decay of Rb^{82} appears not to have been confirmed, however.

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Analysis of the Beta Spectrum and Branching in Ho^{166}

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The shape of the β spectrum of the $Ho^{166}\rightarrow Er^{166}$ ground state $(0-\rightarrow 0+)$ transition and the branching to the first excited state $(2+)$ has been investigated. Corrections due to the variation of the lepton wave functions over the nucleus have also been taken into account, and it is shown that under certain conditions these corrections can be sufficient to explain the experimental findings.

OUR interest in the problem of beta spectrum of the $Ho^{166}\rightarrow Er^{166}$ ground state $(0-\rightarrow 0+)$ transition and in the branching ratio to the first excited state (at 80 kev, $2+$) was aroused by the experimental results of the Canadian group.[1] Their decay scheme is given in the Nuclear Data Cards NRC 55 GO7–55–3–94. The ground-state transition shows approximately an allowed shape, $p_1=52\%$ and $\log ft=8.2$. While the transition to the $2+$ first excited state seems to have predominantly a unique $\Delta I=2$ (yes) shape with $p_2=47\%$ and $\log ft=8.0$.

The usual $(0-\rightarrow 0+)$ tensor correction factor rises 37% over the energy region of the electron and one would expect $\log ft\approx 6.0$, because the matrix elements $\langle\beta\boldsymbol{\sigma}\cdot\mathbf{r}\rangle$ and $\langle\beta B_{ij}\rangle$ are supposed to be of the same order of magnitude. The ratio $p_2/p_1\approx 1/100$ owing to the presence of the large Coulomb factor in the $\langle\beta\boldsymbol{\sigma}\cdot\mathbf{r}\rangle$ correction factor.

Our intention was to see under which assumptions one can get a large ft value and an allowed spectrum. We investigated the following two possibilities:

(a) Destructive interference of pseudoscalar and tensor coupling.[2]

(b) Small value of the $\langle\beta\boldsymbol{\sigma}\cdot\mathbf{r}\rangle$ tensor matrix element.

There are two main corrections which can play an important role in these cases: i.e., the finite size of the nucleus,[3] and the corrections due to the fact that the emission of the leptons can take place over the whole nucleus and not only at the boundary as is usually assumed.[4]

[1] Graham, Wolfson, and Clark, Phys. Rev. **98**, 1173(A) (1955).

[2] A. G. Petschek and R. E. Marshak, Phys. Rev. **85**, 698 (1952).

[3] M. E. Rose, Phys. Rev. **82**, 389 (1951); M. E. Rose and D. K. Holmes, Oak Ridge National Laboratory Report ORNL–1022 (1951); Rose, Perry, and Dismuke, Oak Ridge National Laboratory Report ORNL–1459 (1953).

[4] M. Yamada, Progr. Theoret. Phys. Japan **10**, 245 (1953); M. Yamada Progr. Theoret. Phys. Japan **10**, 241 (1953); M. R. Nataf, Compt. rend. **238**, 1012 and 1117 (1954); H. Takebe, Progr. Theoret. Phys. Japan **12**, 561 (1954).

Following the method of Rose *et al.*[3] we have computed the effect of the finite nuclear size on the wave function of the electron at the boundary $r=r_0$. The corrections are given in Fig. 1.

The second correction, up to the quadratic terms in energy W, can be written in the form

$$\langle\mathcal{O}(\mathbf{r})\varphi(r)\rangle=\langle\mathcal{O}(\mathbf{r})\frac{\varphi(r)}{\varphi(r_0)}\rangle \times\varphi(r_0)=\langle\mathcal{O}(\mathbf{r})\rangle\varphi(r_0)(\alpha+\beta W+\delta W^2),\quad(1)$$

where $\mathcal{O}(\mathbf{r})$ is the usual operator and $\varphi(r)$ is essentially the radial electron wave function. The ratio $\varphi(r)/\varphi(r_0)$ was calculated by using Rose's expansion of the Dirac

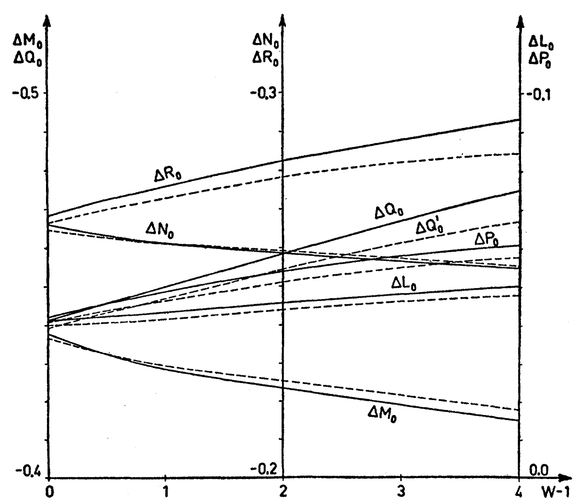

FIG. 1. The corrections ΔL_0, ΔM_0, ΔN_0, ΔP_0, ΔQ_0, ΔR_0 are plotted against the kinetic energy of the electron. The corrections refer to $\rho(r_0)/\rho(0)=1+\epsilon=1.05$ and $r_0=\frac{1}{2}\alpha A^{\frac{1}{3}}$ (full lines) and $r_0'=0.8r_0$ (dashed lines). The influence of ϵ on the corrections is small.

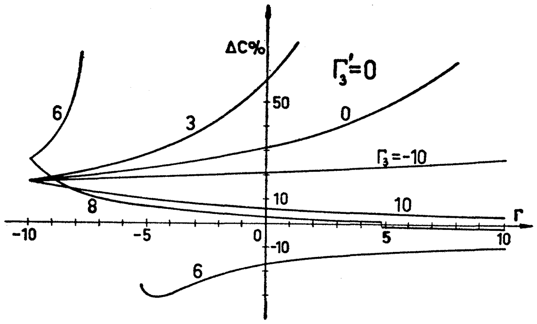

FIG. 2. The maximum variation $\Delta C = \{[C(W_0) - C(W)]/C(W_0)\}_{\max}$ over the whole energy range of the tensor-pseudoscalar correction factor is given as a function of Γ for different values of Γ_3, $\Gamma_3' \approx 0$, and $r_0 = \frac{1}{2}\alpha A^{\frac{1}{3}}$.

wave function.[3] The correction coefficients α, β, and δ contain ratios of the matrix elements $\Gamma_{2n+1} = \langle \mathcal{O}(\mathbf{r})x^{2n}\rangle / \langle \mathcal{O}(\mathbf{r})\rangle$, $n=1, 2, 3 \cdots$, $x=r/r_0$, as new parameters. The usual approach is to put all $\Gamma_{2n+1}=1$, in which case the correction is reduced to unity ($\alpha=1$, $\beta=\gamma=0$); and the correction will be still negligible if $\Gamma_3 \approx 1$ because the influence of higher Γ_{2n+1} terms can be neglected. To obtain a large energy correction, it is necessary to have $|\Gamma_3| \gg 1$. It seems that this can be achieved if the usual matrix element is much smaller than it normally is.

We first examined case (a). The main terms in the correction factor for tensor pseudoscalar mixture can be written in the form[2]

$$C = (\tfrac{1}{9})q^2L_0' + \tfrac{2}{3}qN_0' + M_0' + 2\Gamma(\tfrac{1}{3}qL_0' + N_0') + \Gamma^2 L_0', \quad (2)$$

$$\Gamma = g_5\langle f(r)\boldsymbol{\sigma}\cdot\mathbf{r}\rangle / g_3\langle \beta\boldsymbol{\sigma}\cdot\mathbf{r}\rangle, \quad (3)$$

where $f(r)$ is a function measuring the influence of nuclear forces,[5] q is the momentum of the neutrino, and all the other symbols have the usual meaning.

The destructive interference ($\Gamma \approx 10$), even when corrected by additional terms[6,7]

$$-\frac{1}{M}\frac{g_5}{g_3}\Gamma[UN_0' + R_0' + \tfrac{1}{3}q(UL_0' + 4N_0' - P_0')], \quad (4)$$

where L_0', M_0', $\cdots$ have been corrected for the finite de Broglie wavelength and the second correction, failed to reproduce the experimental findings. $U = W + (\alpha Z/r_0) - q$ and M is the nucleon mass. So we are left with case (b), the case of a singular matrix element.

[5] Alaga, Kofoed-Hansen, and Winther, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 3 (1953). E. J. Konopinski, Phys. Rev. **94**, 492 (1954); R. W. King and D. C. Peaslee, Phys. Rev. **94**, 1284 (1954); R. F. Herbst, Phys. Rev. **96**, 372 (1954).

[6] M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1315 (1954).

[7] Formula (4) represents only the cross term between the two parts of the nonrelativistic form of the pseudoscalar coupling. The other terms necessary to complete the expression can be found in reference 6.

In this case, the second correction will play an important role. The aforementioned small corrections can be neglected. From Figs. 2 and 3 it can be seen that there exist values of Γ_3, Γ_3', and Γ for which the variation of the correction factor over the whole energy region is less than 6%. $\Gamma_3 = \langle\beta\boldsymbol{\sigma}\cdot\mathbf{r}x^2\rangle / \langle\beta\boldsymbol{\sigma}\cdot\mathbf{r}\rangle$ is the ratio of the tensor matrix elements, while $\Gamma_3' = g_5\langle f(r)\boldsymbol{\sigma}\cdot\mathbf{r}x^2\rangle / g_3\langle\beta\boldsymbol{\sigma}\cdot\mathbf{r}\rangle$ is essentially the corresponding ratio of the pseudoscalar and the tensor matrix element. The most critical of the parameters is Γ_3, its lowest acceptable value being approximately 6. The different values of Γ_3' change slightly the value of Γ_3 for which the variation ΔC is less than 6%, but affect the value of Γ rather seriously. $\Gamma_3' < 0$ decreases slightly the lowest acceptable Γ_3. The effect of $\Gamma_3' > 0$ is the opposite.

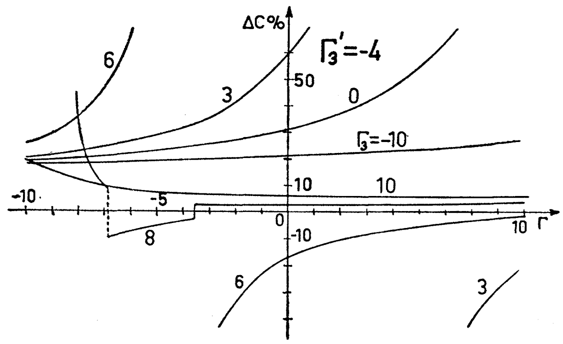

FIG. 3. The maximum variation ΔC as a function of Γ, as in Fig. 2 but with $\Gamma_3' = -4$.

The radius of the nuclear charge distribution is taken to be $r_0 = \frac{1}{2}\alpha A^{\frac{1}{3}}$. The alternative value $r_0' = 0.8r_0$ was taken to determine the influence of the radius on the energy dependence of the correction factor. It was found that the influence is negligible in the interesting region $\Gamma_3 \gtrsim 6$.

As shown, it seems that in this way it is possible to explain the experimental findings on $Ho^{166} \rightarrow Er^{166}$ under the assumption of a small tensor matrix element. For a more precise determination of the parameters involved here, and the influence of the pseudoscalar coupling in this case, one needs a more precise measurement of the shapes and possibly the electron-neutrino angular correlations.

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