within the energy intervals appropriate to Σ and Λ^0 hyperon reactions, yields Σ/Λ^0 about 2.

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Modulation of Primary Cosmic-Ray Intensity*

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It is assumed that the cosmic-ray particles observed at the earth are of galactic origin, except for the occasional bursts from solar flares. With this interpretation the 11-year variation of the cosmic-ray intensity and the Forbush decreases represent depressions of the steady galactic intensity. The observed rigidity dependence of the depression indicates that magnetic fields are responsible. A quantitative investigation of the possible motion and configuration of magnetic fields capable of producing the observed effects is carried out. It is shown that, within the limitations imposed by what we think we know today of the galactic magnetic field, of solar activity, and of interplanetary fields, serious difficulties are encountered by any mechanism, such as Morrison's interplanetary cloud model, modulating the galactic cosmic-ray intensity throughout the solar system.

It is proposed that the modulation of the intensity is produced locally, within a few earth's radii, by interplanetary magnetic gas clouds captured by the terrestrial gravitational field. Such a model seems to produce the observed effects on the basis of the known facts about solar activity. The most straightforward test of this geocentric model, independent of inferences from cosmic-ray effects, is the question of whether the absorption of the captured magnetic hydrogen gas can be detected as a narrow line in the center of the broad solar L_α emission line.

I. INTRODUCTION

FOR some years it has been known that the cosmic-ray intensity in the atmosphere of the earth changes with time, but it has been only the last few years that it could be shown that the changes were due to variations in the primary cosmic-ray intensity and therefore not meteorological in origin or induced by changes in the geomagnetic field. At the same time it has become clear that the variations in the primary spectrum are related somehow to solar activity, though apparently many effects occur simultaneously and the solar relation is not a simple one: low-energy studies show that the variations are a function only of the particle rigidity, the variations being larger for smaller rigidities.

It is indeed fortunate that the theoretical study of the dynamical properties of ionized gases, plasma dynamics, has been pushed ahead in the last decade, because the electromagnetic fields associated with plasma motions afford the only known coupling between cosmic-ray particles and the matter throughout space, except, of course, for short-range nuclear forces that come into play in nuclear collisions. Naturally the attempts to account for the observed variations in the cosmic-ray intensity have appealed to plasma motions; observations would seem to indicate that the most of space is occupied by streaming gases carrying magnetic fields. The high electrical conductivity and relatively slow variations in the gas suggest that the displacement current and the inertial separation of electrons and protons may be neglected, leading to the hydromagnetic approximation of the electromagnetic field equations[1] wherein the gas is treated as a classical conducting fluid.

In this paper we shall concern ourselves with the hydromagnetic processes which might be expected to produce a modulation effect in a pre-existing steady primary cosmic-ray spectrum. In particular, we shall be interested in schemes by which the sun could modulate the galactic cosmic-ray spectrum within the confines of the solar system.

Several interesting hydromagnetic modulating devices are already well known and may be found in the literature. Alfvén has made use of the fact that the magnetic field carried in a rapidly moving beam or jet of ionized gas in interplanetary space will give rise to an electric field for an observer in a fixed frame of reference; he suggests[2] that the resulting electrostatic accelaration of

* Assisted in part by the Office of Scientific Research and the Geophysics Research Directorate, Air Force Cambridge Research Center, Air Research and Development Command, U. S. Air Force.

[1] W. M. Elsasser, Phys. Rev. **95**, 1 (1954).
[2] H. Alfvén, *Cosmical Electrodynamics* (Clarendon Press, Oxford, 1950).

the cosmic-ray particles passing through the beam might account for some of the observed primary cosmic-ray fluctuations. Morrison[3] has suggested that large ionized gas clouds carrying tangled magnetic fields may move outward from the sun and engulf the earth, thereby shielding it from the incident galactic cosmic-ray particles.

Such mechanisms as the above center about the sun and hereafter will be designated as *heliocentric*. The mechanisms must preserve the observed isotropy of the cosmic-ray particle distribution while at the same time varying the intensity by large amounts. Thus they offer the common difficulty that they must operate throughout a large fraction of interplanetary space, which requires immense amounts of matter and magnetic fields from the sun. Apart from this difficulty, however, one finds that Alfvén's beam would introduce complete anisotropy into the perturbation component of the primary cosmic rays, which is simply not observed. And the beam produces negligible acceleration effects because the beam must be thin enough to allow the particles to penetrate all the way through. It would seem that Morrison's mechanism would require either interplanetary cloud velocities of the order of 10^4 km/sec or disordered interplanetary magnetic fields of the order of 0.5×10^{-2} gauss in order to explain the abrupt decreases (as small as 4 hours) in cosmic-ray intensity sometimes observed at Earth; if his idea of disordered fields is extended to account for the observed relative dearth of primary cosmic-ray particles with energies below about 1 Bev, it is found that the sun must eject the magnetic gas cloud in all directions rather than just near the equatorial plane as is usually assumed.[4]

Nagashima[5] has suggested that a geoelectric field, a *geocentric* mechanism, may be responsible for some of the observed modulations of the primary cosmic-ray beam. However the existence of a geoelectric field is doubtful because of the high electrical conductivity of the ionosphere and of interplanetary space, and the effect on the primary cosmic-ray spectrum would be to shift the energy of each incoming particle by a fixed amount, whereas most fluctuations in the cosmic-ray intensity seem to represent a change in the total number of incoming particles[6]; this latter objection can be raised against Alfvén's beam.

Faced with the difficulties expressed above, we shall attempt to construct an alternative primary modulating mechanism in light of contemporary hydromagnetic theory, solar properties, and recent cosmic-ray observations. To avoid having to operate throughout the immense volume of interplanetary space, we shall consider a mechanism which is centered about the earth but is not confined to the terrestrial atmosphere or to regions of dense geomagnetic field; thus the mechanism will be geocentric but not really terrestrial. We suggest that gas, ejected from the sun and carrying with it disordered magnetic fields, may occasionally be captured by the terrestrial gravitational field. Such a model has the virtue of producing the observed dearth of low-energy primary particles (the so-called *low-energy cutoff*) even though the sun may eject matter and magnetic fields only into its equatorial plane. The decay time of the captured magnetic fields is of the order of months, explaining why the cutoff does not vary violently with the day to day solar activity but does vary over a period of years with the mean level of solar activity. However, the capture of new interplanetary matter is a process that would take place sufficiently rapidly to produce the abrupt Forbush-type decreases in the cosmic-ray intensity observed at the earth.

With the presently rapid development of knowledge of cosmic-ray phenomena, solar physics, and plasma dynamics, one expects that the model which we shall construct will sooner or later undergo serious revision, but at present it appears to explain on simple physical principles most of the gross modulation effects.

In this paper we shall develop in a quantitative way some of the difficulties with existing models for modulation of the primary cosmic-ray intensity. We shall then demonstrate that the geocentric model avoids these same difficulties. We shall find that a magnetic storm effect should be associated with the geocentric model, and this is discussed only qualitatively in order to suggest that the observed temporary decrease of the horizontal component of the geomagnetic field can occur. We shall find, ultimately, that it may be possible to test the geocentric model, independently of inferences from cosmic-ray observations, by observing the absorption effects of captured interplanetary gas in the solar spectrum.

II. OBSERVATIONS AND MODELS

We describe the energy spectrum of the primary cosmic rays in terms of the differential spectrum $j(E)$, so that $j(E)dE$ represents the number of cosmic-ray particles per cm^2 per sec per steradian with kinetic energy (per nucleon) in the interval $(E, E+dE)$. $j(E)$ will be taken to refer to the cosmic-ray spectrum that would be observed at the top of the atmosphere in the absence of the geomagnetic field; we shall not concern ourselves with the geomagnetic cutoff and related phenomena, due to the dipole magnetic field of the earth.

The dependence of $j(E)$ on E is discussed at length in a review article by Biermann[7]; for $E>20$ Bev per nucleon $j(E)$ is fairly well represented by E^{-n}, where

[3] P. Morrison, Proceedings of the Guanajuato International Conference on Cosmic Rays, 1955 (unpublished); Phys. Rev. **101**, 1397 (1956).

[4] P. Meyer, Proceedings of the Guanajuato International Conference on Cosmic Physics, 1955 (unpublished).

[5] K. Nagashima, J. Geomag. Geoelec. **5**, 141 (1953).

[6] P. Meyer and J. A. Simpson, Phys. Rev. **99**, 1517 (1955).

[7] L. Biermann, *Annual Review of Nuclear Physics* (Annual Reviews, Inc., Stanford, 1953), p. 336.

$n \cong 2.7$. Below 20 Bev $j(E)$ becomes less steep, with n falling to about 2.0 in the vicinity of 5 Bev. The existing evidence is not inconsistent with the assumption that $j(E)$ reaches a maximum near 1 Bev and then decreases uniformly to zero at $E=0$; the decrease is called the low-energy cutoff.

The variations of the spectrum with time are largest at low energies and decrease monotonically as one observes higher and higher energy particles. This suggests, of course, that magnetic fields are involved in the modulation. The variations with time have been treated extensively in the literature[6,8–10] and will be but briefly described here. The major recognized variations are

(a) A small diurnal variation with an amplitude of the order of 0.5% of the total cosmic-ray intensity. The maximum usually occurs near solar noon, but the phase sometimes wanders sufficiently that the maximum falls near midnight.[11] The small amplitude of the diurnal variation implies that the primary cosmic-ray particles are very near isotropy.

(b) Superposed but independent sequences of 27-day variations with amplitudes as high as approximately 15% of the total intensity. The variation is generally associated with the period of solar rotation.[9]

(c) Forbush-type decreases wherein the cosmic-ray intensity may drop as much as 20–30% in the course of a few hours, recovering slowly and irregularly, with perhaps further sharp decreases, over the next few days or weeks. Forbush-type decreases and magnetic storms occur sometimes simultaneously and sometimes quite independently.[8,10]

(d) Eleven or twenty-two year variation of the cosmic-ray intensity with the general cycle of solar activity. The full character of this long period variation is still tentative, being based on ion-chamber measurements at higher energies,[8] where the effect is small, over the past 20 years and on observations of the low-energy end of the spectrum with neutron detectors since about 1950. It seems that the cosmic-ray intensity increases at all particle energies when the general level of solar activity is low. This increase is largest at the low energies but extends up to 20 or 30 Bev. At higher energies any changes in intensity are too small to be observed. The energy at which the cosmic-ray intensity is a maximum (usually near 1 Bev) was observed to decrease between the years 1948 and 1951[6] and between 1951 and 1954[12]; 1954 was the year of sunspot minimum. By 1954 there was no indication that $j(E)$ went to zero at zero energy.[12,13] At the same time the intensity in the vicinity of the spectrum maximum increased so much that below 4 or 5 Bev $j(E)$ became a noticeably steeper function of E; whereas $j(E) \propto E^{-2.0}$ for $E<5$ Bev prior to 1948, by 1951 it was found[6] that $j(E) \propto E^{-2.5}$ over the same range. Rapid fluctuations at low energies were found in 1948–1949[14] and again in 1954–1955.[12] One presumes that, with the increase in solar activity following the deep minimum of 1954, the cosmic-ray spectrum will return to approximately the form and intensity it possessed in 1948, and it is on this interpretation of the observations that we shall proceed.

On the basis of the observations it seems that we are looking for a quasi-steady state magnetic condition, set up in the solar system by solar activity, which depresses the density of low rigidity galactic cosmic-ray particles at the earth. Now in order to depress the particle density in a region (solar system) immersed in the supposedly uniform and isotropic galactic cosmic-ray field, it is necessary to have a barrier surrounding the region in order to impede the entrance of particles into the region; it is also *necessary* that there be some device for *removing* particles from within the region once they have entered. Otherwise, no matter how slowly particles may enter the region, the region will eventually fill up to the density of the external cosmic-ray field. Clearly, the more effective the removal mechanism, the less effective need be the barrier to depress the average particle density a given amount.

With the above requirements in mind, consider a model in which magnetic field bearing ion clouds ejected from the sun and sweeping outward through the solar system serve to produce a cosmic-ray cutoff in the inner solar system. The tangled magnetic fields carried by the clouds deflect the lower rigidity particles so that their passage into the solar system is random walk; hence the clouds serve as a barrier. The outward motion of the clouds sweeps back the low-rigidity particles; hence the outward motions produce a convective removal: If we associate the clouds with the clouds from the sun that produce magnetic storms, then they would be expected to travel out from the sun with velocities of the order of 2000 km/sec; the orbit of the earth should be swept out once every day. Morrison[3] was the first to consider such a mechanism, though he was mainly interested in the Forbush decreases.

In the next section it is shown that the outward sweeping clouds can produce a striking low-energy cutoff effect. From the random walk of the incoming cosmic-ray particles it is obvious that isotropy is preserved by the mechanism, and it is also obvious that the amplitude of the effect will depend in the proper

[8] S. E. Forbush, Terrestrial Magnetism and Atm. Elec. **42**, 1 (1937); **43**, 203 (1938). S. E. Forbush, Bull. Int. Union Geod. and Geophys., No. 11, 438 (1940); J. Geophys. Research **59**, 525 (1954).

[9] R. A. Millikan and H. V. Neher, Phys. Rev. **56**, 491 (1939).

[10] J. A. Simpson, Phys. Rev. **94**, 426 (1954); Ann. geophys. **11**, 305 (1955); Proceedings of the Guanajuato International Conference on Cosmic Physics, 1955 (unpublished).

[11] Firor, Fonger, and Simpson, Phys. Rev. **94**, 1031 (1954).

[12] H. V. Neher, Proceedings of the Guanajuato International Conference on Cosmic Rays, 1955 (unpublished); Phys. Rev. **103**, 228 (1956).

[13] E. P. Ney, Proceedings of the Guanajuato International Conference on Cosmic Rays, 1955 (unpublished).

[14] J. A. Simpson, Phys. Rev. **83**, 1175 (1951).

way on solar activity. But there are several difficulties encountered:

(a) The required number of outgoing clouds *seems* to imply a solar mass loss of the order of 10^{14} g/sec, which is very much in excess of the estimates of 5×10^{10} g/sec obtained from the study of the dynamics of comet tails.[15,16]

(b) In order for outward rushing magnetic clouds of solar origin to screen the earth in all directions from galactic particles, it is necessary that the clouds be ejected in all directions from the sun. One day is required for the clouds to sweep out the orbit of the earth, but a relativistic particle can make the trip in 8 minutes; hence a leak of very small dimensions in the screening clouds may be serious. Both visual and magnetic observations of the sun indicate that the most violent aspects of solar activity (e.g., sunspots, flares, surges, etc.), with which we might associate the ejection of the necessary high velocity magnetic clouds, do not occur near the poles.[17] This suggests that above both poles of the sun there would be large gaps in the outward moving cloud barrier.

(c) Since the outward rushing cloud mechanism has a characteristic time of one day, it is not clear why the cosmic-ray spectrum does not vary markedly with the appearance of the active solar regions, which are responsible for such phenomena as magnetic storms.

(d) The fact that from solar flares we observe (low-rigidity) cosmic-ray intensity increases with sharp fronts and confined to geomagnetic impact zones on the earth[18,19] implies that the space inside the orbit of the earth is relatively free from scattering magnetic fields; the assumption of enough outward rushing clouds to produce either a Forbush decrease or the observed low-energy cutoff excludes such direct observation of flares particles.

(e) The Forbush-type decreases in the cosmic-ray intensity sometimes show declines with characteristic times as little as four hours and relaxation times up to many months. Presumably the abrupt decreases are due to the appearance of either higher velocity interplanetary clouds, so that the removal rate is increased, or more and magnetically denser clouds, so that the barrier they form is less permeable. As will be shown later a decrease time of 5 hours requires that we have cloud velocities of 10^4 km/sec or tangled interplanetary cloud fields as high as 0.5×10^{-2} gauss. Those examples of Forbush decreases which take weeks or months to recover require that the cloud velocity and magnetic density occur uniformly throughout much of the solar system and be maintained for several months.

In the next section we give a more quantitative discussion of the aspects of this mechanism. For the present, let us go on to see what alternative models exist. Quite generally we may state that a magnetic barrier, for the purpose of impeding galactic cosmic-ray particles in entering the solar system, may be composed of disordered and tangled magnetic fields or, on the other hand, may be a single large-scale ordered field. We have already discussed outward rushing clouds, which seems to make the most efficient use of disordered fields. If we turn our attention to ordered fields, the most obvious model is a heliocentric dipole field; such a field would be expected to turn back galactic cosmic ray particles according to the familiar Störmer theory. The first objection to be raised against the model is the observational fact that the fields on the surface of the sun have an average value of only a few gauss, which, if extrapolated as a dipole to the orbit of the earth give the insufficient field density of about 10^{-7} gauss. There is a further objection, however, which we shall discuss in connection with both the heliocentric dipole and an alternative solar model suggested by Davis.[20]

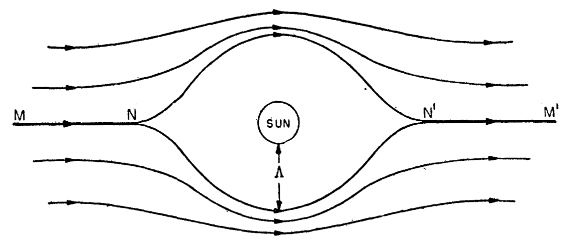

FIG. 1. Schematic diagram of geocentric mechanism for modulating the primary cosmic-ray spectrum, showing the lines of force of the disordered magnetic fields of the captured interplanetary gas in and around the geomagnetic field.

Davis has pointed out that the outward pressure due to the steady streaming of matter from the solar corona may produce a cavity in the galactic arm field (Fig. 1). It is of interest to inquire whether such a cavity might contribute to the low-energy cutoff. If the cavity were to have smooth homogeneous walls it would have barrier properties for low-rigidity particles; a particle spiraling along a line of force of the galactic field would be diverted around the solar system because the galactic lines of force are so diverted (by the effusing solar coronal gas).

To develop our objection to both the heliocentric dipole and the cavity we note that, in conjunction with these ordered barrier models, the most effective removal mechanism seems to be the absorption of cosmic-ray particles by the sun: The stimultaneous removal by outward sweeping clouds does not seem possible because the clouds would, if they swept outward, punch holes and otherwise disorganize the ordered fields; absorption by the interplanetary matter does not seem plausible because it would require an interplanetary density of 10^{-17} g/cm^3 in order to compete with the sun, and the removal by nuclear collisions would not allow the

[15] L. Biermann, Z. Astrophys. **29**, 274 (1951).
[16] H. C. van de Hulst, *The Sun*, edited by G. Kuiper (University of Chicago Press, Chicago, 1953).
[17] Payne, Scott, and Little, Australian J. Sci. Research **5**, 32 (1952).
[18] J. Firor, Phys. Rev. **94**, 1017 (1954).
[19] P. Meyer and J. A. Simpson, Phys. Rev. (to be published).
[20] L. Davis, Phys. Rev. **100**, 1440 (1955).

observed heavy nuclei to reach the earth.[21,22] The observed interplanetary densities are of the order of 10^{-21} to 10^{-19} g/cm^3.[23–25] We suppose then that collision with the sun is responsible for removing the cosmic-ray particles.

Now the disk of the sun occupies about 2×10^{-5} of the area enclosed by the orbit of the earth. Hence, a particle may expect to make 5×10^4 trips through the orbit of the earth before being absorbed, requiring 1.5 years at the speed of light. This is to be contrasted with the much more effective removal time of one day for outward sweeping clouds. Therefore, in order to significantly depress the particle density, an ordered barrier must be about 500 times more impenetrable than a disordered barrier of outward sweeping clouds. A large scale interplanetary dipole field would have to be so nearly perfect in form that less than one in 5×10^4 incoming particle trajectories serves to take the particle into a region forbidden by Störmer theory; the galactic field near the wall of Davis's cavity would have to be so free of gradients parallel to the wall that fewer than one in 5×10^4 of the particles that would otherwise be diverted around the cavity(by a cavity field of perfect symmetry) succeeds in entering the cavity. In view of the distortions that would be produced in an ordered field by the irregular ejection of matter from the sun, as indicated by magnetic storms, aurorae, etc., it seems reasonable to expect that neither a heliocentric dipole nor a heliocentric cavity in the galactic field will possess the required remarkable degree of symmetry and smoothness. This problem is discussed again below.

Looking back over all the arguments given above, we see that one of the basic sources of difficulty in producing a cutoff may be traced to the large size of the solar system: If we assume an outward sweeping disordered barrier, we require an immense solar investment in both magnetic fields and matter, because the cosmic rays must be excluded over an area not less than 4π square astronomical units. If we use a static barrier we have only absorption by the sun to remove particles; the sun occupies so small a fraction of interplanetary space that the removal is verv slow and the cooperating barrier, be it ordered or disordered fields, must be more impermeable than seems reasonable.

In closing this criticism of existing modulation models, it is only proper to emphasize that the *apparent* difficulties which we have suggested arise as much from ignorance as from knowledge; the existing dynamical theory of ionized gases in the presence of magnetic fields cannot begin to cope with some of the high velocities directly observed in solar activity, nor can it even hint as to how ions can be accelerated from thermal energies to the relativistic velocities of cosmic rays. Thus we should not be surprised if the same dynamics does not readily produce a means for modulating the primary cosmic-ray intensity. The above objections to existing cut-off models were raised in order to exhibit the theoretical obstacles that have yet to be overcome, and to give some basis for judging the relative merits of the geocentric mechanism which we shall later propose.

III. PASSAGE OF COSMIC-RAY PARTICLES THROUGH DISORDERED MAGNETIC FIELDS

A. General Theory

To represent in an approximate way how a cosmic-ray particle diffuses through a barrier of disordered magnetic fields we shall apply the statistical concepts of elementary kinetic theory. We let η represent the ratio of the kinetic energy to the rest energy of a given particle. If W represents the total and W_0 the rest energy, then

$$\eta=(W-W_0)/W_0. \tag{1}$$

If the particle velocity is w, then it is readily shown that

$$\frac{w^2}{c^2}=\frac{\eta(\eta+2)}{(\eta+1)^2},\quad 1-\frac{w^2}{c^2}=\frac{1}{(\eta+1)^2}. \tag{2}$$

We let $j(x,y,z;\eta)$ represent the number of particles per unit volume with energy η. We use $L(x,y,z;\eta)$ to represent the distance over which the velocity of a particle becomes uncorrelated with earlier velocities as a consequence of the scattering centers. If $\mathbf{f}(x,y,z;\eta)$ represents the flux of particles due to a gradient in $j(x,y,z;\eta)$, then from elementary kinetic theory

$$\mathbf{f}(x,y,z;\eta)=-\tfrac{1}{3}L(x,y,z;\eta)\nabla[wj(x,y,z;\eta)]. \tag{3}$$

If in addition to the diffusion there is a general drift of the scattering centers through which the particles are diffusing, then (3) is valid in the frame of reference moving with the centers. In a fixed frame of reference we have, then,

$$\mathbf{f}=\mathbf{v}j-\tfrac{1}{3}L\nabla(wj), \tag{4}$$

where $\mathbf{v}$ is the drift velocity of the scattering centers, and for convenience we have dropped the argument $(x,y,z;\eta)$ of $\mathbf{f}$, $\mathbf{v}$, L, and j. If there is no acceleration or absorption of particles, so that the number of particles in any given energy range is conserved, we have

$$\begin{aligned}\partial j/\partial t&=-\nabla\cdot\mathbf{f}\\&=-\nabla\cdot(\mathbf{v}j)+\tfrac{1}{3}\nabla\cdot[L\nabla(wj)].\end{aligned} \tag{5}$$

We are interested in the steady state conditions with a spherically symmetric geometry (heliocentric or geocentric). Thus $\mathbf{v}$ has only an r component, which we denote by v. Equation (5) reduces to

$$\frac{\partial}{\partial r}\left[r^2\left(jv-\frac{1}{3}Lw\frac{\partial j}{\partial r}\right)\right]=0.$$

[21] E. Fermi, Astrophys. J. **119**, 1 (1954).
[22] Morrison, Olbert, and Rossi, Phys. Rev. **94**, 440 (1954).
[23] A. Unsold and S. Chapman, Observatory **69**, 219 (1949).
[24] A. Behr and H. Siedentopf, Z. Astrophys. **32**, 19 (1953).
[25] L. R. O. Storey, Trans. Roy. Soc. (London) **A246**, 113 (1954).

Integrating, we find that

$$jv-\tfrac{1}{3}Lw\partial j/\partial r=C/r^2,$$

where C is the constant of integration. Comparing with (4) we see that the net outward flux $f(r,\eta)$ is

$$f(r,\eta)=f(r_0,\eta)(r_0/r)^2 =j(r,\eta)v(r)-\tfrac{1}{3}L(r,\eta)w\partial j(r,\eta)/\partial r, \quad (6)$$

where $f(r_0,\eta)$ represents the flux density at $r=r_0$.

1. *Static Barrier*

Suppose that the scattering centers do not drift, so that $v=0$, but that the sphere $r=R$ represents a perfect absorber of particles (supposedly representing the sun or the earth). Then at $r=R$ there are only particles that move toward the origin, none away, and the flux density is $-\frac{1}{3}wj(R,\eta)$. Equation (6) reduces to

$$\frac{\partial j(r,\eta)}{\partial r}=\frac{j(R,\eta)R^2}{L(r,\eta)r^2}.$$

Integrating from r to ∞, we have

$$j(\infty,\eta)-j(r,\eta)=j(R,\eta)R^2\int_r^\infty \frac{dr}{r^2L(r,\eta)}. \quad (7)$$

Putting $r=R$, solving for $j(R,\eta)$, and eliminating $j(R,\eta)$ from (7), we have for $v=0$

$$j_1(r,\eta)=j(\infty,\eta)\frac{1+I_1(r,\eta)}{1+I_1(\infty,\eta)}, \quad (8)$$

where

$$I_1(r,\eta)=R^2\int_R^r \frac{dr}{r^2L(r,\eta)}. \quad (9)$$

The subscript 1 on $j(r,\eta)$ indicates that it is the $j(r,\eta)$ resulting from $I_1(r,\eta)$.

2. *Outward Sweeping Barrier*

If, on the other hand, v does not vanish and there are no sinks, then $f(r,\eta)=0$ and (6) gives

$$j_2(r,\eta)=j(\infty,\eta)\exp[-I_2(r,\eta)] \quad (10)$$

upon integration, where

$$I_2(r,\eta)=\frac{3}{w}\int_r^\infty dr\frac{v(r)}{L(r,\eta)}. \quad (11)$$

The subscript 2 on $j(r,\eta)$ indicates that it is the $j(r,\eta)$ resulting from $I_2(r,\eta)$.

B. Scattering Length

Our next job is to determine the dependence of the scattering length $L(r,\eta)$ on the particle energy η. We define a scattering center to be a region over which the magnetic field does not change sign. We let $l(r)$ represent the diameter of a scattering center at r, and $B(r)$ the average field density. We let $L_0(r)$ be the distance traveled by the particle between scattering centers. The radius of curvature P of the trajectory of a particle of rest mass m, charge q (esu), and velocity w in the scattering field $B(r)$ is

$$P(r,\eta)=\frac{mwc}{B(r)q[1-(w^2/c^2)]^{\frac{1}{2}}} =\frac{W_0[\eta(\eta+2)]^{\frac{1}{2}}}{qB(r)}. \quad (12)$$

We must now introduce a distinction between scattering centers which are dense enough and large enough to actually reflect or turn back incident particles and centers which are sufficiently small and dilute to allow particles to penetrate all the way through, deflecting the trajectory of the particle by less than $\pi/2$ radians. The former we call *reflecting* scattering centers; they are defined by $l(r)B(r)$ being sufficiently large that $l(r)>P(r,\eta)$. The latter we call *transmitting* scattering centers; they are defined by $l(r)B(r)$ being sufficiently small that $l(r)<P(r,\eta)$. It should be noted that *reflecting* and *transmitting* are terms which may be applied to the scattering centers only when the particle energy is specified; from (12) it is clear that no matter what the value of $l(r)B(r)$, $P(r,\eta)$ will be larger than $l(r)$ for sufficiently large particle energies and smaller for sufficiently small particle energies.

It is obvious that the scattering length $L(r,\eta)$ is just equal to $L_0(r)$ for thick scattering centers. If the scattering centers are transmitting, the trajectory is deflected through an angle θ upon passing through a center, where

$$\theta\cong l(r)/P(r,\eta). \quad (13)$$

After n such random deflections, the mean total deflection is of the order of $\theta\sqrt{n}$. As a working definition of the scattering length $L(r,\eta)$, we assume that the particle has lost all correlation with its initial motion by the time that the mean total deflection is as large as $\pi/2$. Thus, it is readily shown that

$$L(r,\eta)=\left[\frac{\pi P(r,\eta)}{2l(r)}\right]^2 L_0(r).$$

We use the combination form

$$L(r,\eta)=L_0(r)\left\{1+\left[\frac{\pi P(r,\eta)}{2l(r)}\right]^2\right\} \quad (14)$$

as an approximate representation of $L(r,\eta)$ for both the case that $l\geq P$ and $l\leq P$.

From elementary kinetic theory, we have that the mean free path of a particle, traveling among scattering centers of scale $l(r)$ and spaced such that there are

$N(r)$ per unit volume, is

$$L_0(r)=\frac{1}{\pi l^2(r)N(r)}. \tag{15}$$

Using (12), (14), and (15), we may rewrite (9) as

$$I_1(r,\eta)=\int_R^r dr\left(\frac{R}{r}\right)^2 N(r)\left\{\frac{1}{\pi l^2(r)}+\frac{\pi^3 W_0{}^2\eta(\eta+2)}{4q^2\Phi^2}\right\}^{-1}, \tag{16}$$

where Φ is the total flux $\pi l^2(r)B(r)$ making up the scattering center. As we have already remarked, the electrical conductivity is sufficiently large that the magnetic lines of force move with the matter and Φ may be taken as constant.

High conductivity implies that the individual scattering centers preserve their identity, so that we may write the continuity equation

$$\partial N/\partial t=-\nabla\cdot(N\mathbf{v}).$$

For a spherically symmetric steady state, we have

$$N(r)v(r)=N(r_0)v(r_0)(r_0/r)^2 \tag{17}$$

upon integration. Thus (11) may be rewritten as

$$I_2(r,\eta)=\frac{3v(r_0)N(r_0)}{c}\int_r^\infty dr\left(\frac{r_0}{r}\right)^2 \times\left\{\frac{[\eta(\eta+2)]^{\frac{1}{2}}}{\pi l^2(r)(\eta+1)}+\frac{\pi^3W_0{}^2[\eta(\eta+2)]^{\frac{3}{2}}}{4q^2\Phi^2(\eta+1)}\right\}^{-1}. \tag{18}$$

(16) and (18) give I_1 and I_2 in terms of the characteristics N, v, and Φ, of the scattering centers regardless of the relative values of $P(r,\eta)$ and $l(r)$.

The first term in the braces in (16) and (18) represents the contribution of reflecting scattering centers, the second term represents transmitting scattering centers. From (16) and (18) we see that transmitting centers produce the sharper low-energy cutoff as η goes to zero. Or, to state the matter differently, reflecting scattering centers depress the density of the high-energy particles more than do transmitting centers for a given reduction of density of low-energy particles. The extreme is given by (16) for reflecting centers, in which case $j_1(r,\eta)/j(\infty,\eta)$ is independent of η. Hence, if we wish to produce a low-energy cutoff with absorption playing the role of the removal mechanism, we must assume that the scattering centers are transmitting ($P>l$) down to the lowest observable particle energy.

C. Comparison of Removal Mechanisms

Having developed the expressions for the depression $j(r,\eta)/j(\infty,\eta)$ of the cosmic-ray density by a disordered magnetic barrier in conjunction with either absorptive or convective removal, let us now compare the effectiveness of the two removal mechanisms in depressing the density. Let us suppose that only very thin scattering centers are present so that I_1, $I_2\ll 1$. Then (8) and (10) reduce to

$$j_1(r,\eta)/j(\infty,\eta)=1+I_1(r,\eta)-I_1(\infty,\eta)+O^2(I_1),$$
$$j_2(r,\eta)/j(\infty,\eta)=1-I_2(r,\eta)+O^2(I_2).$$

The ratio, ζ, of the depression produced by absorptive removal to the depression produced by convective removal becomes

$$\begin{aligned}\zeta&=\frac{j(\infty,\eta)-j_1(r,\eta)}{j(\infty,\eta)-j_2(r,\eta)}\\&=\frac{I_1(\infty,\eta)-I_1(r,\eta)}{I_2(r,\eta)}\\&=\frac{1}{3}\left(\frac{R}{r_0}\right)^2\frac{c}{v(r_0)}\frac{[\eta(\eta+2)]^{\frac{1}{2}}}{\eta+1}\\&\quad\times\left[\int_r^\infty\frac{drN(r)}{r^2h(r)N(r_0)}\right]\left[\int_r^\infty\frac{dr}{r^2h(r)}\right]^{-1},\end{aligned} \tag{19}$$

where

$$h(r)=\frac{1}{\pi l^2(r)}+\frac{\pi^3}{4}\left[\frac{W_0}{q\Phi}\right]^2\eta(\eta+2). \tag{20}$$

If we suppose that we may write

$$h(r)\cong h(r_0)(r_0/r)^\mu, \tag{21}$$

$$N(r)\cong N(r_0)(r_0/r)^\nu, \tag{22}$$

over the range of r in which $h(r)$ and $N(r)$ are important, then

$$\zeta=\frac{\kappa}{3}\left(\frac{R}{r_0}\right)^2\left(\frac{r_0}{r}\right)^\nu\left(\frac{c}{v(r_0)}\right)\frac{[\eta(\eta+2)]^{\frac{1}{2}}}{\eta+1}, \tag{23}$$

where

$$\kappa=(\mu-1)/(\mu-1-\nu). \tag{24}$$

We may put $r=r_0$ without loss of generality.

If we apply (23) to the solar system, then R is the radius of the sun, 0.6×10^6 km, and r or r_0 is the radius of the orbit of earth, 1.5×10^8 km. Then (23) reduces to

$$\zeta=\frac{1.6\kappa}{v(r_0)}\frac{[\eta(\eta+2)]^{\frac{1}{2}}}{\eta+1}$$

with $v(r_0)$ in km/sec. For a 1-Bev proton ($\eta=1.07$), we have $\zeta=1.4\kappa/v(r_0)$. Since κ is of the order of unity and the outward cloud velocity may be as large as 2000 km sec, we see that ζ will probably be very much less than unity. Hence, if we set about to decrease the cosmic-ray particle intensity with a barrier composed of disordered scattering centers or magnetic clouds, we find that convective removal is far more effective than solar absorption.

If we apply (23) to a geocentric barrier, then R is the radius of the earth, 0.6×10^4 km, and r_0 is the inner

radius of the barrier, say $r_0=1.5R$. Then $\zeta=3.9\times10^4\kappa/v(r_0)$ for 1-Bev protons, and $v(r_0)$ would have to exceed some 4×10^4 km/sec before the outward sweeping would be as effective as the absorption by the earth. Of course, the earth does not eject magnetic clouds; the calculation serves to illustrate how effective is terrestrial absorption inside a geocentric barrier as compared to the solar absorption inside a heliocentric barrier.

IV. MODEL FOR A HELIOCENTRIC DISORDERED BARRIER

A. Interplanetary Field Densities

As a first step in constructing a model for a heliocentric barrier of disordered fields to impede the passage of galactic cosmic-ray particles into the inner solar system, we must obtain some estimate of the disordered interplanetary fields at our disposal. Observations indicate that the mean electron density in the orbit of the earth is of the order of $500/\text{cm}^3$,[24,25] with occasional transient increases to $5\times10^4/\text{cm}^3$.[23] Such low densities exposed to the ultraviolet radiation of the sun suggest that the interplanetary material should be almost completely ionized. Hence, the interplanetary density ρ should be given approximately by $N_\epsilon M$, where N_ϵ is the electron density and M is the mass of a hydrogen atom.

Consider a volume of highly conducting gas ejected from a region in the sun where the mean magnetic field density is B_s and the material density is ρ_s. If the ejected gas expands isotropically, then the magnetic field varies as $\rho^{\frac{2}{3}}$. Insofar as the magnetic lines of force carried with the gas are not intertwined and tangled, the gas will expand only perpendicular to the lines of force in response to the magnetic stresses; expansion perpendicular to the lines of force leads to B varying directly with ρ. Thus

$$B\leq B_s(\rho/\rho_s)^{\frac{2}{3}} \tag{25}$$

gives an upper limit on B after expansion of the solar gas.

Suppose now that the velocity of the gas after ejection is v. If all of the kinetic energy of ejection came from the magnetic energy contained within the gas, we would equate the initial magnetic energy per unit volume $B_0^2/8\pi$ to the final kinetic energy of the same amount of gas, $\frac{1}{2}\rho_s v^2$ (the residual magnetic energy after expansion into interplanetary space is negligible). If the energies of fields outside the ejected volume of gas contributed to the final kinetic energy, then B_s would not be as large as indicated by the equality of magnetic and kinetic energies. Hence we write the more general condition that

$$B_s^2/8\pi\leq\tfrac{1}{2}\rho_s v^2, \tag{26}$$

giving an upper limit on B_s.

If we use (26) to eliminate B_s from (25), we have

$$B\leq 2\pi^{\frac{1}{2}}\rho^{\frac{2}{3}}v/\rho_s^{1/6} \tag{27}$$

as an upper limit on the interplanetary field density B. We note how insensitive B is to the initial density ρ_s. We put $v=2000$ km/sec, suggested by the delay of the terrestrial effects of a solar flare; with $N_\epsilon=500/\text{cm}^3$ we have the interplanetary density $\rho=10^{-21}$ g/cm^3; we choose the photospheric value 10^{-8} g/cm^3 for ρ_s. Then (27) gives $B\leq1.4\times10^{-4}$ gauss as the upper limit on the mean interplanetary field density at the orbit of the earth.

B. Necessary Barrier Thickness

In order to estimate the number of scattering centers required in an interplanetary barrier of sufficient thickness to produce the observed low energy cutoff in the galactic cosmic-ray spectrum, we shall assume that the galactic differential spectrum is of the form

$$j(\infty,\eta)=\text{constant}/(a+\eta^p), \tag{28}$$

and put $a=3$, $p=2.5$.[5]

For outward sweeping interplanetary magnetic fields, which we have shown to be much more effective in producing a cutoff than a static barrier, we have from (10) that the differential spectrum $j_2(r,\eta)$ observed at the earth will be a maximum at the energy η for which

$$\frac{\partial I_2(r,\eta)}{\partial\eta}=\frac{1}{j(\infty,\eta)}\frac{\partial j(\infty,\eta)}{\partial\eta}. \tag{29}$$

If the scattering centers are reflecting ($P<l$), then from (18) we have that

$$\frac{\partial I_2}{\partial\eta}=-\frac{A_1}{[\eta(\eta+2)]^{\frac{3}{2}}}, \tag{30}$$

where

$$A_1=3\pi r_0^2N(r_0)\frac{v(r_0)}{c}\int_r^\infty\frac{dr}{r^2}l^2(r). \tag{31}$$

If, on the other hand, the centers are transmitting ($P>l$), then

$$\frac{\partial I_2}{\partial\eta}=-A_2\frac{2\eta^2+4\eta+3}{[\eta(\eta+2)]^{5/2}}, \tag{32}$$

where

$$A_2=\frac{12}{\pi^3}r_0^2N(r_0)\frac{v(r_0)}{c}\left(\frac{q\Phi}{W_0}\right)^2\frac{1}{r}. \tag{33}$$

The observed spectrum $j_2(r,\eta)$ has a maximum at energies of the order of $\eta\cong1$ (0.93-Bev protons). Putting (30) and (32) into (29) we find that $A_1=3.25$ (for reflecting scattering centers) and $A_2=1.08$ (for transmitting scattering centers). If the maximum is taken at $\eta=2$, then $A_1=18.5$ and $A_2=7.8$.

We must now deduce the necessary outward cloud velocity $v(r_0)$ from the above values of A_1 and A_2. To

estimate the value of the integral in (31) we remember that we have assumed that the total flux Φ in a scattering center is constant, and that a reflecting center is one where the angle θ, as defined in (13), is in excess of $\pi/2$. As a cloud moves out from the sun, it can be shown that the thickness $l(r)$ of the scattering center increases in proportion to the available room, and hence as $r^{\frac{2}{3}}$ if $v(r)$ is constant. Using (12), we may write (13) as

$$\theta=\frac{l(r)B(r)q}{W_0[\eta(\eta+2)]^{\frac{1}{2}}}=\frac{\Phi q}{\pi W_0 l(r)[\eta(\eta+2)]^{\frac{1}{2}}}. \tag{34}$$

Thus, θ eventually becomes small as $r^{-\frac{2}{3}}$, and what may have been reflecting scattering centers at the orbit of the earth become transmitting, i.e., $\theta<\pi/2$, farther out. Hence, the scattering which shields the earth at $r=r_0$ extends out to some effective distance $r=nr_0$ $(n>1)$ beyond which θ is too small to be effective. Hence, we shall eveluate the integral in (31) putting by $l(r)\propto r^{\frac{2}{3}}$ and cutting off the integration at $r=nr_0$.

We obtain

$$\int_{r_0}^{\infty}\frac{dr}{r^2}l^2(r)=3(n^{\frac{1}{3}}-1)\frac{l_0^2(r_0)}{r_0} \tag{35}$$

for the shielding of the earth at r_0.

We have no way of deducing what might be the closeness of packing of the scattering centers, and the relative magnitudes of P and $l(r)$ are undetermined. Thus we let

$$N(r)=\nu/l^3(r), \tag{36}$$

so that ν is a measure of the closeness of packing ($\nu\leq 1$). We use θ, as defined in (13), as the parameter determining the relative magnitudes of P and $l(r)$, i.e., whether the scattering centers are relatively reflecting or transmitting.

Using (12), (13), (35), and (36), we may rewrite (31) as

$$A_1=\left(\frac{\nu}{\theta}\right)\frac{9\pi(n^{\frac{1}{3}}-1)qr_0v(r_0)B(r_0)}{W_0c[\eta(\eta+2)]^{\frac{1}{2}}}; \tag{37}$$

with $r=r_0$, (33) becomes

$$A_2=(\theta\nu)\frac{12qr_0v(r_0)B(r_0)[\eta(\eta+2)]^{\frac{1}{2}}}{\pi cW_0} \tag{38}$$

with the aid of (12), (13), and (36).

We now use (27), assuming that $v=v(r_0)$, to eliminate $B(r_0)$ from (37) and (38). Solving for $v(r_0)$, we have

$$v(r_0)>\left(\frac{\theta}{\nu}\right)^{\frac{1}{2}}A_1^{\frac{1}{2}}\left(\frac{W_0c}{qr_0}\right)^{\frac{1}{2}}\frac{[\eta(\eta+2)]^{\frac{1}{4}}\rho_s^{1/12}}{(n^{\frac{1}{3}}-1)^{\frac{1}{2}}\mu^{\frac{1}{4}}\rho^{\frac{1}{3}}} \tag{39}$$

for reflecting clouds ($\theta>\pi/2$), and

$$v(r_0)>\frac{A_2^{\frac{1}{2}}}{(\theta\nu)^{\frac{1}{2}}}\frac{\pi^{\frac{1}{4}}}{2\sqrt{6}}\left(\frac{W_0c}{qr_0}\right)^{\frac{1}{2}}\frac{\rho_s^{1/12}}{\mu^{\frac{1}{4}}\rho^{\frac{1}{3}}[\eta(\eta+2)]^{\frac{1}{4}}} \tag{40}$$

for transmitting clouds ($\theta<\pi/2$). We see how extremely insensitive $v(r_0)$ is to ρ_s.

The minimum values of $v(r_0)$ in (39) and (40) are obtained for $\theta=\pi/2$. Now, at some energy η, θ will have this optimum value of $\pi/2$. Since $v(r_0)$ varies with η only as $[\eta(\eta+2)]^{\frac{1}{4}}$, $v(r_0)$ does not depend critically on our choice of η. Thus we arbitrarily consider a model in which $\theta=\pi/2$ at $\eta=1$ (0.93-Bev protons). We consider protons, so that $W_0=1.44\times10^{-3}$ erg and $q=4.8\times10^{-10}$ esu. Then, incidentally, if the interplanetary magnetic field $B(r_0)$, is of the order of 10^{-4} gauss, we find from (12) and (13) that the thickness of the scattering centers is of the order of 1.0×10^{11} cm or 0.002 a.u. (astronomical units).

We give ρ_s the photospheric value of 10^{-8} g/cm^3; as alternatives to 10^{-8} we might assign the chromospheric value of 10^{-12} g/cm^3 or the value 10^{-4} g/cm^3, appropriate to the upper edge of the convective zone, but such variations make a difference of only $10^{\frac{1}{3}}$ or a factor of two in $v(r_0)$. The choice of n in (39) is not critical; the values $n=2$, 8, 27 give $(n^{\frac{1}{3}}-1)^{\frac{1}{2}}=0.5$, 1.0, 1.4 respectively. We shall put $n=8$ as a reasonable median value. The density of the interplanetary medium at the orbit of the earth we take to be $\rho=10^{-21}$ g/cm^3 (500 hydrogen atoms/cm^3; we shall see that larger values serve only to give a larger solar mass loss. We put $r_0=1.5\times10^{13}$ cm for the orbit of earth. Supposing, then, that the maximum in the observed cosmic-ray differential spectrum occurs at $\eta=1$, so that $A_1=3.25$ and $A_2=1.08$, we obtain as lower limits on $v(r_0)$,

$$v(r_0)>(500/\sqrt{\nu})\ \text{km/sec} \tag{41}$$

from (39), and

$$v(r_0)>(280/\sqrt{\nu})\ \text{km/sec} \tag{42}$$

from (40). It is to be kept in mind that $\nu\leq1$. The discrepancy between (41) and (42) arises from the approximation introduced in (14) when we disregard one or the other of the comparable terms in the braces. The mass loss to the sun is

$$M=4\pi r_0^2\rho v(r_0). \tag{43}$$

Using (42), we obtain

$$M=(0.76\times10^{14}/\sqrt{\nu})\ \text{g/sec}.$$

As noted earlier, this solar mass loss is a factor of 10^3 larger than previous estimates from other considerations.

We see, now, to what extent the large size of interplanetary space makes it difficult for a barrier composed of disordered magnetic fields to depress the cosmic-ray

intensity at the orbit of the earth. The sun must eject magnetic clouds in all directions, whereas observations suggest that most of the solar ejection is near the equatorial plane. The immense amount of magnetic clouds required for the barrier represents a mass loss to the sun a thousand times larger than the usual estimates. The steady low-energy cutoff in the cosmic-ray spectrum requires that the solar output of magnetic clouds not vary with the appearance and disappearance of sporadic solar outbursts, generally associated with the ejection of solar matter.

Morrison estimates that tangled interplanetary fields of 1.5×10^{-3} gauss are sufficient to produce a decrease of the cosmic-ray intensity of the order of magnitude of the Forbush decrease. We note that this field density is a factor of ten larger than the upper limit of 1.4×10^{-4} gauss which we estimated from (27), and is a factor 10^3 too large to permit the sharp cosmic-ray intensity increases due to solar flares. The conditions become more serious when we attempt to explain the observed time dependence of the Forbush decrease; Morrison considered decreases which take place over a period of a day or two, and recover only slightly more slowly; sometimes, however, the intensity will drop abruptly by 6% in 5 hours,[26] then level off and require days or weeks to recover. It is not clear just what sort of interplanetary magnetic cloud configuration might produce such an effect. If the sun were to eject magnetic clouds with velocities of 10 000 km/sec and continue the ejection at a uniform rate for days or weeks, then we could account for the abrupt onset and slow relaxation, but 10 000 km/sec and continued ejection are drastic assumptions. If we argue that the outward velocity is not in excess of the conventional 2000 km/sec, then the 5-hour time of onset can be accounted for by assuming that the particles have diffused only 3.6×10^{10} cm into the front of the cloud, requiring tangled fields of the order 0.5×10^{-2} gauss; but in this case, or in any variation of it, we cannot account for both the sharp leveling off after only 5 hours, and the intensity remaining low for days or weeks following the decrease.

V. GEOCENTRIC LOW-ENERGY CUTOFF

It was suggested in the introduction that we should consider the possibility that the low-energy cosmic-ray cutoff is produced locally in the vicinity of the earth. Presumably the earth would be surrounded by a nebulous magnetic cloud, shown schematically in Fig. 1, composed of gas captured from the interplanetary medium by the terrestrial gravitational field. The high electrical conductivity of the gas results in the gas retaining the magnetic field present before the ejection of the gas from the sun. We shall now investigate the necessary characteristics of such a hypothetical geocentric magnetic cloud.

[26] J. A. Simpson, December 5, 1955 (unpublished).

A. Electrical Properties

Presumably the solar ultraviolet radiation maintains at least a partial ionization of the gas of a geocentric nebula. The electrical conductivity may be approximated by[27]

$$\sigma=2\times10^{-14}T^{\frac{3}{2}}\text{ emu.}$$

If we assume that $T\geq2000°\text{K}$, then $\sigma\geq2\times10^{-9}$ emu. The decay time τ for a magnetic field of scale b in a medium of electrical conductivity σ is of the order of

$$\tau\cong b^2\mu\sigma.$$

With $b=500$ km, we have $\tau\geq2$ months. It follows that the disordered magnetic field contained in a geocentric cloud built up from bits and scraps of captured interplanetary magnetic matter will be sufficiently long lived to give a fairly uniform effect on the cosmic-ray intensity even though the solar outbursts which supply the cloud material may occur at irregular intervals. The high conductivity of the captured interplanetary matter also means that after being captured it will be some time before it can diffuse into the geomagnetic field; thus, it may be that the geomagnetic field supports much of the weight of the nebula. We shall return to this point at a later time to show that the geomagnetic field is sufficiently dense to furnish the necessary support.

B. Cutoff Requirements

We consider first the necessary conditions in order that a geocentric magnetic cloud may produce the observed low-energy cosmic-ray cutoff. Since the removal mechanism associated with a static geocentric barrier is terrestrial absorption, it is necessary that the disordered magnetic fields in the captured interplanetary material form transmitting scattering centers, in the sense that $P>l$, in order that the scattering produce a low-energy cutoff; if the centers are sufficiently thick that $P<l$, then, as may be seen from (16), the cosmic ray intensity will be depressed by the same factor at all energies. For transmitting clouds, (16) reduces to

$$I_1(r,\eta)=\frac{C(r)}{\eta(\eta+2)},\tag{44}$$

where

$$C(r)=\frac{4R^2q^2\Phi^2}{\pi^3W_0^2}\int_R^r dr\frac{N(r)}{r^2}.\tag{45}$$

The surface of the earth, where we observe the cosmic-ray spectrum, is also the surface of the absorbing region $r=R$. Thus we observe the differential spectrum $j(R,\eta)$ given by (8) as

$$j(R,\eta)=j(\infty,\eta)\frac{\eta(\eta+2)}{C(\infty)+\eta(\eta+2)}.\tag{46}$$

[27] T. G. Cowling, *The Sun* (The University of Chicago Press, Chicago, 1953).

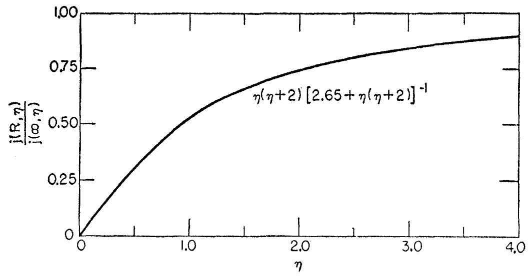

FIG. 2. The cosmic-ray particle density at the surface of the earth compared to the density in the galaxy for the condition that the earth is surrounded by disordered magnetic fields of sufficient density to produce a cutoff below 1 Bev [$C(\infty)=2.65$].

It is readily shown that $j(R,\eta)/j(\infty,\eta)$ depends only on the particle rigidity $\mathcal{R}$,

$$\mathcal{R}\equiv pc/q=(W_0/q)[\eta(\eta+2)]^{\frac{1}{2}}. \tag{47}$$

We have

$$j(R,\eta)=j(\infty,\eta)\frac{\mathcal{R}^2}{[W_0^2C(\infty)/q^2]+\mathcal{R}^2}. \tag{48}$$

$W_0^2C(\infty)/q^2$ is independent of W_0/q. Hence the cutoff effect for a static geocentric barrier depends only on particle rigidity. As one might expect, the effects of outward sweeping scattering centers in Morrison's heliocentric model do not at low energies depend on rigidity alone.

In order to estimate $C(\infty)$ from the observed cosmic-ray spectrum, so that we may compute the number of scattering centers needed, we note that

$$\frac{1}{j(R,\eta)}\frac{\partial j(R,\eta)}{\partial\eta}=\frac{1}{j(\infty,\eta)}\frac{\partial j(\infty,\eta)}{\partial\eta}+\frac{2C(\infty)(\eta+1)}{\eta(\eta+2)[C(\infty)+\eta(\eta+2)]}. \tag{49}$$

With $j(\infty,\eta)$ given by (28) and the maximum of $j(R,\eta)$ occurring near $\eta=1$, we find that $C(\infty)=45/17=2.65$. The resulting $j(R,\eta)/j(\infty,\eta)$ is shown in Fig. 2 and $j(R,\eta)$ in Fig. 3.

In order to express $C(\infty)$ in terms of the thickness $l(r)$ and magnetic density $B(r)$ of the disordered magnetic fields in a geocentric magnetic cloud, it is necessary to construct a definite idealized model of the cloud. To make the problem tractable, we shall suppose that the cloud of fields does not extend below $r=R_1$ and has perfect spherical symmetry; thus below $r=R_1$ there will be no scattering of cosmic-ray particles, and we have only the geomagnetic field which will continue to produce the usual latitude effect. We further assume that the scattering centers above $c=R_1$ are closely packed so that the number of scattering centers per unit volume is

$$N(r)\cong l^{-3}(r). \tag{50}$$

The field density b of the geomagnetic dipole field with density b_E at $r=R$ in the equatorial plane is

$$b=b_E(R/r)^3(3\cos^2\theta+1)^{\frac{1}{2}} \tag{51}$$

in spherical coordinates. Since we have assumed a spherically symmetric model, we idealize (51) to

$$b=b_E(R/r)^3. \tag{52}$$

We have no direct method for calculating the mean density B of the magnetic fields of the captured magnetic clouds, but we can obtain a rough estimate by noting that for equilibrium the total pressure within a magnetic cloud must not exceed the total pressure outside. Presumably the external gas pressure is negligible so that the external pressure is just the pressure $b^2/8\pi\mu$ of the geomagnetic field. Hence, whatever the internal gas pressure, we conclude that the internal magnetic pressure $B^2/8\pi\mu$ does not exceed $b^2/8\pi\mu$, and that $B\lesssim b$. On the other hand, it can be shown that B cannot be much less than the geomagnetic field b if B is to turn back cosmic-ray particles with energies of the order of a few Bev. These two limitations on B lead to the conclusion that $B\cong b$ and, hence, that the pressure of the cloud gas does not exceed $B^2/8\pi\mu$.

The total flux Φ in each scattering center is assumed to be conserved by the high electrical conductivity of the gas so that $\pi l^2(r)B(r)$ is independent of r. Using (50), we may carry out the integration in (45) to obtain

$$C(\infty)=\frac{8q^2\Phi^{\frac{1}{2}}b_E^{\frac{1}{2}}R}{11\pi^{\frac{1}{2}}W_0^2}\left(\frac{R}{R_1}\right)^{11/2}. \tag{53}$$

We let $l_1=l(R_1)$, so that

$$\Phi=\pi l_1^2b_E(R/R_1)^3. \tag{54}$$

It was mentioned earlier that in order for there to be a cutoff at low energies and not merely a depression of

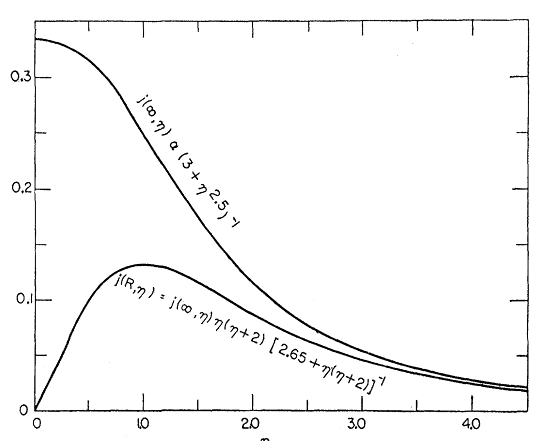

FIG. 3. The assumed galactic cosmic-ray spectrum $j(\infty,\eta)$, given by (28), and the spectrum at the surface of the earth surrounded by disordered magnetic fields of sufficient density to produce a cutoff below 1 Bev [$C(\infty)=2.65$].

intensity over the entire spectrum, we must require that the scattering centers be transmitting, $l_1<P(R_1,\eta)$, for all observable values of η. We let η_0 be the value of η at which

$$l_1=P(R_1,\eta). \tag{55}$$

Using (52), (54), and (55) to eliminate B, Φ, and l_1 from (53), we obtain

$$\frac{R_1}{R}=\left\{\frac{8qb_E R[\eta_0(\eta_0+2)]^{\frac{1}{2}}}{11\pi W_0 C(\infty)}\right\}^{\frac{1}{4}} \tag{56}$$

as the requirement that a geocentric magnetic cloud form a sufficiently impenetrable barrier to produce the observed low-energy cutoff in the cosmic-ray spectrum.

C. Support of Geocentric Clouds

Besides requiring that the geocentric magnetic clouds produce a cutoff, we must be sure that the geomagnetic field can support the weight of the clouds. It can be shown that the potential energy of a cloud of volume V in a large-scale magnetic field of density b is of the order of $V(b^2/8\pi\mu)$; the cloud is buoyed in the direction of decreasing magnetic pressure just as it would be buoyed in the direction of decreasing hydrostatic pressure. It is in this way that we suppose the geomagnetic field to support the individual magnetic clouds. Thus, we must require that the geomagnetic stresses at $r=R$ be at least as large as the weight of the overlying clouds,

$$\frac{b^2(R_1)}{8\pi\mu}\geq\int_{R_1}^{\infty} drg\rho. \tag{57}$$

Here ρ is the material density and g is the acceleration of gravity given by

$$g=g_E(R/r)^2. \tag{58}$$

Using (27) and (52) to express the material density ρ in terms of r, (57) becomes

$$\frac{R_1}{R}<\frac{121 b_E v^3(r_0)}{32\pi^{\frac{1}{2}}R^2\rho_s^{\frac{1}{2}}\mu^{\frac{3}{2}}g_E^2} \tag{59}$$

as the condition that the geomagnetic field be able to support the barrier against the terrestrial gravitational field.

Both (56) and (59) are readily satisfied. We consider protons, so that $q=4.8\times10^{-10}$ esu and $W_0=1.44\times10^{-3}$ erg. Putting the maximum in the observed differential cosmic-ray spectrum at $\eta=1$, we found that $C(\infty)=2.65$. The condition that $l_1<P(R_1,\eta)$ for all observable values of η suggests that $\eta_0\lesssim0.3$ (280-Mev protons) in (68). We use the terrestrial value $R=6.4\times10^8$ cm, $g_E=10^3$ cm/sec^2, $b_E\cong0.5$ gauss, and the solar photospheric value $\rho_s=10^{-8}$ g/cm^3. (56) and (59) reduce to

$$R_1/R<1.7,\quad R_1/R\lesssim3\times10^{-20}v^3(r_0) \tag{60}$$

respectively, to which we add the geometric restriction that $R_1/R>1$. The first condition depends only on the fourth root of $C(\infty)$, and on the eighth root of η_0, and is therefore probably fairly reliable. The second condition, on the other hand, depends on $v^3(r_0)$, and is certainly not as unflexible. However, it is so easily satisfied that it can give little trouble; for instance, the rather low interplanetary velocity of $v(r_0)=50$ km/sec gives $R_1<3.75R$. Larger values of $v(r_0)$ imply a lower material density for a given magnetic density in the cloud. We shall now compute the upper limit on the cloud density.

An upper limit on the weight of the geocentric cloud may now be estimated from (57). Taking $b_E=0.5$ gauss we have that the magnetic stress density at $r=R_1$ is $b^2(R_1)/8\pi\mu=2\times10^{-3}$ dynes/cm^2(0.7×10^{-6} g wt/cm^2) if $R_1=1.7R$. The weight of the cloud cannot exceed this value. If the thickness of the cloud is of the order of 10^4 km, we have that $\rho<0.7\times10^{-15}$ g/cm^3 or 0.4×10^9 atoms/cm^3 as an absolute upper limit on the cloud density. When we come to consider the optical effects, we shall use the density given by (27) for $v(r_0)=2000$ km/sec, suggested by the transit time from the sun of one day; we will find a value about 0.01 the above limit, indicating again how easily (57) may be satisfied.

D. Forbush Decrease

If we suppose that from time to time the terrestrial gravitational field is able to capture magnetic gas from a passing interplanetary cloud, then we would expect that at such times the cosmic-ray intensity at the surface of the earth will decrease. The fraction by which the intensity decreases will be approximately equal to the fraction by which the geocentric magnetic cloud is augmented. The length of time over which the decrease occurs will not be less than the time required for gravitational capture of magnetic clouds, and, of course, the time may be much longer if for several days passing magnetic gas is captured intermittently. Free fall from a distance r to the center of the earth requires a time t, given by

$$t=\frac{\pi}{2}\left(\frac{r^3}{2gR^2}\right)^{\frac{1}{2}},$$

where g is the acceleration of gravity, 980 cm/sec, at the surface of the earth, $r=R$. If r is rather larger than R, t is a good approximation to the time required to fall from r to $\sim R$, for which we intend to use it. Observations of decreases in the cosmic-ray intensity seem to show no fluctuations with characteristic times less than 3 or 4 hours, corresponding to free fall from about 5 times the earth's radius.

We would expect on the basis of the geocentric cloud model that the terrestrial capture of magnetic interplanetary gas would not result in a decrease of the cosmic-ray intensity which was immediately uniform over the earth. It is observed that decreases in intensity

in general do not have entirely the same behavior at widely separated stations. This lack of uniformity can be explained only on the basis of a geocentric mechanism; the earth is too small to be effected by interplanetary inhomogeneities.

Following the capture of magnetic gas, the cosmic-ray intensity should remain depressed until the newly captured fields decay; as was pointed out earlier, this may require as long as several months. Thus the geocentric model seems to account in a natural way for the characteristics of cosmic-ray intensity decreases, and particularly the abrupt Forbush-type decreases, which are so difficult to explain on the basis of heliocentric models.

E. Cosmic Rays from Solar Flares

In evaluating (56) we assumed that $l_1=P(R_1,\eta)$ for some value of η not exceeding 0.3. For $\eta=0.3$ this gives l_1, the diameter of the scattering centers at $r=R_1$, to be 250 km. Consider, now, a 4-Bev proton originating in a solar flare. The radius of curvature of a 4-Bev proton in a field of 0.1 gauss (b at $1.7R$) is 1560 km. The deflection θ upon passing through a scattering center of thickness 250 km is 0.16 radian, or about 9.2°, according to (13). Thus, the particle is deflected but little in each scattering center. If the effective thickness of the geocentric barrier is of the order of 10^4 km, then, provided that the particle goes more or less straight through, the particle undergoes about $n=40$ scatterings. $\theta\sqrt{n}$ becomes 1.0 radians or 57°.

Now it is observed that there are forbidden zones on the surface of the earth for the arrival of cosmic-ray particles originating in solar flares. Firor (1954) has calculated a large number of trajectories through the geomagnetic field for cosmic-ray particles originating at the sun in order to demonstrate the position and extent of the forbidden and allowed impact zones. Comparing the theoretical results with the observed arrival of cosmic-ray particles from solar flares over the surface of the earth, he finds that the particles arrive over much wider regions than straight line propagation between the sun and the geomagnetic field would predict. Firor concludes that scattering of the cosmic-ray particles must take place between the sun and the earth. The random deflection of one radian produced by the geocentric disordered barrier is of the right order of magnitude. The deflection produced by the outward rushing interplanetary clouds in the heliocentric model discussed in Sec. IV is not less than 5 radians and is much too large to be reconciled with the observation of forbidden zones. ($B=10^{-4}$ gauss and $l=10^{11}$ cm gives $\theta=0.4$ for 4-Bev protons. The number of deflections between the sun and the earth is not less than 150, giving a total deflection not less than about ±5 radians).

F. Geomagnetic Effects

The theoretical geomagnetic effects produced at the surface of the earth, $r=R$, by geocentric magnetic clouds appear to be the result of at least two processes. We shall content ourselves here with a physical description of what will happen; the formal development will be presented elsewhere.[28]

The weight of the magnetic clouds forming the geocentric barrier presses down on the geomagnetic field, compressing the field beneath the bottom of the clouds. Thus, we expect that the effect is to increase the geomagnetic field at the surface of the earth.

The presence of nonmagnetic gas clouds, i.e., gas which does not carry an internal magnetic field, may have just the opposite effect. It can be shown that a volume of field free electrically conducting gas encysted in a large-scale magnetic field will successively divide into smaller and smaller fragments until the fragments are so small that they diffuse into the large scale field. Thus, the terrestrial capture of field free interplanetary gas will inflate the geomagnetic field with hydrogen gas, which may push down on the top of the terrestrial atmosphere and up on the lines of force. The result is to lift the geomagnetic field and decrease the field density below the top of the atmosphere.

Ideally suitable combinations of these two effects, while the earth is capturing new interplanetary material, can produce both a cosmic-ray decrease and a magnetic storm, i.e., an initial increase and a more prolonged decrease of the horizontal component of the geomagnetic field. The proportions of magnetic and nonmagnetic gas composing the captured material determine to what extent the cosmic-ray decrease will be accompanied by a magnetic storm[10] and vice versa.

G. Optical Effects

The optical effects of a geocentric distribution of magnetic hydrogen clouds may afford a test for the existence of geocentric clouds which is independent of cosmic-ray inferences. In particular, we must look into the problem of the absorption and emission effects that the clouds may contribute to the solar spectrum and to the light of the night sky.

To estimate the density of the gas, we use (27). Taking twice the radius of the earth as a mean value for the radius of the spherical shell occupied by the clouds, we find that the mean value of the geomagnetic field, and hence the cloud fields, is 0.06 gauss. Using the photospheric density $\rho_s=10^{-8}$ g/cm^3 and the conventional figure of 2000 km/sec for the velocity of the clouds when they were first ejected from the sun, Eq. (27) gives the lower limit on the geocentric cloud density as $\rho\geq8.3\times10^{-18}$ g/cm^3 or $N\geq5\times10^6$ hydrogen atoms/cm^3, comparable to the density in the outer corona.

Now if N is as small as 5×10^6 hydrogen atoms/cm^3 and the clouds form a barrier 5×10^3 km thick, then

[28] E. N. Parker, J. Geophys. Research (December, 1956).

there are 2.5×10^{15} hydrogen atoms per cm^2 in the line of sight. But, as we have already mentioned, the interplanetary density is of the order of 10^3 hydrogen atoms/cm^3, or 1.5×10^{16} atoms/cm^2 between here and the sun. Thus, the geocentric clouds may contribute only a small fraction of the extrasolar absorption in the solar spectrum. The situation is further complicated by the fact that the gas, both in the geocentric clouds and in interplanetary space, is probably far from thermodynamic equilibrium because of (a) the high color temperature (6000°K) and low blackbody temperature (300°K) at the orbit of the earth, (b) the strong solar emission lines in the ultraviolet, and (c) collisional excitation processes are probably rather unimportant. Thus, we hardly know how to proceed to calculate the emission and absorption effects.

At present there seem to be only two facts to go on: Chapman,[29] using the standard expressions for the thermal conductivity of an ionized gas, has shown that if the temperature of the solar corona is as high as 2×10^6 °K, then a kinetic temperature of 4×10^5 °K in the interplanetary medium may be communicated as far as the orbit of the earth; if the solar corona is at 10^6 °K or less, then the high temperatures are not communicated as far as the earth. If the interplanetary temperatures are as high as Chapman's calculations indicate they may be, then the interplanetary gas will show no line absorption or emission and it may not mask the effects of the geocentric clouds.

Calculations of emission and absorption assuming thermodynamic equilibrium may be expected to overestimate the populations of the intermediate levels of the gas atoms. Hence they overestimate the absorption in the Balmer series and in all but the first line of the Lyman series; they overestimate the emission in both series. Assuming that the temperature of the clouds does not exceed 2×10^4 °K, the calculations show that in thermodynamic equilibrium the geocentric clouds would absorb too much L_α to be reconciled with the fact that rocket-borne spectrometers have detected[30] the solar L_α line, and emit too much H_α to be reconciled with the lack of H_α in the light of the night sky. The same difficulty obviously exists with the interplanetary medium unless Chapman's high temperatures are assumed.

The expected departures from thermodynamic equilibrium might well reconcile the geocentric hydrogen clouds to the observations. Much rests on future rocket-borne spectrographs obtaining a detailed profile of the solar L_α line, which will indicate whether the expected narrow absorption line (half-width as small as 0.04 A) of the geocentric clouds is present. If there is no trace of such a line, then the existence of geocentric hydrogen is doubtful.

[29] S. Chapman, Monthly Notices Roy. Astron. Soc. (to be published, 1956).
[30] W. A. Rense, Phys. Rev. **91**, 299 (1953).

VI. CONCLUDING REMARKS

On the basis of the present rudimentary state of the theory of hydromagnetics, we have developed in a quantitative way the existing models for modulating the galactic cosmic-ray intensity at the earth. We have found that the heliocentric models for doing the modulation have yet to overcome fundamental obstacles: The heliocentric models using disordered magnetic field barriers presumably rely on the isotropic ejection of magnetic fields from the sun and involve immense and continuing expenditures of matter and fields; they have difficulty explaining both the steadiness of the low-energy cutoff and the abrupt Forbush decreases. The heliocentric models involving ordered fields require symmetry to a degree that seems to be more than one can reasonably expect, and of course static ordered fields offer no explanation for the Forbush decreases.

To get around the seeming difficulties with the heliocentric models, we have suggested that perhaps the earth captures material from passing interplanetary magnetic clouds to form a geocentric barrier of disordered magnetic fields. The small scale of the model vastly reduces the amount of magnetic gas that is needed, eliminates the necessity for the sun to eject clouds in directions far from its equatorial plane, reduces the characteristic time to the observed value of a few hours for augmenting the barrier and producing a Forbush decrease, and readily accounts for the steady low-energy cutoff in the face of only sporadic solar ejection of clouds.

After suggesting the general idea that the modulation of the cosmic-ray intensity is local and geocentric, we developed a geocentric model using the same hydromagnetic formulation as with the heliocentric models. We assumed that there are available clouds of partially ionized gas carrying magnetic fields. We assumed that the clouds are of solar origin, having been ejected from the sun by hydromagnetic processes, and we constructed (27) giving the field density in the gas by equating the initial magnetic energy to describe the process of the ejection. The details of the terrestrial capture of magnetic interplanetary matter presumably would be handled by similar hydromagnetic procedures, were the problem not so difficult as to be beyond our present analytical means.

Now the inadequate and incomplete nature of contemporary hydromagnetic theory is readily demonstrated, independently of the cosmic-ray problem, by the fact that it offers no explanation for the observed ejection of gas from the sun; hydromagnetic theory offers no suggestions as to how we might hope to achieve the necessary immense magnetic energy densities for high-velocity ejection. It follows that we should expect that serious revisions of the hydromagnetic details of our geocentric model are in store. But, on the other hand, we expect that the advantages of the stability

and small scale of a geocentric model are permanently attractive features.

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APPENDIX. EXCLUSION OF PARTICLES BY ORDERED INTERPLANETARY FIELDS

It is well known that an idealized dipole magnetic field, of the form

$$\mathbf{B}=-\nabla\psi_0,$$

where

$$\psi_0=(M/r^3)\cos\theta$$

in spherical coordinates, will exclude an externally incident charged particle of sufficiently low energy from certain regions near the equator of the dipole according to the classical Störmer theory. We have pointed out, however, that if we add small *random* irregularities ψ_1 to ψ_0, then these irregularities will scatter the particle from the Störmer trajectories for the idealized field ψ_0. We suppose that no large absorbers are present. Therefore, if a large number of particles are directed into the perturbed field $\psi_0+\psi_1$, from random initial directions, then we expect eventually to find particles passing near any given position in the field; hence all regions will be accessible. But, then, no matter how tortuous the route of accessibility, statistical equilibrium ultimately will be achieved with uniform particle density throughout, and the dipole-like field $\psi_0+\psi_1$, will not *permanently* exclude low-energy particles from its interior.

A rigorous general treatment of the diffusion of particles into a field deviating a small amount from an idealized dipole is beyond the scope of this paper. It is possible, however, to carry out the problem in two dimensions in sufficient generality to illustrate the general idea.

Consider a magnetic field $B(x,y)$ in the z direction. We let

$$B(x,y)=B_0+B_1(x,y), \tag{1A}$$

where B_0 is a constant and B_1 is a slowly varying function so that its characteristic scale is much greater than P, the radius of curvature of the trajectory of a charged particle of mass m, charge q (esu), and velocity w, moving in the xy plane. The nonrelativistic equations of motion of the particle are

$$\frac{d^2x}{dt^2}=+\frac{q(B_0+B_1)}{mc}\frac{dy}{dt}, \tag{2A}$$

$$\frac{d^2y}{dt^2}=-\frac{q(B_0+B_1)}{mc}\frac{dx}{dt}, \tag{3A}$$

$$\frac{d^2z}{dt^2}=0. \tag{4A}$$

Following a suggestion of Alfvén,[2] we regard the particle as moving in a circle with moving center. We let (X,Y) denote the coordinates of the center of the circle, or *guiding center* as it is called. The position of the particle relative to the guiding center is given by the rectangular coordinates (ξ,η),

$$\xi=x-X,\quad \eta=y-Y. \tag{5A}$$

The large scale of B_1, makes $\nabla B_1\ll(B_0/P)$, (B_1/P). Hence $|dX/dt|\ll|d\xi/dt|$, etc., and

$$\xi\cong+P\cos\Omega t, \tag{6A}$$

$$\eta\cong-P\sin\Omega t, \tag{7A}$$

where $\Omega=(q/mc)(B_0+B_1)$.

X and Y are smoothly varying functions. If we average the equations of motion over several periods of Ω, then oscillating terms such as $d^2\xi/dt^2$, $d\xi/dt$, etc., drop out. We obtain

$$\left\langle\frac{dX}{dt}\right\rangle_{\text{Av}}=-\left\langle\frac{B_1}{B}\frac{d\xi}{dt}\right\rangle_{\text{Av}},$$

$$\left\langle\frac{dY}{dt}\right\rangle_{\text{Av}}=-\left\langle\frac{B_1}{B}\frac{d\eta}{dt}\right\rangle_{\text{Av}},$$

to first order in the variations of $B_1(x,y)$. Expanding $B_1(x,y)$ about the guiding center (X,Y), we have

$$B_1(x,y)=B_1(X,Y)+\frac{\partial B_1}{\partial X}\xi+\frac{\partial B_1}{\partial Y}\eta+O^2(\xi,\eta).$$

Thus, using (6A) and (7A) we readily find that

$$\left\langle\frac{dX}{dt}\right\rangle_{\text{Av}}=-\frac{P^2q}{2mc}\frac{\partial B_1}{\partial Y}, \tag{8A}$$

$$\left\langle\frac{dY}{dt}\right\rangle_{\text{Av}}=+\frac{P^2q}{2mc}\frac{\partial B_1}{\partial X} \tag{9A}$$

as the components of the drift velocity $\mathbf{v}$ of the guiding center. $\mathbf{v}$ may be written in a variety of ways. For instance, from (8A) and (9A) it is obvious that

$$\mathbf{v}=-\frac{P^2q}{2mc}\nabla\times\mathbf{B}_1 \tag{10A}$$

$$=+\frac{P^2q}{2mc}\mathbf{e}_z\times\nabla B_1, \tag{11A}$$

where $\mathbf{e}_2$ is a unit vector in the z direction. Since the current density $\mathbf{j}$ is related to $\mathbf{B}$ according to

$$\mathbf{j}=(c/\mu)\nabla\times\mathbf{B}_1,$$

we may also write

$$\mathbf{v}=-\frac{P^2q\mu}{2mc^2}\mathbf{j}. \tag{12A}$$

From (11A) and (12A) we see that both $\mathbf{v}$ and $\mathbf{j}$ lie along the contours of equal field density. Since $\mathbf{j}$ is a solenoidal field, it follows that the guiding center either drifts to infinity or in a closed path. If $B_1(x,y)$ is a slowly and irregularly varying function of time, we see that the contours of constant B_1 will continually reconnect into new patterns, eventually giving access to almost all portions of the xy plane.

One can see how a low-energy cosmic-ray particle entering the solar system along the axis of a heliocentric dipole field can be diverted away from the axis into otherwise forbidden regions.

Suppose now that we apply (11A) to the entry of galactic cosmic-ray particles into the field-free solar cavity proposed by Davis.[20] Whether the surface of such a cavity will be smooth, or ragged due to the impact of interplanetary clouds, has not been established. Let us assume, however, that the walls are smooth so that we may reasonably hope that the cavity will form a barrier. For an idealized cavity, shown in Fig. 1, wherein the galactic magnetic field vanishes identically inside the cavity surface S and is independent of azimuthal angle φ measured around the axis of the cavity, Davis has pointed out that only those cosmic-ray particles can enter which circle the line of force MN and $M'N'$ lying along the axis of the cavity. If the equatorial radius of the cavity is Λ, it follows that the effective cross section for entry into the cavity has been reduced from the purely geometrical $\pi\Lambda^2$ to πP^2, where P represents the radius of curvature given in (12); for low-energy particles P may be very much less than Λ.

We measure Λ in astronomical units. We have already pointed out that a particle can expect to make $n=5\times10^4$ trips across the interior of the cavity before being stopped by the sun. In order that the barrier be effective, this rate of removal must exceed the permeability of the barrier; we require that $\pi P^2<\pi\Lambda^2/n$, and hence $P<4.5\times10^{-3}$ astronomical units. In order that P be this small for 1-Bev particles, one requires a galactic field of the order of 0.5×10^{-4} gauss, which is a factor of ten more than current estimates.[31]

So much for an idealized cavity. Suppose now that the actual solar cavity is not quite rotationally symmetric. From (11A) we see that a slight dependence of the galactic field B on azimuthal angle φ (measured around the axis of the cavity) will give the drift velocity $\mathbf{v}$ a component perpendicular to the cavity surface S; thus particles which initially are near to, but do not circle, either MN or $M'N'$ may enter the cavity. Suppose, for instance, that we put $\partial B/\partial\varphi\cong B\Lambda/\lambda$, where λ is the characteristic scale of the azimuthal dependence of B on φ, and Λ is still the cavity radius.

We consider a cosmic-ray particle with velocity w spiralling along the galactic field B near the surface of the cavity. The circular velocity ΩP is presumably of the same order as w and as the velocity parallel to B. From (11A) it follows that the guiding center drifts at an angle ϑ to the lines of force of the galatic field where

$$\begin{aligned}\vartheta&\cong v/w\\&\cong\tfrac{1}{2}(\eta+1)(P/\lambda).\end{aligned}$$

For 1-Bev protons we have $\vartheta\cong P/\lambda$. Since the particles drift past the cavity for a distance of the order of 2Λ, we find that particles initially a distance $2\Lambda\vartheta$ from the cavity surface S have a chance to enter the cavity. We expect that $\partial B/\partial\varphi$ will have a sign such that particles will drift toward S only over about half of the surface S. Thus, the total cross-section Q for entry to the cavity is

$$\begin{aligned}Q&=\pi P^2+\tfrac{1}{2}(2\pi\Lambda)(2\Lambda\vartheta)\\&=\pi P^2\left[1+\frac{\Lambda^2(\eta+1)}{\lambda P}\right].\end{aligned} \tag{13A}$$

Davis estimates that Λ may be of the order of 200 atronomical units. Current estimates of the galactic field of 6×10^{-6} gauss give P of the order of 4×10^{-2} a.u. for a 1-Bev proton. Thus the term in brackets in (13A) becomes $(1+2\times10^6/\lambda)$. In order that the entry cross section Q be no more than twice the value πP^2 for an idealized cavity for which $\partial B/\partial\varphi=0$, we must require that $\lambda>2\times10^6$ a.u. or greater than 10 parsecs. Λ/λ must not exceed 10^{-4}. Hence B must not change by more than one part in 10 000 as one circles the cavity.

Since the corpuscular emission from the sun, which would be responsible for maintaining the cavity, is not sufficiently uniform to admit of so regular a cavity, the only alternative seems to be to assume a much larger galactic field and a much smaller cavity. Suppose, for instance that B is a factor of ten larger than the estimated value of 6×10^{-6} gauss. Then, as previously mentioned, P is sufficiently small that an idealized cavity will be a sufficiently good barrier to furnish a cutoff. With such a large value for B, we would expect that Λ be smaller than 200 a.u. Suppose that Λ were only 2 a.u. Then the term in brackets in (13A) becomes $(1+2\times10^3)$. Λ/λ must not exceed 10^{-3} and we are still faced with the restriction that B must not vary by more than one part in 1000 around the cavity.

[31] S. Chandrasekhar and E. Fermi, Astrophys. J. **118**, 113, 116 (1953).