

Decay of  $\text{Eu}^{155}$ ,  $\text{Sm}^{153}$ ,  $\text{Sn}^{125}$ , and  $\text{Br}^{82}\dagger$ 

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(Received April 2, 1956)

The energies of the beta and gamma rays associated with the decay of  $\text{Eu}^{155}$ ,  $\text{Sm}^{153}$ ,  $\text{Sn}^{125}$ , and  $\text{Br}^{82}$  have been determined by scintillation spectrometry, and coincidence relationships have been investigated when possible. In most instances, values of  $\log ft$  for the beta spectra have been determined from a consideration of the decay scheme and the relative intensities of gamma rays. The results obtained have been interpreted as indicating the following excited energy states of the residual nuclei.  $\text{Gd}^{155}$ : 18 and 102 kev;  $\text{Eu}^{155}$ : 100, 170, and 700 kev;  $\text{Sb}^{125}$ : 1070, 1410, 1880, 1970, and 2110 kev;  $\text{Kr}^{82}$ : 770, 1450, 1800, 2060, and 2620 kev.

The results of the measurements have been considered in relation to the single-particle model of the nucleus to ascertain likely spin values.

## INTRODUCTION

IN the present investigation, a group of radio-nuclides have been studied to determine the decay schemes. Coincident scintillation spectrometers employing crystal detectors of  $\text{NaI(Tl)}$  and anthracene have been used throughout. In the cases considered, the radio-elements had been investigated only partially or not at all by scintillation spectrometry. The purpose of the present measurements was to determine more definitely the cascade relationships among, and the energies and relative intensities of, the various gamma rays associated with the de-excitation of the several residual nuclei, to order the levels and determine likely spin values.

## Europium-155

Early investigations<sup>1-3</sup> of the quantum radiations of  $\text{Eu}^{155}$  have shown gamma rays to be present with energies of  $\sim 85$  and  $\sim 100$  kev. More recently,<sup>4-7</sup> additional gamma rays have been reported. Rutledge *et al.*<sup>4</sup> have observed, in a study of conversion lines, gamma rays of energies 60, 87, 106, and 132 kev. In addition to these radiations, Lee and Katz<sup>5</sup> have reported quanta at energies of 18.7 and 136.8 kev. Wilson and Lewis<sup>6</sup> have presented a decay scheme including only gamma rays at 15 and 85 kev in cascade and a crossover transition at 100 kev. Church and Goldhaber<sup>7</sup> have noted the presence of gamma rays at 59.8, 86.3, and 105 kev, and have further concluded that the radiations at  $\sim 130$  kev arise from the presence of  $\text{Eu}^{156}$ .

There has been general agreement<sup>1,4-6</sup> that two groups of beta rays are present of maximum energies  $\sim 150$  and  $\sim 250$  kev. Because of the uncertainties surrounding the nature of the gamma rays, and since it appears that the unconverted quantum radiations of  $\text{Eu}^{155}$  have not been previously viewed by scintillation counters, the beta rays and gamma rays of  $\text{Eu}^{155}$  have been measured in coincident scintillation spectrometers.

A sample of  $\text{Eu}^{155}$ , in the form of  $\text{EuCl}_3$  in  $\text{HCl}$  solution, was obtained from Oak Ridge National Laboratory, and scintillants of  $\text{NaI(Tl)}$  and anthracene were employed to study gamma-gamma and beta-gamma coincidences. The gamma-ray spectrum of  $\text{Eu}^{155}$  is shown in Fig. 1. The composite peak has been resolved into two components at 84 and 102 kev. The 42-kev x-rays of gadolinium are also apparent. In addition to these radiations, gamma rays were also detected at 122,  $\sim 250$ ,  $\sim 730$ , and  $\sim 1280$  kev. The position of the 122-kev gamma ray is indicated, but the others, very weak, are not shown. These quanta of higher energy<sup>8</sup> are

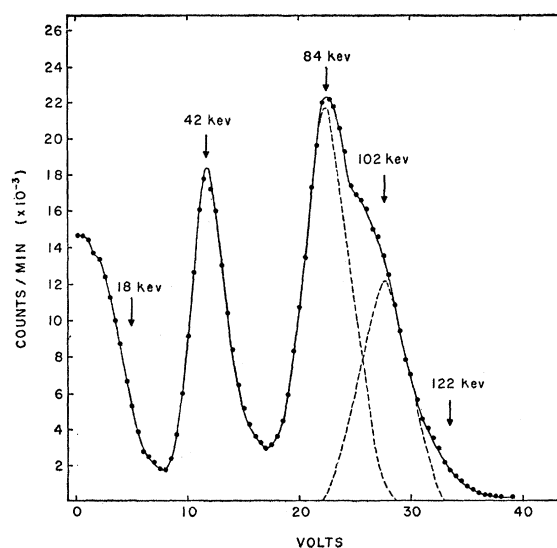


FIG. 1. Gamma rays emitted in the decay of  $\text{Eu}^{155}$ . The evidence for radiation at or near 122 kev is attributed to the presence of  $\text{Eu}^{152,154}$ .

<sup>†</sup> Assisted by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

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<sup>1</sup> Marinsky, Glendenin, and Metzger, Massachusetts Institute of Technology Report NP-1727, July, 1949 (unpublished).

<sup>2</sup> B. H. Ketelle, Oak Ridge National Laboratory Report ORNL-607, March, 1950 (unpublished).

<sup>3</sup> L. Winsberg, *Radiochemical Studies: The Fission Products* (McGraw-Hill Book Company, Inc., New York, 1951). National Nuclear Energy Series, Plutonium Project Record, Vol. 9, p. 1311.

<sup>4</sup> Rutledge, Cork, and Burson, *Phys. Rev.* **86**, 775 (1952).

<sup>5</sup> M. R. Lee and R. Katz, *Phys. Rev.* **93**, 155 (1954).

<sup>6</sup> H. W. Wilson and G. M. Lewis, *Proc. Phys. Soc. (London)* **A65**, 656 (1952).

<sup>7</sup> E. L. Church and M. Goldhaber, *Phys. Rev.* **95**, 626(A) (1954).

considered to arise from the presence of  $\text{Eu}^{152,154}$ . This assignment will be discussed later.

The beta-ray spectrum of  $\text{Eu}^{155}$  was measured in an anthracene counter and resolved into several components obtained from a Fermi-Kurie plot. These spectra are shown in Fig. 2. The maximum energies are  $680 \pm 20$ ,  $300 \pm 10$ ,  $240 \pm 10$ , and  $150 \pm 10$  kev. Very weak contributions to the beta distribution were noted to extend to energies as great as  $\sim 1800$  kev. Taking the two softer spectra to be associated with  $\text{Eu}^{155}$ , their relative intensities are calculated to be 240 kev (23 percent) and 150 kev (77 percent). These results compare favorably with earlier data.<sup>1,5</sup> In making the foregoing measurements, one-tenth drop of the europium chloride solution was evaporated to dryness upon a Nu-skin foil of surface density less than  $0.3 \text{ mg/cm}^2$ . In this manner a thin source was obtained.

In measuring beta-gamma coincidences, the gamma-ray counter was fixed at various photopeaks while the beta-ray counter was moved in differential intervals over the beta-ray energy spectrum. In this manner, it was established that the  $K$  x-rays, the 84-kev gamma ray, and the 102-kev gamma ray are coincident with the 150-kev beta group and noncoincident with beta rays of higher energy. Coincidences were also observed between the 150-kev beta-ray group and quantum radiations in the region of the  $L$  x-rays. In a second experiment, the beta-ray counter was fixed on the energy interval extending from 150 to  $\sim 250$  kev. The gamma-ray counter was moved over the entire gamma-ray spectrum and coincidences were noted with the 42-kev x-rays and the 122-kev gamma ray, as shown in Fig. 3, curve A. When the beta-ray counter was shifted in energy to count only beta rays of energies greater than 250 kev, the beta-gamma coincidences of Fig. 3, curve B were obtained. Since equal numbers of beta rays were counted to obtain either curve, the

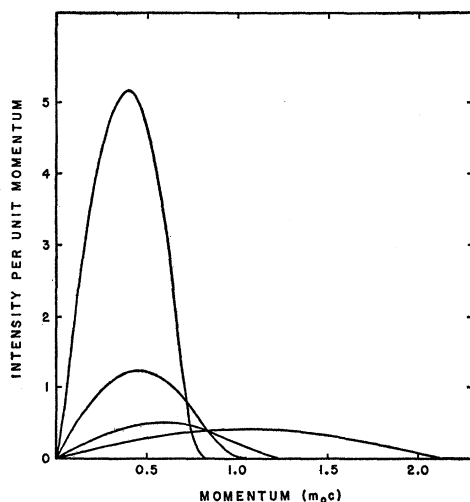


FIG. 2. Beta rays emitted in the decay of radioactive europium. The two spectra of lower energy are associated with  $\text{Eu}^{155}$ .

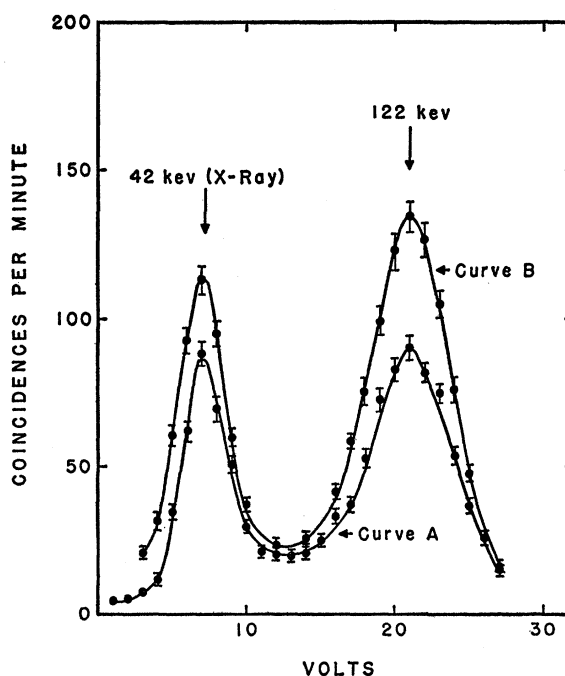
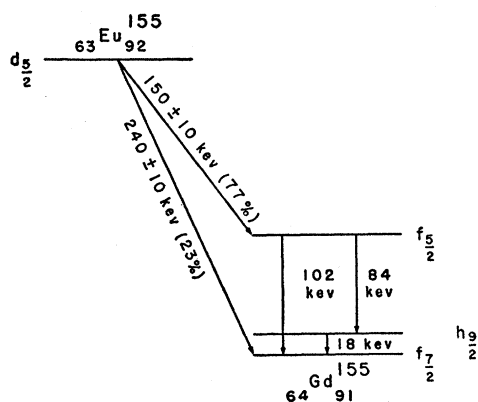


FIG. 3. Beta-gamma coincidence in  $\text{Eu}^{152,154}$ . Curve A, with beta rays on the interval  $150 \text{ kev} < E_\beta < 250 \text{ kev}$ ; Curve B, with  $E_\beta \geq 250 \text{ kev}$ .

increase of coincidences in curve B over those of curve A shows the beta spectrum of maximum energy 250 kev to be noncoincident with the 122-kev gamma ray. Thus, the spectrum of end-point energy 250 kev is coincident with no gamma rays or x-rays. The 122-kev gamma ray is coincident with the hard beta rays presumably emitted in the decay of  $\text{Eu}^{152,154}$ .

Gamma-gamma coincidences were observed by fixing one channel of the coincidence spectrometer at individual photopeaks while moving the other channel over the entire gamma-ray spectrum. With the fixed counter at the 84-kev photopeak, coincidences were observed only in the region of  $L$  x-rays. No coincidences were detected between the  $K$  x-rays or the 102-kev gamma rays on the one hand and the 84-kev radiation on the other. With the fixed counter at the  $K$  x-ray peak, coincidences were detected in the  $L$  x-ray region, at the  $K$  x-ray photopeak, at  $\sim 122$  kev, and with gamma rays of higher energy. No coincidences were detectable between  $K$  x-rays and the 84-kev and 102-kev photopeaks. It was concluded that the coincidences between the  $L$  x-rays, and  $K$  x-rays and the 84-kev photopeak, must arise from a gamma transition which is in cascade with the 84-kev gamma ray and heavily converted in the  $L$  shell. The previously mentioned coincidences between  $K$  x-rays and  $K$  x-ray, between  $K$  x-rays and the 122-kev gamma rays, and between  $K$  x-rays and gamma rays of higher energies are attributed to the presence of  $\text{Eu}^{152,154}$ . From these coincidence measurements and the results of beta-gamma coincidence investigations

FIG. 4. Disintegration scheme of  $\text{Eu}^{155}$ .

previously described, it can be concluded that  $\text{Eu}^{155}$  decays with the emission of two beta-ray groups having end points of  $150 \pm 10$  and  $240 \pm 10$  keV respectively. The 150-keV group terminates at an excited state of  $\text{Gd}^{155}$  which is de-excited by emission of a 102-keV gamma ray or alternately by emission of an 84-keV quantum which is in cascade with a gamma ray of energy  $(102 - 84) \text{ keV} = 18 \text{ keV}$ . This latter gamma ray is evidently highly converted in the  $L$  shell. No gamma ray of energy 59 keV<sup>4,5,7</sup> was observable among the unconverted quantum radiations of  $\text{Eu}^{155}$ . The radiation characteristics of  $\text{Eu}^{155}$  determined by the present measurements are in agreement with the results of Wilson and Lewis.<sup>6</sup> Values of  $\log ft$  for the two beta spectra of  $\text{Eu}^{155}$  have been calculated from their relative intensities as determined in the present measurements and are found to be 7.9 and 7.2 in order of descending energy.

A disintegration scheme of  $\text{Eu}^{155}$  is shown in Fig. 4. The spins of  $\text{Eu}^{151}$  and  $\text{Eu}^{153}$  have been measured<sup>8</sup> as  $5/2$ . According to the shell model of the nucleus, the spin of  $\text{Eu}^{155}$  should also be  $5/2$ . The ground state spin of  $\text{Gd}^{155}$  has been measured by Jenkins and Speck<sup>9</sup> as  $3/2$  and by Murakawa<sup>9</sup> as  $\geq 5/2$ . An attempt was made to construct a decay scheme with the ground state of  $\text{Gd}^{155}$  having a spin of  $3/2+$ ; however, no satisfactory sequence of orbitals could be developed which would account for all aspects of the measured properties of the decay. Accordingly an orbital assignment of  $f_{7/2}$  was taken for the ground state of  $\text{Gd}^{155}$  as indicated by the shell model of the nucleus. The two excited states of  $\text{Gd}^{155}$  have also been given spin values predicted by the single-particle model. These assignments are consistent with the presence of the two first-forbidden beta spectra and with the absence of a spectrum terminating at the first excited state of  $\text{Gd}^{155}$ .

<sup>8</sup> H. Schüller and T. Schmidt, Z. Physik **94**, 457 (1935).

<sup>9</sup> F. A. Jenkins and D. R. Speck, Phys. Rev. **100**, 973(A) (1955); K. Murakawa, Phys. Rev. **96**, 1543 (1954).

### Samarium-153

A large number of measurements<sup>4,5,10-19</sup> have been carried out to determine the energies and relative intensities of the beta and gamma radiations of  $\text{Sm}^{153}$ . Early investigations<sup>10,11,13</sup> have indicated the presence of two beta-ray spectra and gamma rays of energies 110 and  $\sim 600$  keV. Hill<sup>12</sup> has reported in a photographic study of conversion lines the presence of gamma rays at 69.5 and 103 keV. Beta rays of maximum energy 0.70 MeV and gamma rays of energies 70, 103, and 530 keV have been observed by Siegbahn.<sup>14</sup> Sunyar<sup>15</sup> has noted, in addition to the above-cited gamma rays, radiations of quantum energy  $\sim 600$  keV. Rutledge *et al.*<sup>4</sup> have reported quanta at energies of 69.8, 103.4, and 582 keV. The  $K/L$  ratios of the 69- and 103-keV transitions have been reported with widely differing values.<sup>4,14-16</sup>

In later investigations,<sup>5,17-19</sup> attempts have been made to construct a decay scheme of  $\text{Sm}^{153}$ . All of these agree in that more than two beta spectra are considered to be present. A compilation of the results of these more recent studies is shown in Table I. Briefly, it can be said that Bannerman<sup>17</sup> reports no high-energy gamma rays; Lee and Katz<sup>5</sup> find that a 548-keV gamma-ray transition terminates at the ground state of  $\text{Eu}^{153}$  and do not mention the radiations at 170 or 600 keV; Graham and Walker<sup>18</sup> have not indicated a position in the decay scheme for their 520-keV gamma ray nor do they mention radiations of higher energy. Marty<sup>19</sup> has

TABLE I. Radiations of  $\text{Sm}^{153}$ .

Investigators	Beta rays (keV)	Gamma radiations (keV)
Bannerman <sup>a</sup>	800 (15%), 700 (35%), 630 (50%)	69.5 and 101; no $\gamma$ rays of higher energy observed
Lee and Katz <sup>b</sup>	795 (20%), 685 (70%), 620 (<0.6%), 255 (9%)	69, 102, and 548
Graham and Walker <sup>c</sup>	810 (20%), 710 (50%), 640 (30%)	69, 102.5, 170 (weak), and 520
Marty <sup>d</sup>	820 (20%), 720 (40%), 650 (40%)	69, 103, 172, 545, 84 (weak)

<sup>a</sup> See reference 17.

<sup>b</sup> See reference 5.

<sup>c</sup> See reference 18.

<sup>d</sup> See reference 19.

<sup>10</sup> L. C. Miller and L. F. Curtis, Phys. Rev. **70**, 983 (1946).

<sup>11</sup> S. B. Burson and C. O. Muehlhause, Phys. Rev. **74**, 1264 (1948).

<sup>12</sup> R. D. Hill, Phys. Rev. **74**, 78 (1948).

<sup>13</sup> J. M. Hill and L. R. Shepherd, Proc. Phys. Soc. (London) **A63**, 126 (1950).

<sup>14</sup> K. Siegbahn in *M. Siegbahn Commemorative Volume* (Almqvist and Wiksells, Boktryckeri AB, Uppsala, 1951), pp. 237-241.

<sup>15</sup> A. W. Sunyar, private communication to Hollander, Perlman, and Seaborg [Revs. Modern Phys. **25**, 469 (1953)].

<sup>16</sup> F. K. McGowan, Phys. Rev. **80**, 482 (1950); **93**, 163 (1954).

<sup>17</sup> R. C. Bannerman, Proc. Phys. Soc. (London) **A65**, 565 (1952).

<sup>18</sup> R. L. Graham and J. Walker, Phys. Rev. **94**, 794(A) (1954).

<sup>19</sup> N. Marty, J. phys. radium **16**, 458 (1955).

reported a 545-kev transition to terminate at the first excited state and to be coincident with a 103-kev gamma ray; radiation at or near 600 kev was not reported,<sup>19</sup> but weak radiation at 84 kev was observed. In view of the divergent results of the previous studies, it has seemed worthwhile to consider further the problem of the decay scheme of  $\text{Sm}^{153}$ .

For the present investigations, samples of  $\text{Sm}^{153}$ , in the form of  $\text{Sm}_2\text{O}_3$ , were obtained from Oak Ridge National Laboratory on four different occasions, and the radiations were studied in coincident scintillation spectrometers. Scintillants of  $\text{NaI}(\text{Tl})$  and anthracene were employed to study beta-gamma and gamma-gamma coincidences. The crystals of  $\text{NaI}(\text{Tl})$  were 4 cm in thickness and of diameter 3.5 cm. The anthracene crystal had a diameter of 2.5 cm and a thickness of 1.1 cm.

The spectrum of low-energy  $\gamma$  radiation is shown in Fig. 5A. Sufficient carbon absorber was used to stop beta rays of energy as much as 850 kev. The 41-kev  $K$  x-rays of Eu, and gamma rays at 70 kev and 100 kev are clearly indicated by their peaks. In the region of lower energy, escape pulses of the 41-kev x-rays and contributions due to  $L$  x-rays can also be observed. Continuation of the study above 100 kev showed the presence of high-energy  $\gamma$  radiation. By using suitable absorbers to suppress the x-ray and 100-kev radiations and employing a stronger source, it was possible to observe these as shown in Fig. 5B, where 170-, 530-, and 600-kev photopeaks are in evidence. All portions of the spectra of Fig. 5 were found to decay with a half-life of 47 hours. It is of interest to note that a continuous gamma-ray distribution is present, extending from below the position of the 170-kev photopeak

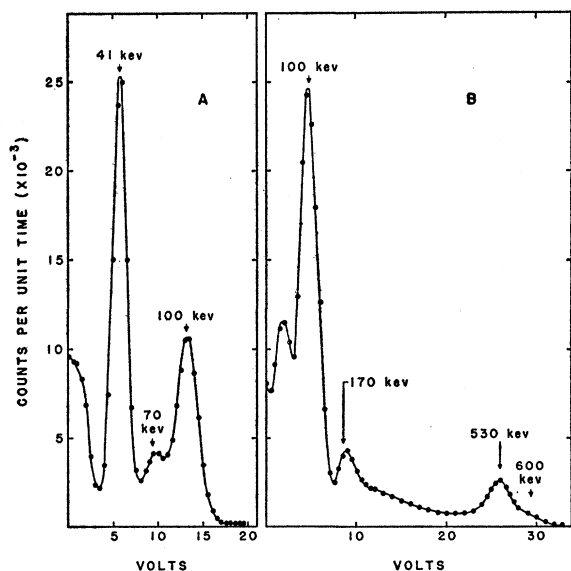


FIG. 5. Curve A, the soft quantum radiations of  $\text{Sm}^{153}$ ; Curve B, the gamma-ray spectrum of  $\text{Sm}^{153}$  *in toto*. The data of Curve B were obtained with the use of absorbers of Pb, Cd, and Cu.

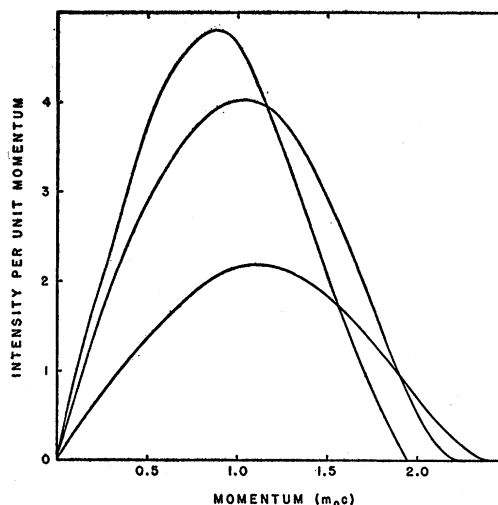


FIG. 6. Spectra of the harder beta rays emitted in the disintegration of  $\text{Sm}^{153}$ .

to higher energies. By observing the intensity of this distribution through various absorbers, it was concluded that it arises from the inner bremsstrahlung of the beta process. This effect is also markedly evident in the work of Marty.<sup>19</sup>

In Fig. 6 are shown three beta-ray groups constructed from a Fermi-Kurie plot of the beta-ray distribution of  $\text{Sm}^{153}$  as observed in an anthracene counter. The maximum energies and relative intensities of these spectra are given in Table II.

Beta-gamma coincidences were measured by placing the gamma-ray counter at the various photopeaks while the beta-ray counter was moved over all energies of the beta-ray spectrum. The results of these coincidence measurements are given in Table III.

The beta-gamma coincidence rate of the particular case of the gamma rays at 530 and 600 kev is shown in Fig. 7. To obtain the data, the gamma-ray counter was set to count all pulses falling in the composite peak of the 530- and 600-kev gamma rays. The channel width of the beta-ray counter was two volts. The beta-gamma coincidence rate clearly indicates a beta-ray end-point energy at  $\sim 8$  volts which, according to calibration of the anthracene crystal by the conversion line of the 661-kev gamma ray of  $\text{Cs}^{137}$ , corresponds to an energy of 130 kev. Additional curves, similar in nature to the data of Fig. 7, were obtained by placing the gamma-ray counter first on the low-energy side, then on the high-energy side of the 530 kev–600 kev

TABLE II. Beta spectra of  $\text{Sm}^{153}$ .

End-point energy (kev)	Relative intensity (percent)
$825 \pm 10$	22
$720 \pm 10$	38
$645 \pm 10$	40

TABLE III. Gamma rays coincident with beta rays of  $\text{Sm}^{153}$ .

$E_\beta$ (max, kev)	Quantum energy (kev)
$825 \pm 10$	None
$720 \pm 10$	100
$645 \pm 10$	70, 170
130	530, 600

peak. The two curves obtained in this manner were identical in end point with the curve of Fig. 7. Thus was established the fact that the 530-kev gamma ray and the 600-kev gamma ray are very probably emitted from the same level in  $\text{Eu}^{153}$ .

To observe gamma-gamma coincidences, one pulse-height analyzer was fixed to accept the pulses of the 100-kev photopeak while the other channel was moved with a width of one volt over the spectral region of lower energy. The coincident relationship between the 100-kev gamma ray and the 70-kev gamma ray and the 41-kev x-ray is shown in Fig. 8A. The coincidences with x-rays arise from internal conversion of the 70-kev gamma ray. Coincidences between the pulses of the 530 kev–600 kev peak and those of the 100-kev peak are shown in Fig. 8B. In Fig. 8C, coincidence rates in the region of low energies are displayed which were obtained by placing one analyzer first at the 530-kev photopeak (Curve I) then at the 600-kev photopeak (Curve II). The difference of shape of the two curves in the vicinity of 70 kev is such as to suggest that the 530-kev gamma ray is coincident with the 70-kev radiation whereas the 600-kev gamma ray is not. This

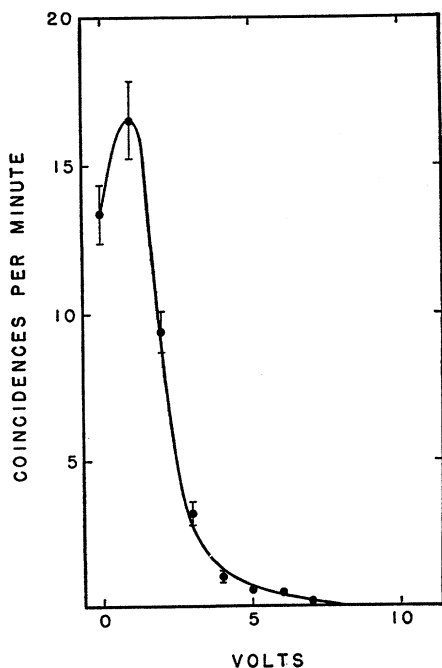


FIG. 7. Beta-gamma coincidences in the decay of  $\text{Sm}^{153}$  with the gamma-ray counter adjusted to count full energy peak pulses in the region 530–600 kev.

result is consistent with the fact that the previously described beta-gamma coincidence measurements show the two hard gamma rays to be emitted from the same level and with the fact that the gamma-gamma coincidences of Fig. 8B show the 100-kev gamma ray coincident with either hard quantum. Thus, a consideration of the energetics of the decay of  $\text{Sm}^{153}$  requires the presence of a 530 kev–70 kev–100 kev cascade.

From the data of Fig. 8A, the  $K$ -shell conversion coefficient of the 70-kev radiation may be obtained. In order to calculate  $[\alpha_K]_{70}$ , the area under the 41-kev x-ray peak was corrected for escape of iodine x-rays in the detection of the 41-kev x-rays, for the presence of pulses arising from the escape of iodine x-rays generated

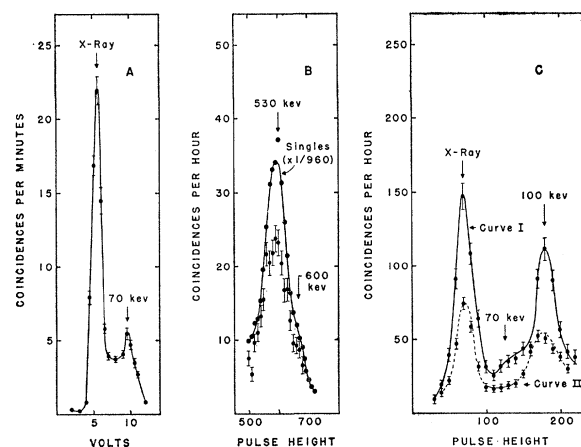


FIG. 8. Gamma-gamma coincidences in decay of  $\text{Sm}^{153}$ ; Curves A and B with one gamma-ray counter fixed at 100 kev; Curve C(I) with one counter at 530 kev; Curve C(II) with one counter at 600 kev.

in the detection of the 70-kev gamma rays, and for the fluorescence yield. The area under the 70-kev photopeak was increased to take into account iodine x-ray escape. The ratio of the corrected area of x-ray peak to the corrected area of the photopeak of the gamma ray was taken to be the  $K$ -shell conversion coefficient of the 70-kev gamma ray. The value of  $[\alpha_K]_{70}$  so calculated is  $4.4 \pm 0.4$  which is to be compared with previously reported values<sup>16,18</sup> of  $3.8 \pm 0.2$  and  $5.7 \pm 1.0$ .

From the data of Fig. 5A and the  $K$ -shell conversion coefficient of the 70-kev gamma ray, the  $K$ -shell coefficient  $[\alpha_K]_{100}$  may be calculated. The areas under the 41-kev photopeak, the 70-kev photopeak, and the 100-kev photopeak in Fig. 5A were accurately determined and properly corrected for increase or diminution arising for iodine x-ray escape as the case might be, and for fluorescence yield. The contribution of the 70-kev gamma ray to the area of the x-ray peak could be calculated from the previously determined value of  $[\alpha_K]_{70}$ . The ratio of the properly corrected area under the 41-kev photopeak to the corrected area under the 100-kev photopeak was taken as a measure of  $[\alpha_K]_{100}$ .

which was found to be  $1.19 \pm 0.15$ . The value may be compared with earlier values<sup>16,19</sup> of  $1.14 \pm 0.2$  and 1.2.

The total conversion coefficient of the 100-keV gamma ray could be calculated from the value of  $[\alpha_K]_{100}$  given above and from previously measured values of the  $K/L$  ratio.<sup>5,14</sup> A knowledge of the total conversion coefficient of the 100-keV gamma ray allows ultimately an estimate of the intensity of the 130-keV beta spectrum which was indicated by the beta-gamma coincidences of Fig. 7. From the total conversion coefficient of the 100-keV gamma ray, the data of Fig. 5B, and the known percentage of beta ray coupled with the 100-keV gamma ray, the number of disintegrations in which the 530 keV–600 keV emitting level is excited could be determined. Thus the percent of transitions associated with the 130-keV beta spectrum was estimated. In this calculation, considerations of the conversion coefficients of the 530 keV–600 keV transitions were omitted, because they have been determined to be small.<sup>5</sup> The percentage of beta disintegrations in the 130-keV group was found to be 0.06. Table IV lists the values of  $\log ft$  of the

TABLE IV. Values of  $\log ft$  for beta spectra of  $\text{Sm}^{153}$ .

Energy (keV)	$\log ft$
825	7.2
720	7.0
645	6.8
130	$\sim 7.2$

various beta spectra of  $\text{Sm}^{153}$  as calculated from the measured branching ratios of Table II and this measured intensity of the 130-keV group. According to these values of  $\log ft$ , all of the beta spectra of  $\text{Sm}^{153}$  could fall in the first forbidden category.

The relative intensities of the several gamma rays emitted in the decay of  $\text{Sm}^{153}$  are given in Table V. The relative intensities of the unconverted quantum radiations were initially determined, corrected for crystal detection efficiency and the photopeak area to Compton distribution ratio at the various energies. In making these intensity comparisons, the effect of the presence of various absorbing materials preceding the gamma-ray counter was properly taken into account. Finally, a correction was introduced to take into account internal conversion of the gamma ray in question. The relative transition intensities as well as the transition probability per disintegration are shown in Table V.

The total conversion coefficient of the 170-keV gamma ray is taken to be the theoretically expected value<sup>20</sup> which is  $\sim 0.45$  for either an  $E2$  or  $M1$  transition at that energy. The conversion coefficients of the radiations of the 530 keV–600 keV region are taken as that of the 548-keV gamma ray of Lee and Katz.<sup>5</sup> The total conversion coefficients of the 70- and 100-keV gamma rays

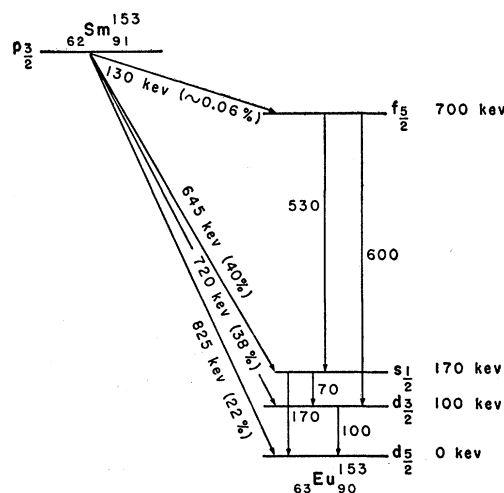
TABLE V. Characteristics of the gamma-ray transitions of  $\text{Sm}^{153}$ .

Quantum energy (keV)	70	100	170	530	600
Assumed total conversion coefficient	5.55	1.38	0.45	0.009	0.009
Relative intensity of unconverted quantum radiations	0.25	1.0	$7.3 \times 10^{-4}$	$1.5 \times 10^{-3}$	$3.6 \times 10^{-4}$
Relative transition intensity	0.68	1.0	$4.5 \times 10^{-4}$	$6.5 \times 10^{-4}$	$1.5 \times 10^{-4}$

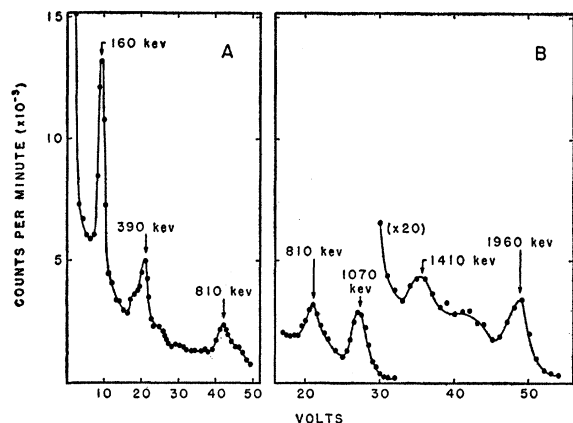
were calculated from values of  $\alpha_K$  measured in the present investigation, corrected for previously determined<sup>5</sup>  $K/L$  ratios.

The disintegration scheme of  $\text{Sm}^{153}$  is shown in Fig. 9. The spin of the ground state of  $\text{Eu}^{153}$  has been previously measured<sup>8</sup> as  $5/2$ . This result is consistent with a shell model prediction of  $d_{5/2}$ . Orbitals of  $d_{3/2}$  and  $s_{1/2}$  are indicated for low-lying excited states in the same shell. These spin values are also in agreement with the conclusion that the three gamma rays of low energy are mixtures of  $M1$  and  $E2$  transitions. The values of  $\log ft$  for the three more abundant beta transitions are such as to suggest them to be first forbidden ( $\Delta I=0, 1$ ; yes). The forbiddenness of these three spectra would require an orbital value of  $p_{3/2}$  for the ground state of  $\text{Sm}^{153}$ . Such an assignment is not inconsistent with shell theory considerations.

If a pattern of spin values indicated by the single-particle model is to be maintained in considering the spin of the level at 700 keV, it is necessary to enter the fifth shell where no orbitals are available which would be consistent with a first forbidden transition having an initial state of orbital  $p_{3/2}$ . The value of  $\log ft$  of the 130-keV transition also permits it to be classified as  $l$  forbidden ( $\Delta I=1, \Delta l=2$ ; no), allowing an orbital assignment of  $f_{5/2}$  for the 700-keV level in  $\text{Eu}^{153}$ . According to these orbital assignments, the 530- and 600-keV gamma rays could be mixtures of  $M2$  and  $E3$  transitions, and  $E1$  and  $M2$  transitions, respectively.

FIG. 9. Disintegration scheme of  $\text{Sm}^{153}$ .

<sup>20</sup> Rose, Goertzel, Sprinrad, Harr, and Strong, Phys. Rev. 83, 79 (1951).

FIG. 10. Gamma-ray spectrum of  $\text{Sn}^{125}$ .

### Tin-125

The early investigations<sup>21-27</sup> of the decay of the 10-day  $\text{Sn}^{125}$  yield good agreement in the fact that it decays mainly by emission of a 2.4-Mev beta spectrum. However, a weak beta branch of lower energy, associated with gamma radiation, is also indicated in some of the investigations.<sup>22,23,26,27</sup> Hayward<sup>25</sup> reports the presence of two beta groups of end-point energies 2.37 Mev ( $\sim 95$  percent) and 0.40 Mev ( $\sim 5$  percent) and finds no evidence of gamma radiation. Boyd and Ketelle<sup>26</sup> have reported the presence of 1.90-Mev gamma radiation as measured in a scintillation spectrometer and by lead absorption. Mandeville *et al.*<sup>27</sup> have confirmed the presence of a high-energy gamma transition at  $\sim 1.67$  Mev and of a beta spectrum at  $\sim 0.5$  Mev with an intensity of  $\sim 0.1$  per disintegration. Beta-gamma coincidences were observed in the course of these latter measurements.

In a recent investigation, Burson *et al.*<sup>28</sup> have reported, in the decay of a 9.7-day  $\text{Sn}^{125}$ , three beta groups of end-point energies  $\sim 0.4$ ,  $\sim 1.3$ , and 2.3 Mev, associated with seven gamma rays of energies  $1.98 \pm 0.02$ ,  $1.41 \pm 0.02$ ,  $1.07 \pm 0.01$ ,  $0.90 \pm 0.01$ ,  $0.81 \pm 0.01$ ,  $0.47 \pm 0.01$ , and  $0.34 \pm 0.01$  Mev and have further reported the decay to excite four states in  $\text{Sb}^{125}$  at energies 1.07, 1.41, 1.88, and 1.97 Mev.

The present investigation was commenced prior to publication of the last-mentioned report<sup>28</sup> and has been

continued to construct, insofar as is possible, a complete decay scheme of  $\text{Sn}^{125}$ . For the present investigations,  $\text{Sn}^{124}$ , isotopically enriched to 90.26 percent, was twice irradiated in the Brookhaven reactor. The reaction  $\text{Sn}^{124}(n, \gamma)\text{Sn}^{125}$  also yields the 9.5-minute  $\text{Sn}^{125}$  which decays to the 2.7-year  $\text{Sb}^{125}$ . Before study of the irradiated material, chemical purifications were performed for the removal of Sb, impurities, etc. The purified 10-day activity so obtained was studied with the use of scintillants of NaI(Tl) and anthracene.

Figure 10 shows the gamma-ray spectrum observed with two different gain settings of the linear amplifier of the scintillation spectrometer. Figure 10A is a plot of the low-energy region with high gain and Fig. 10B represents the high-energy region with reduced gain. The composite peak at 810 keV is included in both sets of curves in order to compare the relative intensities of the gamma rays at all energies. The prominent photopeaks at 160 keV and 390 keV arise from the presence of the 14-day  $\text{Sn}^{117m}$  and the 112-day  $\text{Sn}^{113}$ . Photopeaks at 810, 1070, 1410, and 1960 keV are clearly present. The composite peak at 390 includes 340- and 470-keV photopeaks which are resolved in coincidence measurements to be discussed later. The peak at 810 keV appears to be likewise complex, and when the instrumental resolution is taken into account, the presence of a 900-keV gamma ray is exhibited.

The beta spectrum was observed with an anthracene counter and the end-point energy was found to be  $\sim 2.4$  Mev.

Gamma-gamma coincidences in the decay of  $\text{Sn}^{125}$  were measured by fixing one gamma-ray counter on a single photopeak and moving the remaining counter over the entire gamma-ray spectrum. The coincidence rates so obtained are plotted in Fig. 11.

The gamma-gamma coincidence data are in partial agreement with the results of Burson *et al.* Some aspects of the data which differ with those previously reported are as follows:

(1) The data of Fig. 11A show the 1070-keV gamma ray to be in coincidence with a gamma ray at 230 keV as well as with gamma rays at 340, 470, 810, and 900 keV as previously reported.<sup>28</sup>

(2) In Fig. 11B are shown coincidences between the

TABLE VI. Relative intensities of the gamma rays of  $\text{Sn}^{125}$ .<sup>a</sup>

Energy (keV)	Relative intensity	Transition probability per disintegration
1960	4.00	$9.3 \times 10^{-3}$
1410	1.00	$2.6 \times 10^{-3}$
1070	16.60	$38.1 \times 10^{-3}$
900	5.00	$11.7 \times 10^{-3}$
810	8.30	$18.8 \times 10^{-3}$
470	2.65	$6.2 \times 10^{-3}$
340	1.45	$3.6 \times 10^{-3}$
230	10.95	$25.0 \times 10^{-3}$

<sup>a</sup> In preparing this table of data, it has been assumed that the total conversion coefficients of the gamma rays are small.

<sup>21</sup> J. C. Lee and M. L. Pool, Phys. Rev. **76**, 606 (1949).

<sup>22</sup> A. S. Newton and W. R. McDonell, University of California Radiation Laboratory Report UCRL-395, July, 1949 (unpublished).

<sup>23</sup> J. A. Seiler, *Radiochemical Studies: The Fission Products* (McGraw-Hill Book Company, Inc., New York, 1951), National Nuclear Energy Series, Plutonium Project Record, Vol. 9, p. 910.

<sup>24</sup> Ketelle, Nelson, and Boyd, Phys. Rev. **79**, 242(A) (1950).

<sup>25</sup> R. W. Hayward, Phys. Rev. **79**, 409 (1950).

<sup>26</sup> G. E. Boyd and B. H. Ketelle, Oak Ridge National Laboratory Report ORNL-1053, October, 1951 (unpublished).

<sup>27</sup> Mandeville, Shapiro, Mendenhall, Zucker, and Conklin, Phys. Rev. **88**, 554 (1952).

<sup>28</sup> Burson, Leblanc, and Martin, Phys. Rev. **99**, 660(A) (1955), and Nuclear Science Abstracts **9**, 78 (1955).

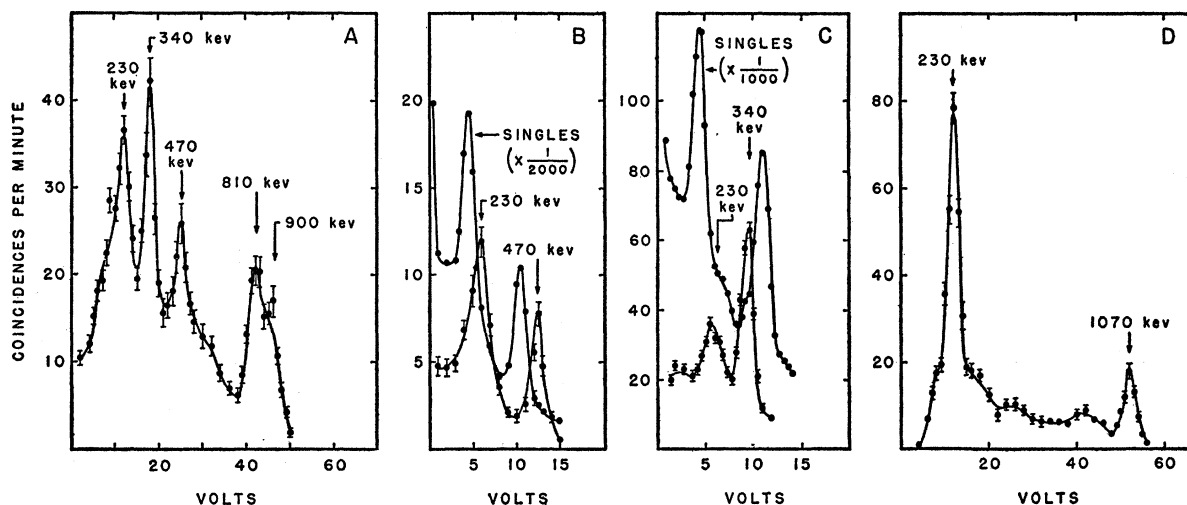


FIG. 11. Gamma-gamma coincidences in the decay of  $\text{Sn}^{125}$ : Curve A, with fixed counter at 1070 keV; Curve B, at 1410 keV; Curve C, at 470 keV; Curve D, at 810 keV.

1410-keV gamma ray and the 230-keV radiation as well as with the 470-keV gamma ray as previously reported.

(3) The coincidence curve of Fig. 11C shows the 470-keV gamma ray to be coincident with the 340-keV and 230-keV gamma rays. When one counter was fixed at 340 keV, a photopeak was observed in the coincidence curve when the moving counter passed through the region of 470 keV.

(4) In Fig. 11D are presented data which show coincidences between the 810-keV gamma ray and the gamma ray at 230 keV as well as with the 1070-keV radiation.<sup>28</sup>

In obtaining the data of Fig. 11, precautions were taken to insure that the 230-keV gamma-ray results from a real transition and is not an "apparent gamma ray" arising from backscatter effects relating to the presence of the high-energy quanta. Although the

geometry of the coincidence experiments was varied, and although certain selective absorption measurements were performed, the 230-keV line persisted in position and shape.

The relative intensities of the gamma rays emitted in the decay of  $\text{Sn}^{125}$  have been estimated from the areas under the full energy peaks of Fig. 10. These estimates are corrected for detection efficiency and the ratio of the number of pulses of full energy absorption to the number of pulses falling in the Compton distribution. These relative intensities are given in Table VI. With the assumption that the gamma rays are associated with five percent of the beta rays of  $\text{Sn}^{125}$ , the transition probabilities per disintegration have also been computed.

The decay scheme of Fig. 12 was constructed largely from a consideration of coincidence studies and energetics. It is also consistent with the measured relative intensities of the gamma rays. The intensity of the 230-keV transition was estimated from the combined intensities of the 470- and 810-keV gamma rays, assuming that the 1880-keV level of the decay scheme is not fed by a beta-ray group terminating at it.

The ground state of the 10-day  $\text{Sn}^{125}$  has been previously shown<sup>26</sup> to have an orbital value of  $h_{11/2}$ . The shell model of the nucleus furthermore predicts  $g_{7/2}$  to be the orbital of the ground state of  $\text{Sb}^{125}$ . The ground-state beta transition ( $E_\beta = 2.4$  MeV) would then conform to the spin change  $\Delta I = 2$ , yes, and so would be regarded as first forbidden. The observed value of  $\log ft$  is 8.9, in agreement with the classification given above.

From the transition probabilities of the gamma rays and the decay scheme in which the gamma rays have been arranged, the presence of beta spectra were indicated which have the transition probabilities and values of  $\log ft$  indicated in Table VII.

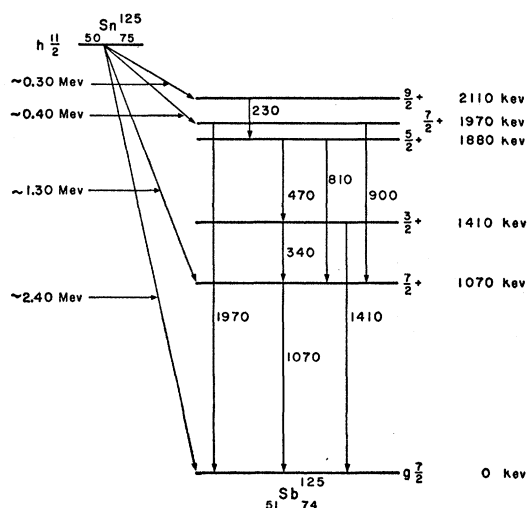
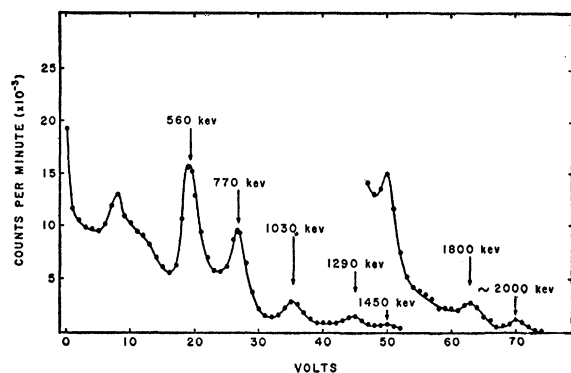


FIG. 12. Disintegration scheme of  $\text{Sn}^{125}$ .



FIG. 13. Gamma-ray spectrum of  $\text{Br}^{82}$ .

These various values of  $\log ft$  are consistent with a first-forbidden classification for all of the beta spectra. The relative intensities of the gamma rays and the

classifications of the beta spectra can lead to some tentative assignments of orbitals of the excited states of  $\text{Sb}^{125}$  as also given in Fig. 12. It is doubtful that the single-particle model of the nucleus can be used in making orbital assignments, because the high excitation energy could lead to excitation of the nuclear core.

### Bromine-82

In early investigations of the decay of the 36-hour  $\text{Br}^{82}$ , Roberts *et al.*<sup>29</sup> have reported the emission of a single-beta spectrum of maximum energy 465 keV followed by gamma rays of quantum energies 545, 787, and 1350 keV in cascade. Deutsch<sup>30</sup> has searched for beta rays of higher energy with negative results.

Siegbahn *et al.*<sup>31</sup> have observed seven gamma rays, obtained from photoelectric peaks and internal conversion lines at quantum energies of 547, 608, 692, 766, 823, 1031, and 1312 keV. In addition to these, Hubert

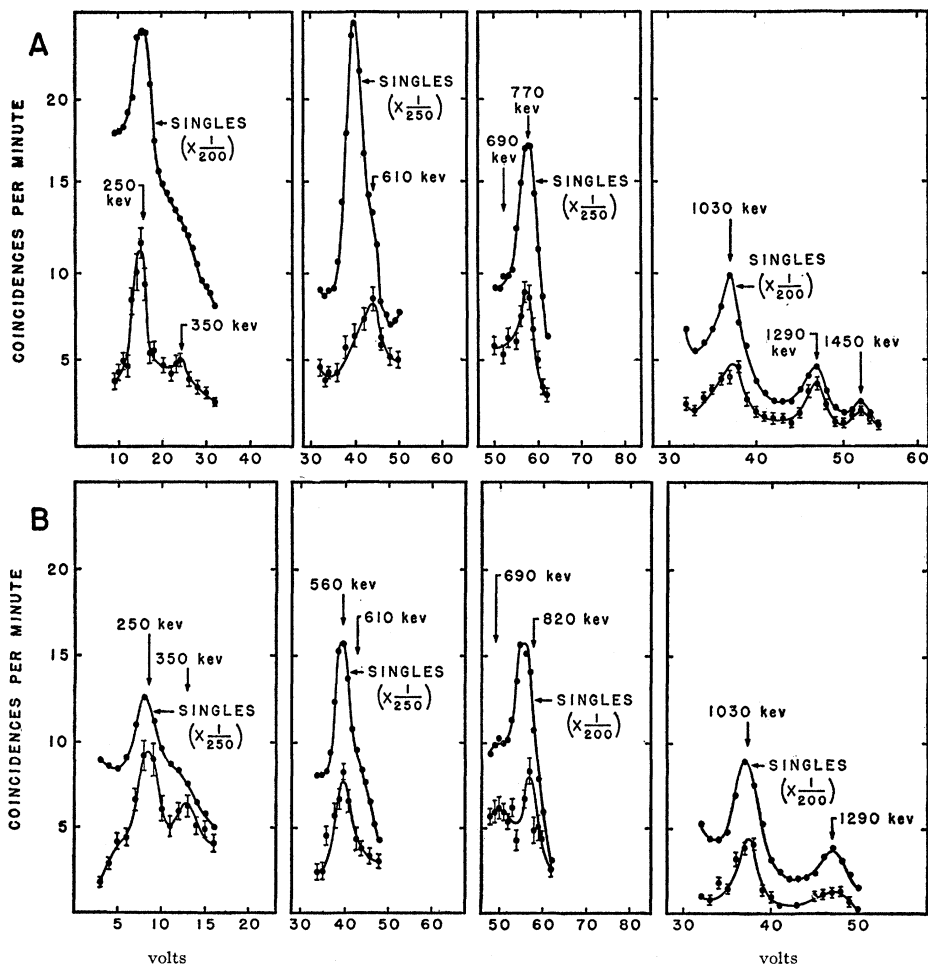


FIG. 14. Gamma-gamma coincidences in the decay of  $\text{Br}^{82}$ . Curves A, with one counter fixed at 560 keV; Curves B, at 770 keV; Curves C, at 1030 keV; Curves D(1,2) at 1290 keV; Curves D(3,4), at 1450 keV.

<sup>29</sup> Roberts, Downing, and Deutsch, *Phys. Rev.* **60**, 544 (1941).

<sup>30</sup> M. Deutsch, *Phys. Rev.* **61**, 672 (1942).

<sup>31</sup> Siegbahn, Hedgran, and Deutsch, *Phys. Rev.* **76**, 1263 (1949).

*et al.*<sup>32</sup> and Dzhelepov *et al.*<sup>33</sup> have reported an additional gamma ray of quantum energy  $\sim 1450$  kev.

Levels of  $\text{Kr}^{82}$  are also excited in the positron ( $K$  capture) decay of  $\text{Rb}^{82}$ . Huddleston and Mitchell,<sup>34</sup> and Easterday,<sup>35</sup> have reported in the decay of  $\text{Rb}^{82}$  six gamma rays in addition to the previously observed<sup>29-33</sup> eight gamma rays of  $\text{Br}^{82}$ . These latter gamma rays were also detected by Lu *et al.*<sup>36</sup> with the use of the "sum peak" technique. More recently, Benczer and Wu<sup>37</sup> have investigated the radiations of  $\text{Br}^{82}$  and  $\text{Rb}^{82}$ . They did not find three of the six additional gamma rays reported in the decay of  $\text{Rb}^{82}$ , and the remaining three were described as being associated with an impurity. They have also studied the beta spectrum and found it to be simple with a maximum energy of 0.435 Mev. From a consideration of all the decay schemes proposed as a result of the above-quoted

TABLE VII. Values of  $\log ft$  of the beta-ray spectra of  $\text{Sn}^{126}$ .

Energy (Mev)	Percent of disintegrations	$\log ft$
$\sim 2.4$	95	8.9
$\sim 1.3$	0.4	10.2
$\sim 0.4$	2.1	7.8
$\sim 0.3$	2.5	7.3

investigations, it was concluded that a further study of  $\text{Br}^{82}$ , particularly by the method of coincident scintillation spectrometers was in order.

For the present investigations  $\text{Br}^{82}$ , in the form of  $\text{KBr}$ , was obtained from Oak Ridge National Laboratory at three different times and was studied with scintillation spectrometers of  $\text{NaI(Tl)}$  and anthracene. In Fig. 13 is given a plot of the  $\gamma$  spectrum taken with the aid of a 3.5-cm diameter and 4 cm thick  $\text{NaI(Tl)}$

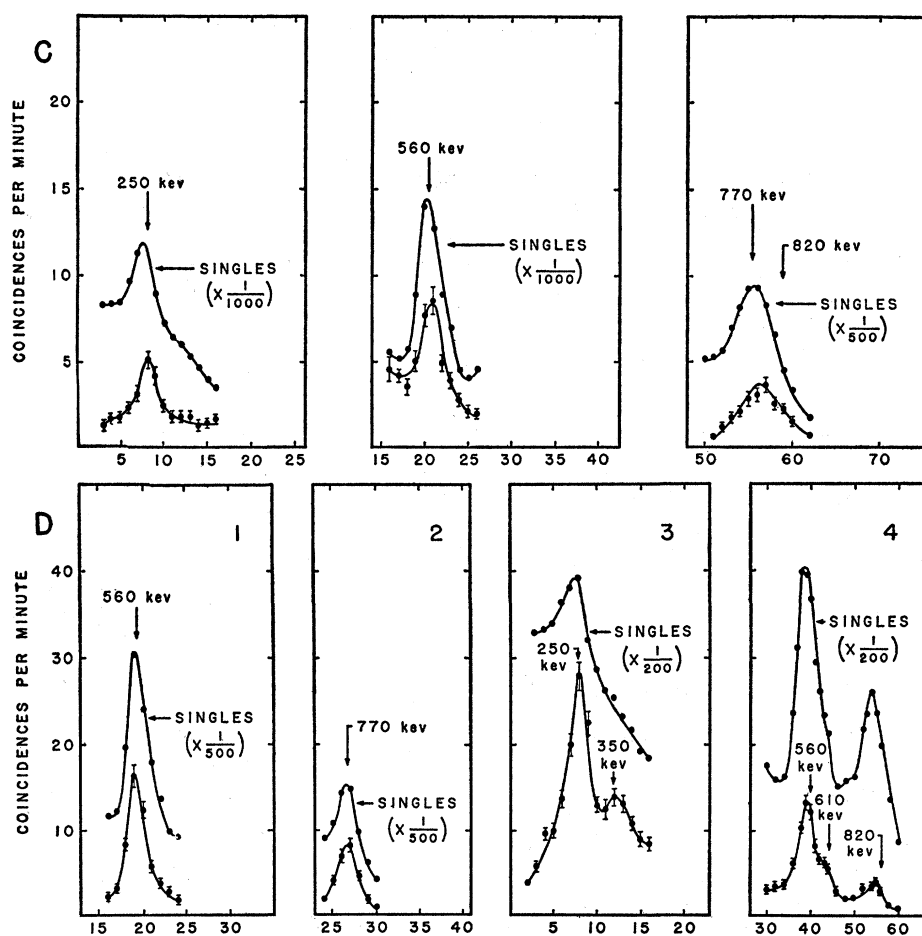


FIG. 14—Continued.

<sup>32</sup> P. Hubert and J. Laberrie-Frolow, *Compt. rend.* **232**, 2420 (1951).

<sup>33</sup> B. Dzhelepov and A. Silant'ev, *Doklady Akad. Nauk S.S.S.R.* **85**, 533 (1952).

<sup>34</sup> C. M. Huddleston and A. C. G. Mitchell, *Phys. Rev.* **88**, 1350 (1952).

<sup>35</sup> H. T. Easterday, University of California Radiation Laboratory Report UCRL-2172, 1953 (unpublished).

<sup>36</sup> Lu, Kelly, and Wiedenbeck, *Phys. Rev.* **95**, 1533 (1954).

<sup>37</sup> N. Benczer and C. S. Wu, *Bull. Am. Phys. Soc. Ser. II*, **1**, 41 (1956). See also "Nuclear level schemes," U. S. Atomic Energy Commission Report TID-5300, June, 1955.

TABLE VIII. Gamma-gamma coincidences.

Fixed channel	Coincident with (kev)	Fig. No.
560 kev	~250, ~350, 610, 690, 770, 1030, 1290, 1450	14-A
770 kev	~250, ~350, 560, 610, 690, 820, 1030, 1290	14-B
1030 kev	~250, 560, 770, 820	14-C
1290 kev	560, 770	14-D
1450 kev	~250, ~350, 560, 610, 820	(1-2) 14-D (3-4)

crystal. Photopeaks at quantum energies of 560, 770, 1030, 1290, and 1450 are clearly resolved. The photopeaks at 550 and 770 seem to be complex. Their structures will be analyzed in a discussion of coincidence studies to follow. In addition to these gamma rays, very weak radiations of quantum energies  $\sim 1.8$  Mev and  $\sim 2.0$  Mev are also observed. The spectra were obtained with the source at different distances from the crystal and it was found that the heights of the peaks of the two high-energy gamma rays remained the same relative to the other photopeaks indicating that they are not sum peaks arising from the simultaneous detection of several gamma rays of low energy. It was also noted that the entire spectrum decayed with a half-life of  $\sim 36$  hours. Precaution was taken to reduce back-scattering effects, but a very prominent peak in the region of  $\sim 250$  kev continued to be observed. Thus is indicated the presence of gamma ray of energy less than 560 kev, the photopeak of which is superposed upon back-scattered radiations and contributions of the Compton distribution of the high-energy gamma rays.

The beta spectrum of  $\text{Br}^{82}$  was observed in an anthracene crystal and when analyzed in a Fermi-Kurie plot was shown to be a simple spectrum of end-point energy  $460 \pm 10$  kev. Beta-gamma coincidences were observed by fixing the gamma counter on various photopeaks and moving the beta counter over the entire spectrum with a channel width of one volt. In each case an end-point energy of 460 kev was noted. This result is in agreement with previous reports<sup>29-33,36,37</sup> and gives no indication of the low-energy beta-ray spectra suggested by Huddleston and Mitchell.<sup>34</sup>

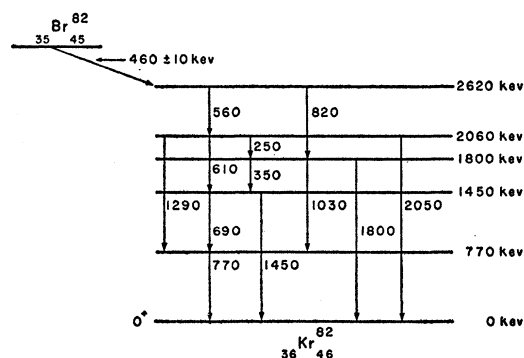
Gamma-gamma coincidences were measured by fixing one gamma-ray counter to accept a single prominent photopeak and moving the other gamma-ray counter over the entire spectrum. The coincidences so observed are shown in Fig. 14 and the results are tabulated in Table VIII. The presence of a gamma ray at  $\sim 350$  kev was also detected.

The disintegration scheme of  $\text{Br}^{82}$  is shown in Fig. 15 where the energy and coincidence measurements of the preceding discussion have been combined. This scheme differs in many aspects from the one most recently presented.<sup>37</sup> The data of Fig. 14A show the 560-kev gamma ray to be coincident with all quanta except the 820-kev gamma ray. It appears that coincidences were not previously detected<sup>37</sup> between the 560-kev gamma ray and the 1030-kev radiation. The detection of such

coincidences in the present investigation and the consequent relationship between those two quanta accounts for many of the differences in the two schemes.† The quanta at 250, 350, 1800, and 2050 kev also appear to be unreported in the work of Benczer and Wu.<sup>37</sup> It should perhaps be remarked that early experiments<sup>38</sup> involving photoneutron production have shown evidence of the latter two high-energy quanta. The location of the high-energy crossover transitions in the disintegration scheme of Fig. 15 is based upon a logical consideration of coincidence studies and energetics. The existence of these transitions makes seem unlikely the rather large spin values which have been assigned previously to levels lying between the first excited state and the topmost level. The value of the spin of the 2.62-Mev state of  $\text{Kr}^{82}$  need not be large, because the beta transition of  $\text{Br}^{82}$  is allowed ( $\Delta I = 0, 1$ ; no), and shell model considerations<sup>39</sup> show that the possible values of the spin of the ground state of  $\text{Br}^{82}$  could range from  $1+$  to  $6-$ ; that is, a small spin value for  $\text{Br}^{82}$  is also acceptable.

If  $\text{Kr}^{82}$  is excited by simple decay<sup>37</sup> of  $\text{Br}^{82}$  and  $\text{Rb}^{82}$ , that is, if there is only a single beta decay in either case, then the relative intensities of the gamma rays do not give sufficient information to determine the order of emission of the gamma rays. In other words, if simple decay occurs, then one cannot say that the 770-kev gamma ray must be at the bottom of the cascade simply because of its high intensity. In this circumstance the entire scheme of the de-excitation of  $\text{Kr}^{82}$  might well be inverted, with the 560-kev gamma ray being at the bottom of the cascade.

On the other hand, if earlier evidence<sup>34</sup> of branching in the decay of  $\text{Rb}^{82}$  is accepted, the intensity of the 770-kev gamma ray would be sufficient to adjudge it to be emitted from the first excited state. The complex

FIG. 15. Disintegration scheme of  $\text{Br}^{82}$ .

† Note added in proof.—This scheme agrees well with that reported recently by R. C. Waddell and E. N. Jensen, Phys. Rev. **102**, 816 (1956).

<sup>38</sup> V. Myers and A. Wattenberg, Phys. Rev. **75**, 992 (1949).

<sup>39</sup> Mayer, Moszkowski, and Nordheim, Revs. Modern Phys. **23**, 315 (1951).

decay of  $\text{Rb}^{82}$  appears not to have been confirmed, however.

#### ACKNOWLEDGMENTS

The writers wish to acknowledge the kind interest of Dr. W. F. G. Swann, Director of The Bartol Research

Foundation. Mr. V. R. Potnis aided in taking some of the data. It is desired also to express appreciation for a fellowship granted by The Bartol Research Foundation which made possible the visit of one of us (V.S.D.) to the United States.

### Analysis of the Beta Spectrum and Branching in $\text{Ho}^{166}$

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(Received March 27, 1956)

The shape of the  $\beta$  spectrum of the  $\text{Ho}^{166} \rightarrow \text{Er}^{166}$  ground state ( $0^- \rightarrow 0^+$ ) transition and the branching to the first excited state ( $2^+$ ) has been investigated. Corrections due to the variation of the lepton wave functions over the nucleus have also been taken into account, and it is shown that under certain conditions these corrections can be sufficient to explain the experimental findings.

OUR interest in the problem of beta spectrum of the  $\text{Ho}^{166} \rightarrow \text{Er}^{166}$  ground state ( $0^- \rightarrow 0^+$ ) transition and in the branching ratio to the first excited state (at 80 kev,  $2^+$ ) was aroused by the experimental results of the Canadian group.<sup>1</sup> Their decay scheme is given in the Nuclear Data Cards NRC 55 G07-55-3-94. The ground-state transition shows approximately an allowed shape,  $p_1=52\%$  and  $\log ft=8.2$ . While the transition to the  $2^+$  first excited state seems to have predominantly a unique  $\Delta I=2$  (yes) shape with  $p_2=47\%$  and  $\log ft=8.0$ .

The usual ( $0^- \rightarrow 0^+$ ) tensor correction factor rises 37% over the energy region of the electron and one would expect  $\log ft \approx 6.0$ , because the matrix elements  $\langle \beta \sigma \cdot \mathbf{r} \rangle$  and  $\langle \beta B_{ij} \rangle$  are supposed to be of the same order of magnitude. The ratio  $p_2/p_1 \approx 1/100$  owing to the presence of the large Coulomb factor in the  $\langle \beta \sigma \cdot \mathbf{r} \rangle$  correction factor.

Our intention was to see under which assumptions one can get a large  $ft$  value and an allowed spectrum. We investigated the following two possibilities:

(a) Destructive interference of pseudoscalar and tensor coupling.<sup>2</sup>

(b) Small value of the  $\langle \beta \sigma \cdot \mathbf{r} \rangle$  tensor matrix element.

There are two main corrections which can play an important role in these cases: i.e., the finite size of the nucleus,<sup>3</sup> and the corrections due to the fact that the emission of the leptons can take place over the whole nucleus and not only at the boundary as is usually assumed.<sup>4</sup>

<sup>1</sup> Graham, Wolfson, and Clark, Phys. Rev. **98**, 1173(A) (1955).

<sup>2</sup> A. G. Petschek and R. E. Marshak, Phys. Rev. **85**, 698 (1952).

<sup>3</sup> M. E. Rose, Phys. Rev. **82**, 389 (1951); M. E. Rose and D. K. Holmes, Oak Ridge National Laboratory Report ORNL-1022 (1951); Rose, Perry, and Dismuke, Oak Ridge National Laboratory Report ORNL-1459 (1953).

<sup>4</sup> M. Yamada, Progr. Theoret. Phys. Japan **10**, 245 (1953); M. Yamada Progr. Theoret. Phys. Japan **10**, 241 (1953); M. R. Nafat, Compt. rend. **238**, 1012 and 1117 (1954); H. Takebe, Progr. Theoret. Phys. Japan **12**, 561 (1954).

Following the method of Rose *et al.*<sup>3</sup> we have computed the effect of the finite nuclear size on the wave function of the electron at the boundary  $r=r_0$ . The corrections are given in Fig. 1.

The second correction, up to the quadratic terms in energy  $W$ , can be written in the form

$$\langle \Theta(\mathbf{r}) \varphi(r) \rangle = \langle \Theta(\mathbf{r}) \frac{\varphi(r)}{\varphi(r_0)} \rangle \times \varphi(r_0) = \langle \Theta(\mathbf{r}) \rangle \varphi(r_0) (\alpha + \beta W + \delta W^2), \quad (1)$$

where  $\Theta(\mathbf{r})$  is the usual operator and  $\varphi(r)$  is essentially the radial electron wave function. The ratio  $\varphi(r)/\varphi(r_0)$  was calculated by using Rose's expansion of the Dirac

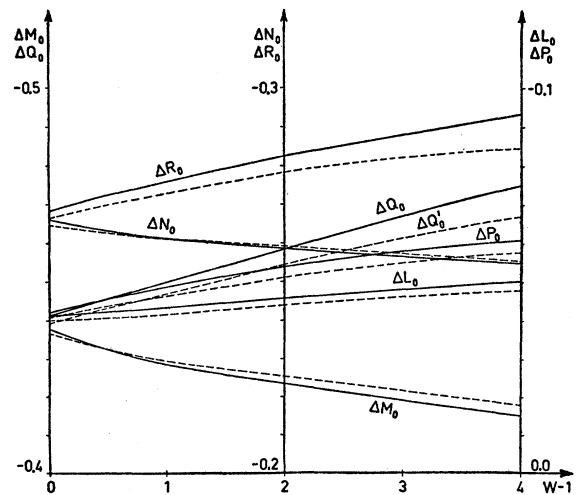


FIG. 1. The corrections  $\Delta L_0$ ,  $\Delta M_0$ ,  $\Delta N_0$ ,  $\Delta P_0$ ,  $\Delta Q_0$ ,  $\Delta R_0$  are plotted against the kinetic energy of the electron. The corrections refer to  $\rho(r_0)/\rho(0)=1+\epsilon=1.05$  and  $r_0=\frac{1}{2}\alpha A^{1/3}$  (full lines) and  $r_0'=0.8r_0$  (dashed lines). The influence of  $\epsilon$  on the corrections is small.