameters of U^{235}†

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The neutron total cross section of $_{92}U^{235}$ has been studied for neutron energies between 1.5 and 60 ev by using the Brookhaven fast chopper. The transmission data were analyzed by the "area" method and the Breit-Wigner parameters obtained for the resonances up to 35 ev. The observed level spacing of 0.65 ev is the smallest yet reported for any isotope. The best available fission data have been combined with total cross-section data to obtain Γ_γ. The values of Γ_γ are approximately constant and are consistent with those of neighboring heavy nuclei, whereas the Γ_n^0's have a very broad distribution similar to that observed in a number of other elements. The distribution of Γ_f's is also rather broad and resembles the neutron widths much more than the radiation widths in this respect. Average values of these parameters are presented, together with a discussion of the implications of these findings for current theories of fission.

I. INTRODUCTION

IN spite of the great importance of the low-energy neutron cross section of U^{235} for the design of various types of reactors, the data available in 1953 were still incomplete. Although it had been established that sharp resonances were present and that the ratio of capture to fission varied from resonance to resonance, the cross-section data were not accurate enough to give parameters of many resonances. Since an active program of the measurement of the parameters of resonances in nonfissionable nuclei was in progress with the Brookhaven fast chopper, it was decided to include U^{235} because of its fundamental nuclear physics interest and the value of the results for reactor design.

It is of interest to review the results that were available at the time the present work was begun. Very early measurements in 1939 by the Columbia group under Fermi,[1] using cadmium and boron filters, showed that the number of fission bursts in a fission counter approximately followed a $1/v$ law. In a theoretical paper in 1939 on the mechanism of nuclear fission, Bohr and Wheeler[2] concluded that fission widths of the resonances in U^{235} must be greater than 10 ev. Since radiation widths were estimated to be $\sim$0.1 ev, it was believed that the absorption of neutrons in U^{235} almost always led to fission, and that, since the level spacing was expected to be less than 10 ev, it was not expected that any sharp resonances would be found.

† Work performed under contract with U. S. Atomic Energy Commission.
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However, work in 1944 by Anderson *et al.*[3] at Los Alamos on both total and fission cross sections, with a pulsed cyclotron and the time-of-flight technique, revealed that sharp resonances, ~0.1 ev wide, were present at low energies. Comparison of the data on the fission and total cross sections indicated that the ratio of capture to fission varied from resonance to resonance. Although fluctuations in the cross sections were found above 2 ev the energy resolution was not sufficient to resolve all the resonances and an analysis of the parameters of the resonances above this energy could not be made.

In 1947, a series of measurements with improved instrumental resolution was undertaken on the total and fission cross sections of U^{235} at Columbia University by Havens *et al.*[4] Their data showed that, although the ratio of fission to capture changed from resonance to resonance, all the low-energy resonances found in the total cross section of U^{235} were also present in the fission cross section. The resolution was still not sufficient to resolve all the resonances above 5 ev, although a few approximate parameters were given for that energy region. In the fall of 1953, soon after the high-resolution fast chopper at Brookhaven was put in operation and rapid methods of analyzing resonances were developed, a program was started to measure the total cross section of U^{235} and to obtain parameters for as many resonances as possible. Additional measurements were also made in 1955.

Since 1953 several groups in the U. S. in addition to the fast-chopper group at BNL have made additional measurements on the low-energy cross sections of U^{235}. In 1954 Leonard, Seppi, and Friesen at Hanford reported measurements of high statistical accuracy on the total and fission cross section up to 0.6 ev obtained with a crystal spectrometer. Yeater, Gaerttner, and Kelly reported fission cross-section measurements at Knolls Atomic Power Laboratory in the kev energy region in 1954 and have very recently made fission measurements in the resonance energy region down to 6 ev. Sailor[5] and Foote[6] at Brookhaven in 1954 and 1955 made accurate measurements of the total and scattering cross section in the low-energy region below 5 ev and for a few resonances from 5 to 10 ev. At higher energies from 9 to 60 ev Simpson *et al.*,[7] at the Materials Testing Reactor, have measured the total cross section with a fast chopper. Some of the results of these measurements are included in the neutron cross-section compilation.[8] The details of the measurements are being prepared for publication by the various groups.

The first published indication that the U^{235} fission cross section might not follow the $1/v$ law was by Keller[9] in 1953. The fission cross-section measurements on U^{235} made at Kjeller in 1954 by Popovic[10] was the first published work to show definitely that the cross section did not follow the $1/v$ law in the thermal region. Popovic and Rajsic[11] later found a small resonance at 0.29 ev in the fission cross section. Independent measurements by Hubert, Vendryes, and Auclair[12] in 1954 showed that the fission cross section of U^{235} did not follow the $1/v$ law and revealed resonances at 0.29, 1.13, and 2.0 ev. From total cross-section measurements they also found a small resonance at 0.29 ev in natural uranium which they attributed to U^{235}. The shape of the fission cross-section curve below 1 ev indicated that a bound level (negative energy resonance) was present close to zero neutron energy. Measurements on the total and fission cross sections of U^{235} also have been made in the United Kingdom and the Soviet Union. The results were presented at the Geneva Conference[13] and are included in the addendum to the neutron cross-section compilation.[8] They have also been summarized in review articles by Harvey and Sanders[14] and Egelstaff and Hughes.[15]

At the present time considerable information has been published on the resonance properties of non-fissionable nuclides. It is well established[16–18] that the radiation widths for various levels in a particular nuclide are constant to 10 or 20 percent. The dependence of the radiation width upon atomic weight, level spacing, and binding energy has been studied and is in agreement with theoretical expectations for electric dipole transitions.[18,19] This constancy of the radiation width from resonance to resonance in a given nuclide is in sharp contrast to the large fluctuations[20] in neutron widths from resonance to resonance. The experimental neutron width distribution has been compared[21] with various possible distribution laws.

[3] Anderson, Lavatelli, McDaniel, and Sutton (unpublished, 1944).
[4] Havens, Rainwater, Wu, Dunning, Melkonian, Perez-Mendez, and Levin (unpublished 1947–1953).
[5] V. L. Sailor, Phys. Rev. **100**, 1249(A) (1955).
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A recent theoretical investigation of the neutron width distribution as well as the near-constancy of the radiation width has been made by Thomas and Porter.[22] According to this study, the neutron width distribution corresponds to a chi-squared distribution with one degree of freedom, whereas the radiation width distribution corresponds to several hundred degrees of freedom. In the case of fission, since there are hundreds of fission products, it might be expected that the fission width distribution would more closely correspond to the narrow radiation width distribution characteristic of gamma emission than the broad neutron width distribution corresponding to neutron emission. It is of obvious importance to determine the fission width distribution, but to do so requires not only precise measurements but careful analysis of the data in order to get a sufficient number of fission widths of reasonable accuracy.

II. DETERMINATION OF FISSION WIDTHS

For nonfissionable nuclides, it has been emphasized[20] that the radiation and neutron widths of low-energy resonances can be obtained from transmission measurements, which give the total cross section. The total cross section of an isolated resonance is expressed in terms of the level parameters by

$$\sigma_T=\pi\lambda\!\!\!^{-2}gE^{\frac{1}{2}}\frac{\Gamma_n{}^0(\Gamma_\gamma+\Gamma_f)}{(E-E_0)^2+(\Gamma/2)^2}+\sigma_s.$$

In this equation, $2\pi\lambda\!\!\!^{-}$ is the wavelength of a neutron with energy E; E_0 is the energy of the neutron at resonance; g is the statistical weight factor; $\Gamma_n{}^0$, Γ_f, Γ_γ, and Γ are the reduced neutron width, the fission width, the radiation width, and the total width of the level; and σ_s is the scattering cross section.

For zero-spin target nuclei, the analysis of the total cross section gives the total width and neutron width of the resonance,[20] and the radiation width for nonfissionable nuclei ($\Gamma_f=0$) is given directly by the difference of these two widths, $\Gamma_\gamma=\Gamma-\Gamma_n$. For target nuclei of nonzero spin, however, the total width and the neutron width times the statistical weight factor, g, are obtained. This statistical weight factor has two possible values, both nearly $\frac{1}{2}$ except for the case where the target nucleus has spin $\frac{1}{2}$ when the two possible values are $\frac{1}{4}$ or $\frac{3}{4}$. However, since low-energy resonances are mainly capture, negligible error is introduced in the radiation width because of the uncertainty in the statistical factor. Thus it is not necessary to measure the capture or scattering cross sections of nonfissionable nuclides to determine the partial widths of the resonance. The additional measurement of the scattering cross section of target nuclei of nonzero spin is useful, however, for it enables one to determine the statistical weight factor, thus the angular momentum of the compound nucleus.

For fissionable nuclides the total width, Γ, and the product $g\Gamma_n$ are obtained from total cross-section measurements. Since the spin of U^{235} is $7/2$, the two possible values of g are $7/16$ and $9/16$. As the neutron widths are very small, this uncertainty in g introduces no significant error in obtaining the sum of the fission width and the radiaion width from the total cross-section measurements. However, in order to separate the quantity $(\Gamma_f+\Gamma_\gamma)$ into its components, additional information is needed. This additional information can be either the fission cross section as a function of energy or knowledge of Γ_γ for each resonance. In the first method, a well-resolved fission cross-section curve gives the total width Γ and the quantity $g\Gamma_n\Gamma_f\Gamma$. Combining these fission results with those of the total cross-section measurements, then gives Γ, Γ_n, Γ_f, and Γ_γ. However, except for the very lowest resonances, where accurate measurements are available, it is not practical to determine Γ_γ or Γ_f this way when Γ_f is much larger than Γ_γ. For example, if $\Gamma_f=5\Gamma_\gamma$, the quantity Γ_f/Γ must be known to an accuracy of better than 4% in order to get an accuracy of better than 20% in Γ_γ. Since the ratio Γ_f/Γ is computed from the quantity $g\Gamma_n$ and $g\Gamma_n\Gamma_f/\Gamma$, it is necessary that each of these be known to at least three percent, which is very difficult because of the difficulty in obtaining highly accurate fission cross-section data. For resonances where $\Gamma_f<\Gamma_\gamma$, however, the fission data are useful in determining Γ_γ and Γ_f. In the present work, four resonances have been analyzed by this method and the data are presented in Sec. IV.

The alternative method is to use an average Γ_γ for all resonances where good fission data do not exist, since the radiation width is known[18] to be almost the same from resonance to resonance in a single nuclide. This average value can be determined from a few very low-energy measurements where both the total and fission data are quite accurate, and from a few higher energy resonances that have little fission in them, i.e., $\Gamma_f<\Gamma_\gamma$. The individual values of Γ_γ obtained in this manner agree well with one another. Their average is accurate to about 10% and agrees with what would be expected from the knowledge of the radiation widths of nearby nuclides. This average Γ_γ is then used for all other resonances to determine the fission widths from the sum $(\Gamma_\gamma+\Gamma_f)$ obtained from total cross-section measurements. This second method is followed in obtaining most of the Γ_f's in the present paper, which gives the details of the transmission measurements and the analysis from which $(\Gamma_\gamma+\Gamma_f)$ is obtained (Sec. III), the determination of the average Γ_γ from the best available fission cross-section measurements (Sec. IV), the computation of the fission widths (Sec. V), and a discussion of the results (Sec. VI).

[22] R. G. Thomas and C. E. Porter (to be published).

III. FAST CHOPPER MEASUREMENTS

During 1954 and 1955, the Brookhaven fast chopper[23] was used for extensive measurements of the total cross section of U^{235}. Because of the particular suitability of the fast chopper for total cross sections and the fact that much information on parameters can be obtained from total cross sections alone, no fission measurements were made with the fast chopper. The fast chopper, neutron detector, and timing equipment were used in essentially the same form as already described.[23] The maximum resolution, as before, was 0.07 μsec/m, attained only above 20 ev when the usual BF_3 detector was used. In the present work the energy region of maximum resolution was extended to the 10-ev to 20-ev region by use of a scintillation detector,[24] in which the flight time is quite small compared to the other timing uncertainties. The scintillation detector was not used for the entire energy region because of a background counting rate higher than that of the BF_3 detector. In the energy region from 2 ev to 10 ev the resolution was 0.2 μsec/m.

Enriched samples of uranium oxide and uranium metal were used for the various runs required to obtain the necessary data. The metal samples were enriched to greater than 90% U^{235}, while the uranium in the oxide samples was 99.86% U^{235}. In general, metal foils were used for thin samples because of the difficulty of preparing oxide samples of accurate thickness; for thick samples, however, the advantage of the higher U^{235} content of the oxide could be exploited.

Transmission measurements were made from 1.5 ev to 60 ev using several different sample thicknesses appropriate for parameter analysis.[25] The resonances up to 35 ev only were analyzed for Breit-Wigner parameters, as the resolution was obviously insufficient to resolve all levels above this energy. A transmission curve for a moderately thick sample in the 8 to 20 ev region is shown in Fig. 1. This curve illustrates the extremely close spacing of the resonances in U^{235} and the resultant difficulty encountered in obtaining accurate measurements of resonance parameters, particularly for the very small resonances. In analyzing these data, it was almost always necessary to correct for the distorting effect of nearby levels in determining the area under a transmission dip due to a given resonance. In addition to the resonances observed at positive energies, the data below 5 ev gave definite evidence of the presence of a "tail" of negative energy resonances. This tail was taken into account in analyzing the resonances in this low-energy region.

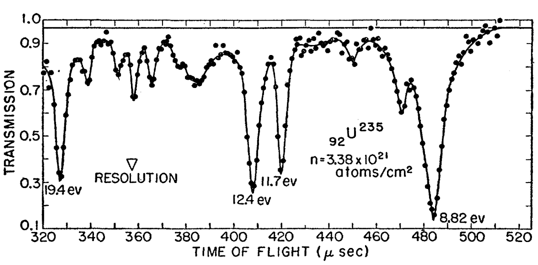

FIG. 1. The transmission as a function of time of flight at 20 m for a 30-mil sample of U^{235} ($n=3.38\times10^{21}$ atoms/cm^2). The horizontal straight line is the base line from which the areas of the resonances are measured. The analysis of these and similar data by the method described in Fig. 3 is given in Table I.

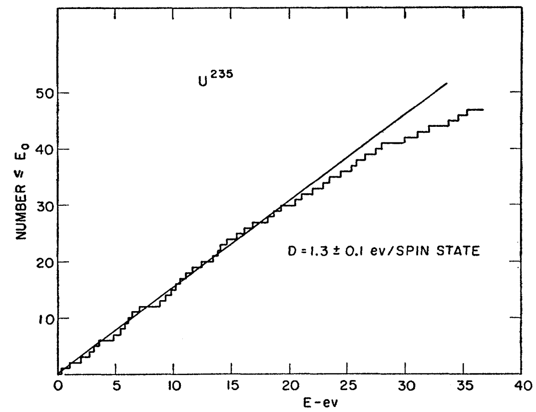

FIG. 2. The number of levels observed in U^{235} as a function of neutron energy. It is apparent from the dropping away of the number of levels from the straight line that levels have been missed above 20 ev.

With such small level spacings, there is the possibility that small levels may be missed or that a transmission dip may not be due to a single resonance. One method of determining whether or not levels are being missed or not resolved is to run several sample thicknesses and observe the manner in which the area of the transmission dips vary with thickness. Only if resonances are correctly identified will the areas vary in accord with the theoretical curves.[25] Another method for determining where levels are starting to be missed is illustrated in Fig. 2. The dropping away of the observed number of levels below the straight line indicates that some levels are missed above 20 ev. The slope of the straight line corresponds to an observed level spacing of 0.65±0.05 ev hence to a spacing of 1.3±0.1 ev for each of the two possible spin states on the assumption that there are the same number of levels in each spin state.

The areas of the transmission dips were measured for all the resonances observed up to 35 ev and the Breit-Wigner parameters were computed by the methods already described.[25] This analysis was carried out with the aid of the analysis curves of Fig. 3, taken from reference 25, which represent a simplification over the curves used in earlier work.[20] Reasonably accurate thin and thick samples were available for ten of the resonances observed below 20 ev, making it possible

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[24] Palevsky, Zimmerman, and Larsson, Rev. Sci. Instr. (to be published).
[25] D. J. Hughes, J. Nuclear Energy **1**, 237 (1955).

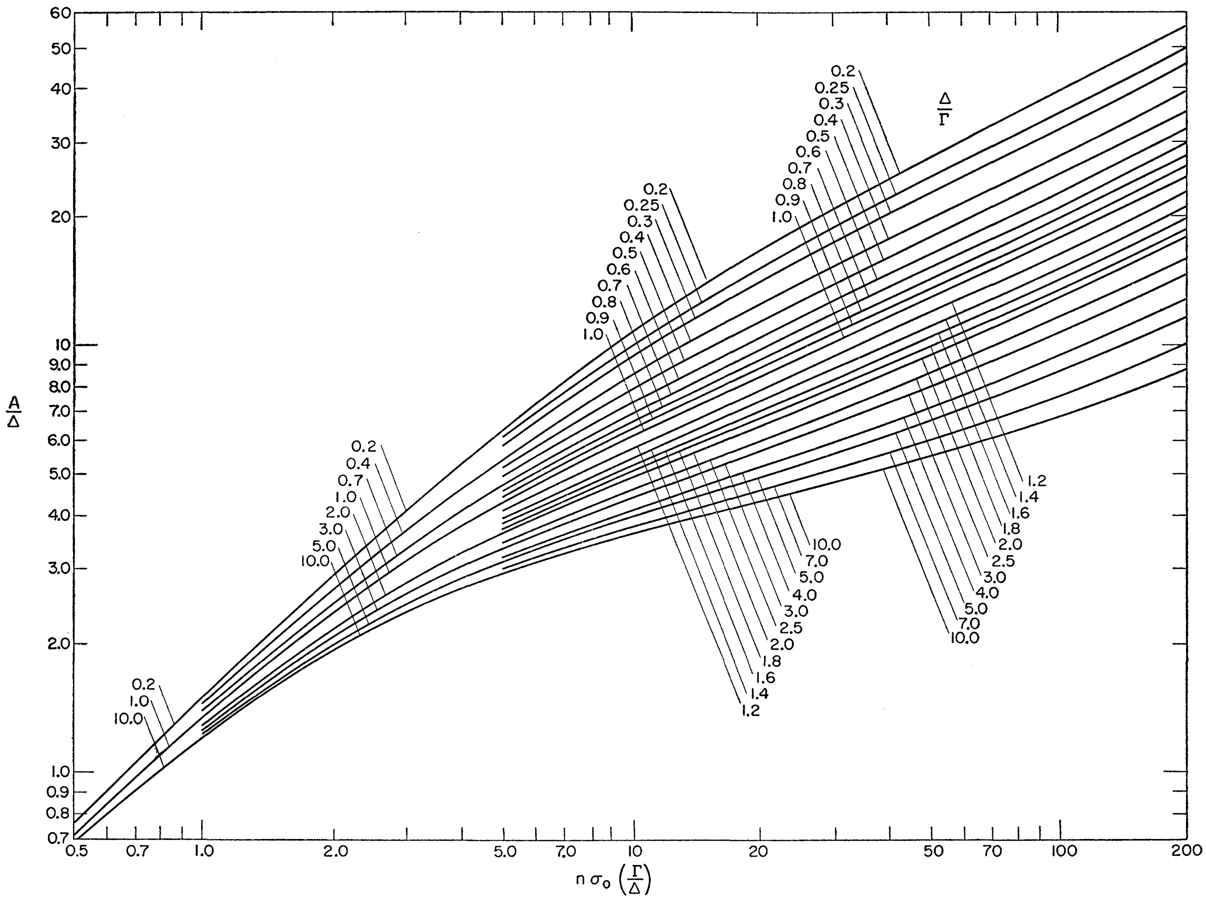

FIG. 3. Curves relating the measured area of transmission dips, A (ev), sample thickness n (atoms/cm²), and Doppler broadening Δ (ev) to the resonance parameters Γ and σ_0 (peak height). The principle of the method of resonance analysis is that values of A, n, and Δ for two sample thicknesses are consistent with only a particular location on a single curve, and this location determines Γ and σ_0. Because the desired parameters appear combined with the measured quantities, the method involves successive approximations.

to obtain $g\Gamma_n$ and Γ for these cases. The errors in the measured Γ's are rather large, due primarily to the fact that really thick samples could not be used because of overlapping in the wings of the resonances.

In most of the cases studied here only thin sample data were available, due to the overlapping of resonances. Fortunately, it is possible to obtain the quantity $g\Gamma_n$ for a resonance from thin sample data alone, with no knowledge of Γ, since the area of a thin sample transmission dip gives $\sigma_0\Gamma$ directly (with σ_0 the peak height of the total cross section) and this product is proportional to $g\Gamma_n$. If the sample is of intermediate thickness there is a slight dependence of the measured $g\Gamma_n$ upon Γ. For those cases in which Γ could not be measured directly, an average Γ of 0.1 ± 0.02 ev was used to obtain $g\Gamma_n$ from the experimental data. In all cases the error in $g\Gamma_n$ due to lack of knowledge of Γ was much less than that due to the error in A.

Table I summarizes the results for the resonances up to 35 ev and lists the pertinent experimental data used to obtain the parameters. In this table the statistical weight factor, g, is assumed to be $\frac{1}{2}$, ($I=7/2$ for U^{235}), and the "reduced width" is computed from the relation $\Gamma_n^0=\Gamma_n/\sqrt{E_0}$. The parameters of the 0.29- and 1.13-ev resonances are discussed in Sec. IV. The reduced neutron widths of the resonances vary over a very wide range (approximately a factor of 400 in the energy region below 20 ev, where we have concluded from Fig. 2 that all the resonances have been resolved). The average value of $2g\Gamma_n^0$ for the levels below 20 ev is 0.09 ± 0.02 mv. This value of $2g\Gamma_n^0$ is very small compared to the values found for most nuclei,[20] as would be expected from the small level spacing.

In order to study the distribution of reduced widths, the integral of the number of levels *versus* width is plotted in Fig. 4. An exponential distribution having the same average value as the reduced widths is represented by the straight line of Fig. 4. The distribution of reduced neutron widths has a shape similar to that for other nuclides,[21] but deviates from an exponential distribution considerably more than other nuclides studied.

The $\bar{\Gamma}_n^0/D$ ratio or "strength function," of importance to nuclear structure theory, is obtained from Fig. 5,

TABLE I. Resonance parameters of U^{235}. The results for the levels where no Γ's are listed are based on an average Γ of 100±20 mv. (1 mv=10^{-3} ev.)

E_0 (ev)	Δ (ev)	Sample thickness $n\times10^{-21}$ (atom/cm²)	Area (% error) (ev)	Γ_n (% error) (mv)	Weighted Γ_n (mv)	Reduced width Γ_n^0 (mv)	Γ (mv)
0.290±0.005					0.0037±0.0001	0.0069±0.0002	138±5
1.13±0.01					0.0146±0.0006	0.0138±0.0006	142±7
2.04±0.03	0.030	3.38	0.021 (5)	0.0066 (7)	0.0066±0.0003	0.0046±0.0002	43±5
		7.45	0.043 (15)	0.0065 (18)			
		10.84	0.063 (10)	0.0070 (13)			
		28.0	0.13 (10)	0.0070 (17)			
2.82±0.05	0.035	7.45	0.014 (25)	0.0026 (25)	0.0026±0.0007	0.0015±0.0004	
3.14±0.02	0.037	1.42	0.021 (15)	0.029 (16)	0.028±0.002	0.016±0.001	150±40
		3.38	0.051 (5)	0.025 (6)			
		7.45	0.11 (10)	0.030 (11)			
3.61±0.02	0.040	3.38	0.087 (5)	0.054 (7)	0.053±0.003	0.028±0.002	80±20
		7.45	0.16 (8)	0.053 (13)			
4.84±0.02	0.046	0.691	0.016 (15)	0.055 (15)	0.054±0.005	0.025±0.002	46±20
		1.74	0.035 (15)	0.053 (16)			
		7.45	0.12 (10)	0.055 (15)			
5.45±0.10	0.049	7.45	0.056 (20)	0.022 (20)	0.022±0.004	0.009±0.002	
5.83±0.10	0.051	7.45	0.040 (20)	0.016 (20)	0.016±0.003	0.0066±0.001	
6.12±0.10	0.052	7.45	0.063 (15)	0.027 (15)	0.027±0.004	0.011±0.002	
6.40±0.05	0.053	0.691	0.062 (10)	0.32 (10)	0.30±0.03	0.119±0.012	53±10
		7.45	0.310 (5)	0.29 (10)			
7.10±0.05	0.056	0.691	0.018 (30)	0.091 (30)	0.11±0.02	0.041±0.006	56±15
		7.45	0.17 (10)	0.116 (14)			
8.82±0.07	0.062	0.691	0.16 (10)	1.3 (13)	1.3±0.1	0.43±0.04	95±15
		1.74	0.31 (10)	1.3 (20)			
		3.38	0.43 (5)	1.3 (13)			
9.3±0.1	0.064	3.38	0.075 (15)	0.11 (17)	0.11±0.02	0.036±0.007	
9.8±0.1	0.066	3.38	0.019 (20)	0.026 (20)	0.025±0.004	0.008±0.001	
		7.45	0.036 (15)	0.024 (16)			
10.2±0.1	0.067	3.38	0.046 (15)	0.072 (15)	0.065±0.008	0.020±0.002	
		7.45	0.084 (10)	0.063 (12)			
10.6±0.1	0.068	3.38	0.029 (20)	0.045 (20)	0.033±0.007	0.010±0.002	
		7.45	0.038 (15)	0.027 (16)			
11.1±0.1	0.070	3.38	0.026 (20)	0.042 (20)	0.045±0.007	0.014±0.002	
		7.45	0.062 (15)	0.049 (18)			
11.7±0.1	0.072	0.691	0.081 (7)	0.79 (10)	0.79±0.08	0.23±0.02	42±15
		3.38	0.24 (10)	0.78 (20)			
		7.45	0.35 (10)	0.77 (28)			
12.4±0.1	0.074	0.691	0.128 (8)	1.4 (10)	1.4±0.1	0.39±0.04	61±15
		3.38	0.37 (5)	1.4 (16)			
		7.45	0.49 (5)	1.2 (19)			
13.4±0.1	0.077	3.38	0.04 (40)	0.08 (40)	0.08±0.02	0.021±0.006	
		7.45	0.08 (40)	0.08 (43)			
13.8±0.1	0.078	3.38	0.070 (20)	0.15 (20)	0.15±0.03	0.040±0.008	
		7.45	0.13 (20)	0.14 (23)			
14.1±0.1	0.079	3.38	0.083 (15)	0.19 (18)	0.20±0.03	0.053±0.008	
		7.45	0.17 (15)	0.20 (18)			
14.6±0.2	0.080	3.38	0.074 (10)	0.17 (12)	0.17±0.02	0.044±0.004	
		7.45	0.14 (10)	0.17 (12)			
15.5±0.2	0.082	3.38	0.11 (8)	0.27 (10)	0.27±0.03	0.069±0.008	
16.1±0.2	0.084	3.38	0.11 (10)	0.29 (13)	0.29±0.04	0.07±0.01	
16.8±0.2	0.086	3.38	0.10 (10)	0.28 (13)	0.29±0.04	0.07±0.01	
18.1±0.2	0.089	0.691	0.032 (25)	0.43 (25)	0.46±0.07	0.11±0.02	
		3.38	0.15 (10)	0.47 (13)			
18.7±0.2	0.091	0.691	0.022 (30)	0.30 (30)	0.20±0.06	0.046±0.014	
		7.45	0.079 (40)	0.11 (40)			
19.3±0.2	0.092	0.691	0.18 (5)	3.2 (7)	3.2±0.3	0.73±0.07	120±20
		3.38	0.55 (10)	3.2 (12)			
		7.45	0.78 (5)	4.4 (22)			
20.6±0.2	0.095	7.45	0.19 (2)	0.36 (26)	0.36±0.11	0.08±0.02	
21.1±0.2	0.096	7.45	0.37 (15)	0.93 (23)	0.9±0.3	0.20±0.06	
22.1±0.3	0.098	7.45	0.057 (25)	0.09 (25)	0.09±0.02	0.019±0.005	
22.9±0.3	0.100	3.38	0.15 (10)	0.58 (13)	0.65±0.08	0.14±0.02	
		7.45	0.34 (10)	0.85 (17)			
23.5±0.3	0.102	3.38	0.30 (8)	1.5 (12)	1.8±0.2	0.37±0.04	
		7.45	0.61 (10)	2.4 (22)			
24.4±0.3	0.104	3.38	0.11 (10)	0.44 (12)	0.44±0.06	0.09±0.01	
25.3±0.3	0.105	3.38	0.093 (15)	0.38 (17)	0.38±0.07	0.076±0.014	
25.8±0.3	0.106	3.38	0.075 (15)	0.31 (17)	0.31±0.06	0.06±0.01	
26.5±0.3	0.108	3.38	0.11 (15)	0.49 (17)	0.49±0.09	0.097±0.017	
27.3±0.3	0.109	3.38	0.025 (40)	0.11 (40)	0.11±0.04	0.021±0.008	
27.9±0.3	0.111	3.38	0.16 (10)	0.78 (13)	0.78±0.11	0.15±0.02	
30.7±0.4	0.116	3.38	0.087 (15)	0.44 (14)	0.44±0.07	0.079±0.012	
31.1±0.4	0.117	3.38	0.088 (15)	0.45 (16)	0.45±0.08	0.081±0.014	
32.1±0.4	0.119	3.38	0.20 (10)	1.1 (13)	1.2±0.2	0.20±0.03	
33.6±0.4	0.121	3.38	0.31 (10)	2.2 (16)	2.2±0.4	0.38±0.08	
34.6±0.5	0.123	3.38	0.47 (12)	4.5 (18)	4.5±0.9	0.76±0.15	
35.3±0.5	0.124	3.38	0.62 (8)	7.7 (16)	7.7±1.5	1.3±0.3	

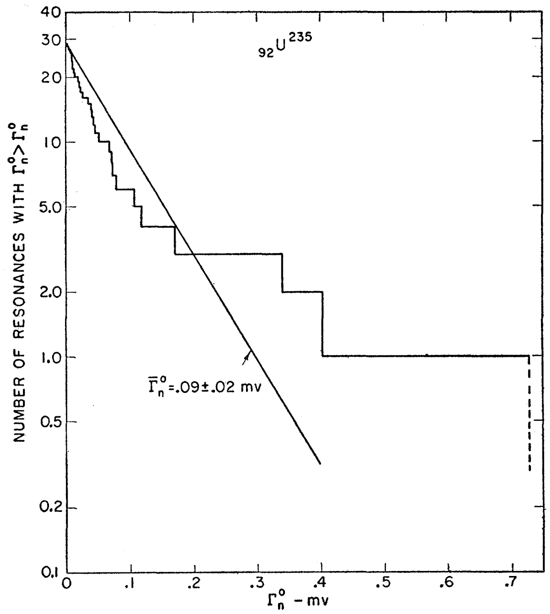

FIG. 4. The distribution of reduced neutron widths Γ_n^0 of the first 30 levels in U^{235}. The number of resonances with widths exceeding a particular value is plotted against this value. The straight line represents an exponential distribution with an average Γ_n^0 of 0.09 mv, which is the arithmetic average of the 30 widths.

which is a plot of the sum of the reduced neutron widths *versus* energy. The slope of the straight line gives a value for the ratio $\bar{\Gamma}_n^0/D$ of $(0.9\pm0.2)\times10^{-4}$. This value is in agreement with measurements on other nuclei in this region of atomic weight but considerably greater than that predicted by the cloudy crystal ball model.[26] Although the level spacing is obtained from the levels below 20 ev only, a wider energy range can be used to determine $\bar{\Gamma}_n^0/D$. The wider usable energy range results from the fact that this ratio is simply the sum of $g\Gamma_n^0$ for all the resonances in a certain energy range, divided by the range. Thus, if small resonances are missed above 20 ev, they have a very small effect on the sum, hence on $\bar{\Gamma}_n^0/D$. Also, for thin samples, if a transmission dip is really a combination of two unresolved resonances, the value of $g\Gamma_n^0$ computed in the analysis is just the sum of the two individual values of $g\Gamma_n^0$.

All of the results presented in the present section are obtained from the total cross section alone. In order to separate the total width into radiation and fission, however, it is necessary to determine the average Γ_γ. This determination, which involved the fission as well as the total cross section, will be made in the next section.

IV. DETERMINATION OF THE AVERAGE RADIATION WIDTH

As explained in Sec. II, the radiation widths of resonances can be obtained by analyzing both the total and fission cross-section data for those few cases in which precise measurements are available. For U^{235}, the data for four resonances are sufficiently accurate to permit determination of the radiation widths to an accuracy of about 20%. The fission data on other resonances have not been obtained with sufficiently high resolution to determine radiation widths to better than about 40%. The four resonances for which careful analysis is possible are those at 0.29, 1.13, 2.04, and 4.84 ev. In the present paper we shall analyze the best world data as summarized by Harvey and Sanders[14] in order to derive values for Γ_γ. The 0.29- and 1.13-ev resonances are quite broad ($\Gamma\approx0.14$ ev), and it is only because the data on both the total and fission cross sections are accurate to a few percent that values for Γ_γ can be obtained.

Figure 6 is a plot of the absorption (radiative capture plus fission) cross section of U^{235} multiplied by $\sqrt{E}$, taken from reference 14. The data are plotted as $\sigma_{abs}\times\sqrt{E}$ rather than σ_{abs} to eliminate the approximate $1/v$ nature of absorption cross sections at low energy. The points are obtained from the best United States, United Kingdom, and Union of Soviet Socialist Republics results by subtracting the appropriate scattering cross section.[14] From this figure it is obvious that there are well-resolved levels at 0.29 and 1.13 ev, and that these resonances are superimposed on a "tail" of negative-energy resonances, whose contribution to σ_{abs} does not follow a $1/v$ law in the thermal energy region. It has already been pointed out in Sec. III that evidence for a tail of negative-energy resonances had been obtained from the total cross-section measurements below 5 ev. The fission cross

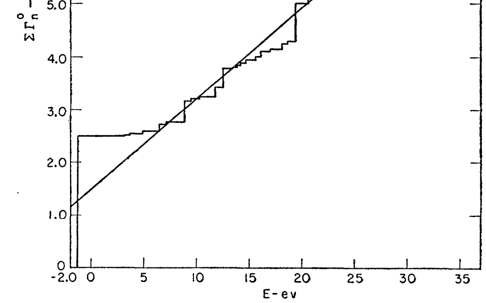

FIG. 5. The integrated reduced neutron with as a function of neutron energy, which is used to get the strength function $\bar{\Gamma}_n^0/D$. The energy interval is here extended to -2 ev to include the strong negative level at -1.4 ev (reference 14).

[26] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

section in this low-energy region, multiplied by $\sqrt{E}$, is shown in Fig. 7. This cross section is quite similar in shape to the absorption cross section but is some 20 percent lower. A complete discussion of the fit to this low-energy region, including negative-energy resonances, both with and without interference in the fission cross section, is given in reference 14.

In order to determine the parameters for the 0.29- and 1.13-ev resonances, it is necessary to establish the contribution from the tails of the negative energy resonances. This was done by computing the shape of the 0.29-ev resonance from 0 to 0.15 ev and from 0.45 to 0.60 ev by use of approximate parameters and subtracting from the observed curve to obtain the background cross section. The background cross section directly under the resonance (0.15 to 0.45 ev) was obtained by joining the computed sections with a simple curve, approximately a straight line. Similarly, the contribution of the 1.13-ev resonance from 0.7 to 0.9 ev and 1.3 to 1.5 ev was computed and subtracted from the observed curve to obtain the background cross section under this resonance.

The original absorption curve, after subtraction of the background cross section, was then analyzed by using the "shape" method of analysis.[23] The corrected fission curve was constructed in the same manner and the quantities $g\Gamma_n$ and $g\Gamma_n\Gamma_f/\Gamma$ computed from the areas under the resonances after applying a "wing" correction. For the 0.29-ev resonance, the values obtained for Γ and $g\Gamma_n$ from the absorption curve are 0.137 ± 0.005 ev and $(1.87\pm0.06)\times10^{-6}$ ev. From the fission curve the values for Γ and $g\Gamma_n\Gamma_f/\Gamma$ are 0.141 ±0.010 ev and $(1.33\pm0.07)\times10^{-6}$ ev. For the 1.13-ev resonance, the values for Γ and $g\Gamma_n$ are 0.142 ± 0.008 ev and $(7.3\pm0.3)\times10^{-6}$ ev from the absorption curve, and the values for Γ and $g\Gamma_n\Gamma_f/\Gamma$ are 0.140 ± 0.015 ev and $(5.5\pm0.3)\times10^{-6}$ ev from the fission curve. Combination of these results for the 0.29-ev resonance gives a value for Γ_γ of 0.039 ± 0.006 ev and for Γ_f of

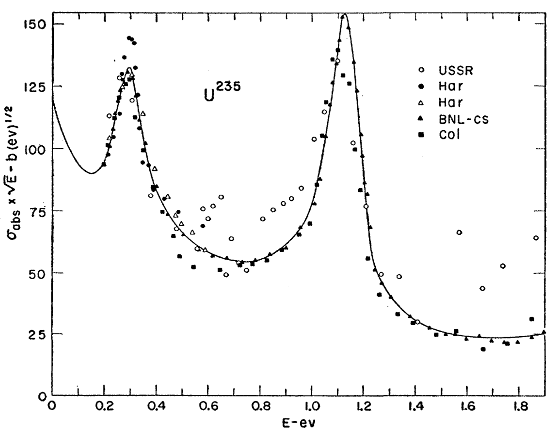

FIG. 6. A plot of $\sigma_{abs}\times\sqrt{E}$ for U^{235} from 0 to 1.9 ev (reference 14).

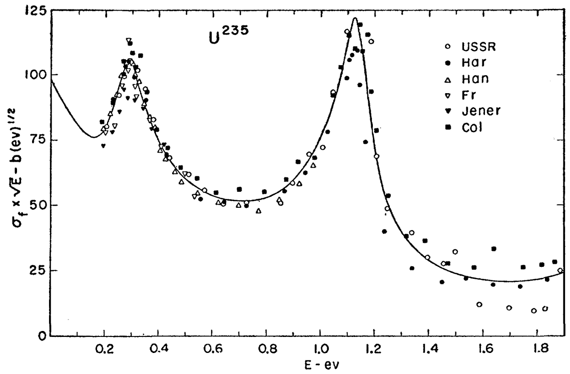

FIG. 7. A plot of $\sigma_F\times\sqrt{E}$ for U^{235} from 0 to 1.9 ev (reference 14).

0.098 ± 0.007 ev. The results for the 1.13-ev resonance give $\Gamma_\gamma=0.035\pm0.008$ ev and $\Gamma_f=0.107\pm0.010$ ev.

The two other levels from which good values for Γ_γ can be obtained are the 2.04- and 4.84-ev resonances, for both of which the fission width is considerably smaller than the radiation width. The fission cross-section data from the review by Egelstaff and Hughes[15] were used to get values of $g\Gamma_n\Gamma_f/\Gamma$ for both resonances, and the total cross-section data of Sec. III were used to compute Γ and $g\Gamma_n$ for the 2.04-ev resonance. The values of $g\Gamma_n$ and Γ for the 4.84-ev level are a result of a re-evaluation by Sailor of previously published[5] results obtained with the Brookhaven crystal spectrometer. For the 2.04-ev resonance, the quantity $g\Gamma_n\Gamma_f/\Gamma$ is $(0.89\pm0.18)\times10^{-6}$ ev from the fission cross section, and $g\Gamma_n$ and Γ are $(3.30\pm0.15)\times10^{-6}$ ev and $(0.043$ $\pm0.005)$ ev from the total cross section. For the 4.84-ev resonance, $g\Gamma_n\Gamma_f/\Gamma$ is $(3.5\pm1.7)\times10^{-6}$, from the fission data, and $g\Gamma_n$ and Γ are $(26\pm3)\times10^{-6}$ ev and (0.029 ± 0.01) ev from the total cross section.

The quantities just listed for the 2.04-ev resonance lead to values for Γ_γ and Γ_f of (0.031 ± 0.005) ev and (0.012 ± 0.003) ev, respectively. Similarly, for the 4.84-ev resonance we obtain $\Gamma_\gamma=0.025\pm0.009$ ev and $\Gamma_f=0.004\pm0.0025$ ev. These two radiation widths and those of the two lower energy resonances already considered (0.040 ± 0.006 and 0.037 ± 0.008 ev) are all consistent with a single value for Γ_γ. The weighted average Γ_γ for the four resonances is (0.035 ± 0.004) ev.

V. FISSION WIDTHS

In order to obtain fission widths for as many levels as possible, the average Γ_γ given in the preceding section (based on four levels) can now be applied to the analysis of further levels, assuming this value to be constant from resonance to resonance. Evidence from nonfissionable isotopes has shown[18] that the variation of Γ_γ from level to level is small and hence the use of the Γ_γ obtained from those few levels that have been completely analyzed can hardly be subject to much uncertainty.

The fission widths of the 0.29-, 1.13-, 2.04-, and

TABLE II. Fission widths for twelve resonances in U^{235}.

E_0(ev)	Γ_f(mv)	E_0(ev)	Γ_f(mv)
0.29±0.005	98±7	6.40±0.05	18±14
1.13±0.01	107±10	7.10±0.05	21±19
2.04±0.3	12±3	8.82±0.07	59±19
3.14±0.02	115±44	11.7±0.1	6±19
3.61±0.02	45±24	12.4±0.1	25±19
4.84±0.02	4±2.5	19.3±0.2	82±24

Average $\Gamma_f=50\pm15$ mv

4.84-ev resonances were derived in Sec. IV from the total and fission cross-section data. The fission width of each of the other levels was obtained from the results described in Sec. III by subtracting $2g\Gamma_n$ and the average Γ_γ from the total width, thus giving a fission width from the total cross section and the average Γ_γ alone. The resulting fission widths are listed in Table II for the 12 levels for which total widths have been measured.

At first sight the present method might seem to give a distinct bias to the distribution of fission widths obtained, because of the possibility that only large fission widths would be observed, the levels of small fission width being missed experimentally. Such an effect is a definite possibility in the experimental determination of the size distribution of neutron widths.[21] However, the fact that fission widths are shown for only certain levels in Table II is not a matter of the magnitude of the fission widths, but almost completely of the neutron widths, for it is Γ_n that determines the strength of a level, and hence the probability that it can be analyzed. The peak height of a level is proportional to $g\Gamma_n/\Gamma$ and, as the total width does not vary greatly from level to level compared to the variation in Γ_n, it is the latter that determines the size of a level, and whether the level can be analyzed. This procedure would introduce a bias in the distribution only if there is a correlation between the size of fission widths and of neutron widths but examination of the results indicates no such correlation.

The distribution in size of the fission widths is plotted semilogarithmically in Fig. 8. Even though the number of levels is not great, it is immediately obvious that the fission widths in their wide distribution of sizes resemble the neutron widths much more than the radiation widths. Relative to the neutron width distribution, there is an additional uncertainty in the fission case, because of the error in the average Γ_γ.

By making use of an average Γ_γ, fission widths can also be obtained from the ratio $1+\alpha$ (where $\alpha=\Gamma_\gamma/\Gamma_f$) deduced from neutron yield experiments. The neutron yield work gives the value of $\eta=\nu/(1+\alpha)$ relative to thermal, where ν is the average number of neutrons emitted per fission. ν is believed to be constant in the resonance energy region. The approach to fission width measurements followed in this paper, however, is undoubtedly preferable in the case of U^{235}, since this method takes full advantage of the high resolution available in total cross-section measurements.

VI. DISCUSSION

It is useful to recall what was expected on a theoretical basis about fission before detailed results of resonance analysis were available. About the time that fission was first being discussed theoretically,[2] the extreme compound nucleus model of nuclear reactions was the basis of the development. It was expected that fission would take place in a compound nucleus much faster than gamma emission, and hence that the fission width would be much larger than the radiation width. As a result, it was something of a surprise when it was discovered that the fission width was of the same order of magnitude as the radiation width, that is, that radiation could effectively compete with fission. According to the early theoretical views, it was also thought that the fission width would be about the same from neutron resonance to resonance. Experiments[3] performed at Los Alamos during the war, however, showed not only that radiation could compete with fission but that the ratio of radiation to fission, usually denoted α, varied markedly from level to level.

The experimental results of the present paper give sufficient detail about neutron widths and fission widths to allow a reasonably detailed comparison with theoretical predictions. Fortunately there has been sufficient work[15–19] on radiation widths for many nonfissionable nuclei so that the general properties of radiation from the highly excited states formed by neutron capture are well known. The radiation widths for various levels of a single nucleus are remarkably con-

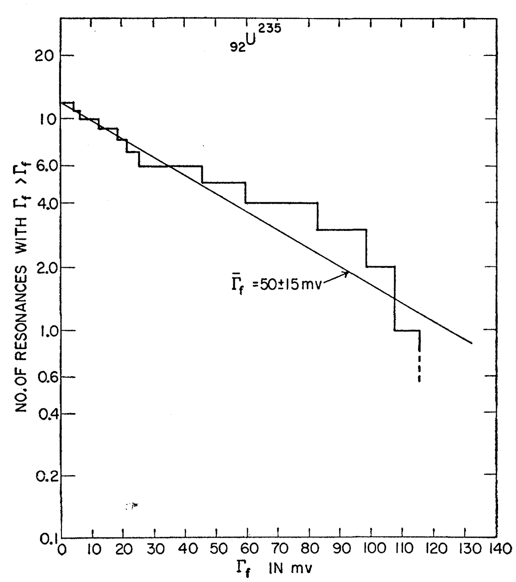

FIG. 8. The size distribution of fission widths for U^{235} (based on 12 resonances) plotted in the same manner as the neutron width distribution of Fig. 4.

stant from level to level, the differences being only of the order of magnitude of experimental error (there may be a small difference in radiation width for levels of different spin, however).[17]

The constancy of radiation width from level to level is not surprising when considered in the light of the usual interpretation of gamma emission. At excitation energies in the region corresponding to neutron binding the level density is large enough so that many levels are available as final states for gamma emission, probably of the order of hundreds of states. With so many final states available, the most rapid type of emission, electric dipole, will almost certainly take place and the observed Γ_γ will be the sum over the partial radiation widths to the individual final states. Because of this averaging over hundreds of final states or exit channels, it is not at all surprising that the observed radiation width is so nearly constant for each level. The existence of many exit channels of course implies that the cross section will not be coherent, that is, that the contributions from different resonance levels at a given point in the cross-section curve will add incoherently. In other words, for neutron capture the cross sections will add rather than their amplitudes.

The observations on neutron widths for many nuclides, on the other hand, show that within a single nuclide the neutron widths of various levels differ by extremely large factors. In contrast to the radiation width, this large spread in magnitude is associated with the unique nature of the neutron emission in each level, with only one final state being available, and as a result only a single exit channel existing for the reaction. Because of the single channel, for neutron emission, the scattering cross section is a coherent process and its value at a given point is made up of coherently added contributions from various levels. Here again, as for the radiation widths, the observed behavior of the widths is consistent with what is expected theoretically, for in the case of neutron emission a single final state is implied for energetic reasons in contrast to the many final states for emission of radiation.

For fission the expectation had been that the process more closely resembled radiation than neutron emission, the reason being that the presence of many possible fission products was similar to the many possible final states in radiation rather than the single final state possible in elastic scattering. However, it is obvious that the fission width distribution of Fig. 8 resembles much more closely the typical neutron width distribution than the single value typical of radiation emission. Judging from the distributions of widths only, it would be concluded that fission is much more nearly like neutron emission than radiation.

Before turning to the question of the manner in which fission can be considered a unique process rather than an averaging over many final states, it is well to consider how closely the fission width distribution resembles that of the neutron widths. The neutron width distribution reported by Hughes and Harvey[18] for a number of nonfissionable nuclides has been discussed in detail by Thomas and Porter.[22] A particular neutron width distribution suggested by Thomas and and Porter seems to fit the experimental distribution of Hughes and Harvey somewhat better than the simple exponential first used as a fit to the data by Hughes and Harvey. This Thomas-Porter distribution is appropriate to the widths for a process in which only one exit channel (one degree of freedom in their notation) is possible, such as is true for the neutron widths corresponding to elastic scattering.

At the other extreme, a process in which several hundred exit channels are possible gives a distribution in which only a single width is found as for radiation widths. The fission width distribution found in the present work is actually slightly concave downward, indicating a slight tendency toward the gamma-ray distribution and leading one to expect a small number of exit channels rather than a single channel. A complete analysis of the observed fission widths of the present paper has been carried out by Thomas,[27] who finds that the most likely number of channels is 4 ± 1. It is certainly true that the number of channels in fission is small but the present data are not sufficiently accurate to fix the number of exit channels with high precision.

Another way of getting at the number of exit channels in fission is by use of a relation given by Wheeler,[28]

$$\bar{\Gamma}_F/D=N/2\pi,$$

where N is the "number of levels available to the nucleus in its critical state," i.e., the number of exit channels. If the numbers found in the present work are substituted into this relationship, it is found that a value of $N=0.24$ is obtained. While the quantitative accuracy of the above relation is probably not to be taken too seriously, the result of this relationship certainly seems to be that the number of exit channels is extremely small.

It should be possible to get more information on the nature of the fission process, in particular the number of exit channels involved, by means of other experiments. In general, if fission were a process that is an average over many exit channels in the normal sense, then it would be expected that particular properties of fission, such as the fission product mass distribution, the number of neutrons per fission, or the number of delayed neutrons, for example, would be the same in each neutron resonance.

Some evidence is available on the fission mass distribution and the delayed neutron yield as functions of neutron energy but both these quantities seem to

[27] R. G. Thomas (private communication).

[28] J. A. Wheeler, *Proceedings of International Conference on the Peaceful Uses of Atomic Energy* (United Nations, New York, 1956), Vol. 2, p. 155.

show no variation with neutron energy, implying that they are the same for each energy level. Recent accurate measurements of η, the number of fission neutrons per neutron absorbed, have been made for several fissionable isotopes. These measurements[29] show large changes in η with energy, but of course this variation is simply a matter of the different values of Γ_F from level to level. The η results combined with fission cross-section measurements seemed for a time to indicate that ν changed with energy in the thermal region for plutonium, but recent direction measurements[30–34] of ν show that it is constant from level to level. Another conclusion to be drawn from the small number of channels in fission that could be checked experimentally is the coherence between fission from level to level that should be present if the number of exit channels is extremely small. Unfortunately the interference effects are extremely difficult to detect. Whereas initial measurements[35] seemed to show the definite occurrence of interference, at the present time the situation is not at all clear.

The fundamental theoretical problem that arises in connection with the experimental results is the explanation of the very small number of exit channels in fission ln spite of the large number of fission products that are formed at each level. The entire fission product mass distribution is formed in each neutron resonance level; yet in spite of this large number of final products, the number of exit channels is certainly extremely small. The recent theories of fission of Wheeler[28] and Bohr[36] picture the process as one that is much simpler than had been envisaged in the past. Instead of a highly excited nuclear state, these new theories depict the fission process as one in which the nuleus has a very simple excitation, one that can be described by a very small number of quantum numbers. Corresponding to this simple excitation there are only a very few exit channels, and in some way it is only after these exit channels are passed that the formation of the hundreds of different fission products take place.

Related to the simplicity of the nuclear excitation, current fission theories predict that the properties of fission should depend strongly on the spin of the compound nucleus. Thus if slow neutrons are above the fission threshold for both spin states, there should be two types of levels involved. If thermal neutrons are above threshold for only one spin state, however, all the observed levels should have the same spin. From the results of the present paper there seems to be no evidence for two types of levels occurring in fission, all the parameters apparently being members of one smooth distribution. The other possibility that only one spin state is present cannot, unfortunately, be checked at the present time. Spin determinations of the various levels can be made if the scattering cross section of the resonances can be measured, but this is very difficult in U^{235} because of the small value of Γ_n and hence the small peak height of the scattering resonances. Another possibility is that the levels of one spin state might have such large fission widths that they lead to a constant background in the fission cross section. Some such background fission cross section has been observed especially in plutonium, but its magnitude is much too low to be ascribed to the other spin state.

While direct measurements on a number of interesting predictions of the current fission theories cannot, unfortunately, be made at the present time because of experimental difficulties, the rapid accumulation of data of various types will certainly prove to be very useful in checking the accuracy of the recent theories.

[29] The η measurements reported by various countries at Geneva in August 1955 are reviewed by J. A. Harvey and J. E. Sanders (reference 14).

[30] Auclair, Landon, and Jacob, Compt. rend. **241**, 1935 (1955).

[31] Leonard, Seppi, and Friesen, Bull. Am. Phys. Soc. Ser. II, **1**, 8 (1956).

[32] Bollinger, Coté, Hubert, LeBlanc, and Thomas, Bull. Am. Phys. Soc. Ser. II, **1**, 165 (1956).

[33] Kalashinikov, Tebedev, Mikallyan, and Pevzner (private communication).

[34] Palevsky, Hughes, Zimmerman, and Eisberg, J. Nuclear Energy (to be published).

[35] V. L. Sailor, *Proceedings of International Conference on Peaceful Uses of Atomic Energy* (United Nations, New York, 1956), Vol. 4, p. 199.

[36] A. Bohr, *Proceedings of International Conference on Peaceful Uses of Atomic Energy* (United Nations, New York, 1956), Vol. 2, p. 151.