obtain this, one may supplement the tables with an asymptotic formula given by Bartlett and Watson, namely

$$(\sigma/\sigma_R-1)\sim\pi\alpha\beta(\cos\chi)(\theta/2)+O(\theta^2), \tag{2}$$

where

$$\cos\chi=\mathrm{Re}\left\{\frac{\Gamma(\frac{1}{2}-iq)\Gamma(1+iq)}{\Gamma(\frac{1}{2}+iq)\Gamma(1-iq)}\right\}$$

is tabulated in Table III. In the course of investigating the adequacy of the 15° interval size, it proved convenient to plot the quantity $(\sigma/\sigma_R-1)/\sin(\theta/2)$.

IV. DISCUSSION OF RESULTS

Table I summarizes intercomparisons with earlier results. By and large the agreement is seen to be excellent. The Feshbach results are extended in this table to energies where they are expected to be borderline in their accuracy. Curr's formulas give remarkable agreement except for the large-angle scattering of positrons in Hg, where even the α^8 term is an appreciable fraction of the total. The values for positron scattering which were obtained from Massey's paper were read from his curves. Not given in this table are sample intercomparisons with Yadav's results for $Z=92$, which also showed satisfactory agreement in most cases. We have performed additional calculations for the Z,β values used by Sherman in his recent work[8] and have obtained agreement to about 1% in all portions of his cross-section tables.

At very low energies and large angles in Hg and U, there were significant discrepancies between the SEAC results and values given by Bartlett and Watson and by Yadav. An investigation revealed that this should be attributed to an inadequate number of terms used by these authors in their calculation of the F_1.

Regarding interpolation between results quoted in Tables IV and V, the mesh is such that for a given Z for which the tabulations exist, it is possible to interpolate graphically to a percent or so except at the largest angles at high energies. Trial interpolations bear this out. The same statements hold regarding interpolation in Z using tabulated values at fixed energy and angle. An interpolation in all three variables would not be so accurate but could be valuable for orientation purposes. When high accuracy is required, it would be better to go back to the SEAC and do another original calculation, which is quite feasible now that the code exists. Requests of this nature should be addressed to the Computation Laboratory of the National Bureau of Standards, Washington, D. C.

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Coulomb Scattering of Relativistic Electrons by Point Nuclei*

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The Mott series for the Coulomb scattering of electrons by point nuclei have been evaluated numerically with the aid of the UNIVAC computer. Calculations of the series for $F(\theta)$ and $G(\theta)$, the scattering cross section, and the polarization asymmetry factor, $S(\theta)=\delta^{\frac{1}{2}}$, were performed for scattering by nuclei of charge Z equal to 80, 48, and 13 at ratios of electron velocity to light velocity, $\beta=v/c$, equal to 0.2, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9. The results are tabulated.

INTRODUCTION

THE Dirac theory of the electron was applied by Mott[1] to the scattering of electrons by nuclei in order to investigate possible polarization effects in double scattering experiments. The theoretical results for the expected polarization and for the single scattering cross sections involve slowly and conditionally convergent series which are not amenable to easy calculation. Mott calculated results for gold ($Z=79$) at 90 degrees. Bartlett and Watson[2] have summed the series numerically for mercury nuclei ($Z=80$) over a range of angles and energies. More recently, other investigators[3–6] have performed numerical calculations. This collection of data is augmented by the results of this paper, in which the Mott series, the polarization, and the differential scattering cross section are evaluated for the scattering of electrons by nuclei of charge $Z=80$, 48, and 13, at energies given by the ratio of electron velocity to light velocity, $\beta=v/c=0.2$, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9, through scattering angles, θ, in 15-degree intervals from 15 degrees to 165 degrees. These calculations were performed with the aid of the UNIVAC computer.

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MOTT SCATTERING FORMULAS

The differential cross section for an unpolarized beam of electrons scattered through an angle θ is

$$\frac{d\sigma}{d\Omega}(\theta)=\lambda\!\!\!^{-2}\{q^2(1-\beta^2)|F|^2\csc^2(\tfrac{1}{2}\theta)+|G|^2\sec^2(\tfrac{1}{2}\theta)\},\quad (1)$$

where $2\pi\lambda\!\!\!^{-}$ is the de Broglie wavelength, $q=\alpha/\beta$, $\alpha=Ze^2/\hbar c$, and $\beta=v/c$. If an unpolarized beam is scattered through an angle θ_1, the scattered electrons will be partially polarized. If this partially polarized beam is scattered again through an angle θ_2, the intensity of twice-scattered electrons will then depend on the azimuth about the direction θ_1. The differential cross section for this double-scattering process is

$$\frac{d\sigma}{d\Omega}(\theta_1,\theta_2,\phi_2)=\frac{d\sigma}{d\Omega}(\theta_1)\frac{d\sigma}{d\Omega}(\theta_2)\{1+\delta(\theta_1,\theta_2)\cos\phi_2\},$$

where $d\sigma(\theta_1)/d\Omega$ and $d\sigma(\theta_2)/d\Omega$ are defined by (1), ϕ_2 is the azimuthal angle about the direction θ_1, and $\delta(\theta_1,\theta_2)$ is the polarization asymmetry. This last quantity can be expressed as a product of two factors, each having the same form, where one is a function of θ_1 only and the other a function of θ_2 only. Thus $\delta(\theta_1,\theta_2)=S(\theta_1)S(\theta_2)$, where

$$S(\theta)=\frac{2\lambda\!\!\!^{-2}q(1-\beta^2)^{\frac{1}{2}}}{\sin\theta d\sigma(\theta)/d\Omega}\{F(\theta)G^*(\theta)+F^*(\theta)G(\theta)\}.\quad (2)$$

The complex functions $F(\theta)$ and $G(\theta)$, which appear in (1) and (2), are defined as follows:

$$F(\theta)=F_0+F_1,$$

$$F_0(\theta)=\frac{i}{2}\frac{\Gamma(1-iq)}{\Gamma(1+iq)}\exp[iq\ln\sin^2(\tfrac{1}{2}\theta)],\quad (3a)$$

$$F_1(\theta)=\frac{i}{2}\sum_{k=0}^{\infty}[kD_k+(k+1)D_{k+1}](-1)^kP_k(\cos\theta),$$

$$G(\theta)=G_0+G_1,$$

$$G_0(\theta)=-iq[\cot^2(\tfrac{1}{2}\theta)]F_0,\quad (3b)$$

$$G_1(\theta)=\frac{i}{2}\sum_{k=0}^{\infty}[k^2D_k-(k+1)^2D_{k+1}](-1)^kP_k(\cos\theta),$$

where Γ is the gamma function and P_k is the Legendre polynomial of order k. D_k is given by

$$D_k=\frac{e^{-i\pi k}}{k+iq}\frac{\Gamma(k-iq)}{\Gamma(k+iq)}-\frac{e^{-i\pi\rho_k}}{\rho_k+iq}\frac{\Gamma(\rho_k-iq)}{\Gamma(\rho_k+iq)},\quad (4)$$

where $\rho_k=(k^2-\alpha^2)^{\frac{1}{2}}$.

APPROXIMATIONS AND SERIES TRANSFORMATIONS

The ratios of gamma functions, which appear in Eq. (4) were evaluated by using the recursion relations for gamma functions and Stirling's approximation as follows:

$$\frac{\Gamma(x-iq)}{\Gamma(x+iq)}=\frac{(x+iq)(x+1+iq)\Gamma(x+2-iq)}{(x-iq)(x+1-iq)\Gamma(x+2+iq)},$$

$$\frac{\Gamma(x+2-iq)}{\Gamma(x+2+iq)}=e^{-2i\tau_x},$$

$$\tau_x=\arg\Gamma(x+2+iq),$$

$$\cong\tfrac{1}{2}q\ln[(x+2)^2+q^2]+(x+\tfrac{3}{2})\arctan\left(\frac{q}{x+2}\right)$$

$$-q\left[1+\frac{1}{12[(x+2)^2+q^2]}-\frac{3(x+2)^2-q^2}{360[(x+2)^2+q^2]^3}+\frac{5(x+2)^4-10q^2(x+2)^2+q^4}{1260[(x+2)^2+q^2]^5}\right].$$

In the last equation, x refers either to k or to ρ_k in Eq. (4). [The gamma-function ratio, which appears in the definition of F_0, can be written

$$\Gamma(1-iq)/\Gamma(1+iq)=e^{-2i\sigma_0},$$

where σ_0 is available in published tables.[7]]

With these approximations the D_k were evaluated. These terms were inserted into (3) and the series F_1 and G_1 were determined numerically. Since these series are conditionally convergent and converge very slowly, two transformations were employed. First the "reduced" series of Yennie, Ravenhall, and Wilson[8] was used to improve the convergence at small angles. This transformation can be applied to any series of Legendre polynomials, given by

$$f(\alpha)=\sum_{l=0}A_lP_l(\alpha),$$

where $\alpha=\cos\theta$. With use of the recurrence relations for Legendre polynomials, this series can be transformed to

$$(1-\alpha)f(\alpha)=\sum_{l=0}A_l^{(1)}P_l(\alpha),$$

or

$$(1-\alpha)^mf(\alpha)=\sum_{l=0}A_l^{(m)}P_l(\alpha),$$

where

$$A_l^{(m)}=A_l^{(m-1)}-\frac{l+1}{2l+3}A_{l+1}^{(m-1)}-\frac{l}{2l-1}A_{l-1}^{(m-1)}.$$

The series for F_1 and G_1 were "reduced" in this manner with $m=3$.

The second transformation was applied to the reduced series. This is the well-known Euler transformation[9]

[7] *Tables of Coulomb Wave Functions*, National Bureau of Standards Applied Mathematics Series 17 (U. S. Government Printing Office, Washington, D. C., 1952), Vol. 1, Table III.

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TABLE I. Calculated results for mercury ($Z=80$).

θ		$\beta=0.2$	0.4	0.5	0.6	0.7	0.8	0.9
15°	Re F	0.478	−0.340	−0.421	−0.0757	0.202	0.364	0.444
	Im F	0.171	0.365	−0.264	−0.487	−0.445	−0.324	−0.196
	Re G	29.1	31.2	−18.3	−28.2	−22.2	−14.1	−7.46
	Im G	−79.3	29.0	28.7	3.90	−10.7	−16.7	−18.2
	$d\sigma/d\Omega$[a]	2.64×10^{8}	1.47×10^{7}	5.35×10^{6}	2.21×10^{6}	9.67×10^{5}	4.10×10^{5}	1.39×10^{5}
	S	2.11×10^{-3}	-4.25×10^{-4}	1.60×10^{-3}	3.45×10^{-3}	4.04×10^{-3}	3.77×10^{-3}	2.79×10^{-3}
30°	Re F	−0.160	−0.174	0.286	0.448	0.466	0.434	0.389
	Im F	−0.465	−0.458	−0.382	−0.143	0.0486	0.180	0.269
	Re G	−19.4	−9.68	−6.42	−1.86	0.884	2.33	3.06
	Im G	6.75	3.32	−5.31	−6.91	−6.29	−5.26	−4.31
	$d\sigma/d\Omega$	1.72×10^{7}	9.30×10^{5}	3.48×10^{5}	1.51×10^{5}	6.89×10^{4}	3.03×10^{4}	1.06×10^{4}
	S	-1.93×10^{-3}	1.53×10^{-2}	1.96×10^{-2}	1.66×10^{-2}	1.14×10^{-2}	6.32×10^{-3}	2.30×10^{-3}
45°	Re F	0.469	0.346	0.450	0.378	0.285	0.205	0.142
	Im F	0.205	−0.203	0.0480	0.257	0.366	0.425	0.458
	Re G	3.34	−2.68	0.532	1.86	2.30	2.39	2.35
	Im G	−7.94	−3.50	−3.69	−2.71	−1.88	−1.30	−0.912
	$d\sigma/d\Omega$	3.63×10^{6}	1.99×10^{5}	7.93×10^{4}	3.59×10^{4}	1.68×10^{4}	7.53×10^{3}	2.66×10^{3}
	S	-9.65×10^{-3}	3.93×10^{-2}	2.01×10^{-2}	2.02×10^{-3}	-1.06×10^{-2}	-1.74×10^{-2}	-1.76×10^{-2}
60°	Re F	−0.184	0.430	0.281	0.131	0.0247	−0.0484	−0.0992
	Im F	0.460	0.0954	0.357	0.452	0.485	0.497	0.500
	Re G	3.93	0.542	1.60	1.76	1.70	1.59	1.48
	Im G	1.88	−2.41	−1.42	−0.715	−0.320	−0.996	0.0243
	$d\sigma/d\Omega$	1.19×10^{6}	7.45×10^{4}	3.09×10^{4}	1.42×10^{4}	6.69×10^{3}	2.99×10^{3}	1.05×10^{3}
	S	5.64×10^{-2}	2.18×10^{-3}	-3.80×10^{-2}	-6.16×10^{-2}	-7.22×10^{-2}	-7.10×10^{-2}	-5.86×10^{-2}
75°	Re F	−0.444	0.257	0.0204	−0.126	−0.214	−0.267	−0.301
	Im F	−0.0131	0.382	0.489	0.497	0.487	0.476	0.468
	Re G	−0.302	1.17	1.31	1.20	1.08	0.977	0.898
	Im G	2.52	−1.02	−0.304	0.0321	0.180	0.242	0.265
	$d\sigma/d\Omega$	5.21×10^{5}	3.81×10^{4}	1.59×10^{4}	7.25×10^{3}	3.37×10^{3}	1.48×10^{3}	5.11×10^{2}
	S	8.20×10^{-2}	−0.104	−0.143	−0.160	−0.162	−0.150	−0.117
90°	Re F	−0.191	0.00426	−0.228	−0.345	−0.406	−0.441	−0.461
	Im F	−0.381	0.515	0.499	0.458	0.428	0.411	0.403
	Re G	−1.42	0.978	0.869	0.740	0.647	0.583	0.536
	Im G	0.779	−0.262	0.118	0.250	0.289	0.292	0.282
	$d\sigma/d\Omega$	2.94×10^{5}	2.35×10^{4}	9.64×10^{3}	4.29×10^{3}	1.94×10^{3}	8.30×10^{2}	2.78×10^{2}
	S	-3.59×10^{-2}	−0.234	−0.261	−0.271	−0.265	−0.242	−0.190
105°	Re F	0.187	−0.237	−0.430	−0.514	−0.554	−0.574	−0.584
	Im F	−0.435	0.537	0.443	0.378	0.345	0.330	0.327
	Re G	−1.03	0.647	0.516	0.425	0.370	0.335	0.312
	Im G	−0.209	0.0481	0.218	0.258	0.257	0.244	0.226
	$d\sigma/d\Omega$	2.10×10^{5}	1.66×10^{4}	6.56×10^{3}	2.81×10^{3}	1.22×10^{3}	5.01×10^{2}	1.60×10^{2}
	S	−0.203	−0.333	−0.356	−0.367	−0.364	−0.340	−0.277
120°	Re F	0.474	−0.432	−0.581	−0.637	−0.662	−0.672	−0.676
	Im F	−0.292	0.493	0.361	0.289	0.258	0.249	0.252
	Re G	−0.487	0.373	0.280	0.228	0.199	0.183	0.172
	Im G	−0.421	0.124	0.190	0.195	0.184	0.170	0.155
	$d\sigma/d\Omega$	1.80×10^{5}	1.29×10^{4}	4.89×10^{3}	2.00×10^{3}	8.27×10^{2}	3.19×10^{2}	94.4
	S	−0.283	−0.372	−0.401	−0.424	−0.436	−0.429	−0.373
135°	Re F	0.634	−0.575	−0.686	−0.723	−0.737	−0.742	−0.741
	Im F	−0.0918	0.436	0.278	0.206	0.181	0.179	0.188
	Re G	−0.167	0.187	0.135	0.110	0.0971	0.0902	0.0859
	Im G	−0.310	0.101	0.122	0.118	0.109	0.0995	0.0904
	$d\sigma/d\Omega$	1.71×10^{5}	1.10×10^{4}	3.95×10^{3}	1.54×10^{3}	5.98×10^{2}	2.13×10^{2}	56.3
	S	−0.262	−0.342	−0.380	−0.418	−0.453	−0.479	−0.464
150°	Re F	0.701	−0.670	−0.754	−0.778	−0.786	−0.788	−0.785
	Im F	0.0813	0.376	0.210	0.142	0.122	0.125	0.139
	Re G	−0.0379	0.0750	0.0532	0.0433	0.0387	0.0364	0.0350
	Im G	−0.148	0.0530	0.0582	0.0547	0.0499	0.0451	0.0409
	$d\sigma/d\Omega$	1.69×10^{5}	9.86×10^{3}	3.42×10^{3}	1.27×10^{3}	4.66×10^{2}	1.52×10^{2}	34.2
	S	−0.188	−0.257	−0.295	−0.337	−0.387	−0.446	−0.505
165°	Re F	0.721	0.724	−0.792	−0.809	−0.814	−0.814	−0.811
	Im F	0.192	0.336	0.166	0.101	0.0848	0.0916	0.109
	Re G	−0.00414	0.0175	0.0123	0.0100	0.00904	0.00857	0.00832
	Im G	−0.0376	0.0143	0.0150	0.0139	0.0126	0.0114	0.0103
	$d\sigma/d\Omega$	1.69×10^{5}	9.31×10^{3}	3.15×10^{3}	1.13×10^{3}	3.97×10^{2}	1.20×10^{3}	22.6
	S	-9.56×10^{-2}	−0.137	−0.161	−0.189	−0.226	−0.281	−0.380

[a] $d\sigma/d\Omega$ is given in barns per steradian.

TABLE II. Calculated results for cadmium ($Z=48$).

θ		$\beta=0.2$	0.4	0.5	0.6	0.7	0.8	0.9
15°	Re F	0.347	0.106	0.399	0.482	0.495	0.478	0.450
	Im F	0.359	−0.484	−0.390	−0.114	0.0310	0.135	0.210
	Re G	36.4	−24.9	−12.6	−3.88	1.00	3.63	5.03
	Im G	−35.2	−5.64	−16.3	−16.9	−15.0	−12.7	−10.8
	$d\sigma/d\Omega$[a]	9.51×10^{7}	5.25×10^{6}	1.95×10^{6}	8.19×10^{5}	3.58×10^{5}	1.51×10^{5}	5.03×10^{4}
	S	-1.14×10^{-4}	1.81×10^{-3}	1.64×10^{-3}	1.22×10^{-3}	7.56×10^{-4}	3.58×10^{-4}	7.35×10^{-5}
30°	Re F	−0.497	0.487	0.464	0.404	0.347	0.300	0.262
	Im F	−0.0337	−0.0565	0.165	0.286	0.356	0.398	0.426
	Re G	−0.935	−0.696	1.72	2.50	2.69	2.67	2.57
	Im G	12.2	−6.28	−4.87	−3.60	−2.71	−2.10	−1.67
	$d\sigma/d\Omega$	6.12×10^{6}	3.54×10^{5}	1.34×10^{5}	5.68×10^{4}	2.50×10^{4}	1.06×10^{4}	3.55×10^{3}
	S	4.37×10^{-3}	2.30×10^{-3}	-1.16×10^{-3}	-3.67×10^{-3}	-5.23×10^{-3}	-5.81×10^{-3}	-5.13×10^{-3}
45°	Re F	−0.0533	0.400	0.292	0.216	0.164	0.128	0.101
	Im F	−0.485	0.286	0.402	0.452	0.476	0.489	0.497
	Re G	−5.13	1.56	1.79	1.70	1.57	1.43	1.31
	Im G	0.471	−2.27	−1.39	−0.906	−0.630	−0.462	−0.354
	$d\sigma/d\Omega$	1.28×10^{6}	7.78×10^{4}	2.96×10^{4}	1.26×10^{4}	5.52×10^{3}	2.33×10^{3}	776
	S	0.0123	−0.0119	−0.0188	−0.0227	−0.0242	−0.0232	−0.0188
60°	Re F	0.371	0.209	0.104	0.0457	0.0110	−0.0109	−0.0255
	Im F	−0.308	0.456	0.497	0.510	0.514	0.516	0.517
	Re G	−1.74	1.31	1.17	1.02	0.906	0.812	0.736
	Im G	−2.09	−0.689	−0.332	−0.176	−0.100	−0.0616	−0.0402
	$d\sigma/d\Omega$	4.50×10^{5}	2.78×10^{4}	1.05×10^{4}	4.43×10^{3}	1.93×10^{3}	803	264
	S	7.70×10^{-4}	−0.0427	−0.0510	−0.0550	−0.0554	−0.0515	−0.0410
75°	Re F	0.488	0.0220	−0.0566	−0.0930	−0.111	−0.120	−0.124
	Im F	0.0263	0.515	0.519	0.517	0.516	0.515	0.515
	Re G	0.0222	0.860	0.716	0.612	0.536	0.479	0.434
	Im G	−1.59	−0.117	0.00157	0.0371	0.0547	0.0450	0.0412
	$d\sigma/d\Omega$	2.12×10^{5}	1.30×10^{4}	4.86×10^{3}	2.01×10^{3}	862	352	113
	S	−0.0372	−0.0832	−0.0919	−0.0959	−0.0953	−0.0886	−0.0710
90°	Re F	0.410	−0.133	−0.184	−0.201	−0.206	−0.205	−0.202
	Im F	0.297	0.516	0.505	0.501	0.501	0.502	0.503
	Re G	0.503	0.524	0.426	0.363	0.319	0.286	0.261
	Im G	−0.827	0.0666	0.0909	0.0859	0.0749	0.0639	0.0542
	$d\sigma/d\Omega$	1.21×10^{5}	7.23×10^{3}	2.65×10^{3}	1.08×10^{3}	0.449	178	55.5
	S	−0.080	−0.123	−0.133	−0.139	−0.139	−0.131	−0.108
105°	Re F	0.256	−0.253	−0.281	−0.285	−0.279	−0.271	−0.262
	Im F	0.461	0.488	0.476	0.476	0.479	0.484	0.488
	Re G	0.478	0.303	0.246	0.211	0.187	0.169	0.155
	Im G	−0.342	0.102	0.0946	0.0799	0.0662	0.0549	0.0458
	$d\sigma/d\Omega$	7.99×10^{4}	4.58×10^{3}	1.64×10^{3}	646	261	99.6	29.5
	S	−0.112	−0.153	−0.166	−0.175	−0.180	−0.176	−0.151
120°	Re F	0.0994	−0.342	−0.353	−0.347	−0.334	−0.321	−0.308
	Im F	0.541	0.451	0.443	0.448	0.457	0.465	0.473
	Re G	0.323	0.164	0.135	0.117	0.104	0.0952	0.0878
	Im G	−0.106	0.0842	0.0711	0.0578	0.0471	0.0386	0.0321
	$d\sigma/d\Omega$	5.91×10^{4}	3.22×10^{3}	1.12×10^{3}	428	166	60.1	16.5
	S	−0.122	−0.163	−0.180	−0.195	−0.208	−0.213	−0.197
135°	Re F	−0.0306	−0.404	−0.405	−0.392	−0.375	−0.358	−0.342
	Im F	0.568	0.415	0.413	0.424	0.437	0.449	0.460
	Re G	0.177	0.0806	0.0670	0.0589	0.0532	0.0487	0.0452
	Im G	−0.0171	0.0531	0.0431	0.0345	0.0279	0.0228	0.0188
	$d\sigma/d\Omega$	4.77×10^{4}	2.48×10^{3}	840	310	115	39.0	9.60
	S	−0.110	−0.151	−0.170	−0.190	−0.211	−0.231	−0.238
150°	Re F	−0.123	−0.445	−0.439	−0.422	−0.402	−0.383	−0.366
	Im F	0.570	0.385	0.390	0.405	0.422	0.437	0.449
	Re G	0.0753	0.0323	0.0272	0.0242	0.0220	0.0203	0.0189
	Im G	0.00414	0.0249	0.0199	0.0158	0.0127	0.0104	0.0857
	$d\sigma/d\Omega$	4.15×10^{4}	2.07×10^{3}	684	245	87.2	27.4	5.87
	S	−0.0822	−0.116	−0.134	−0.153	−0.177	−0.208	−0.247
165°	Re F	−0.178	−0.468	−0.459	−0.439	−0.418	−0.398	−0.379
	Im F	0.564	0.367	0.375	0.394	0.413	0.429	0.443
	Re G	0.0181	0.00759	0.00645	0.00576	0.00527	0.00488	0.00455
	Im G	0.00261	0.00639	0.00505	0.00400	0.00321	0.00262	0.00216
	$d\sigma/d\Omega$	3.83×10^{4}	1.85×10^{3}	606	212	73.1	21.7	4.00
	S	−0.0435	−0.0630	−0.0736	−0.0863	−0.103	−0.128	−0.176

[a] $d\sigma/d\Omega$ is given in barns per steradian.

TABLE III. Calculated result for aluminum ($Z=13$).

θ		$\beta=0.2$	0.4	0.5	0.6	0.7	0.8	0.9
15°	Re F	0.496	0.322	0.264	0.222	0.192	0.168	0.150
	Im F	0.0563	0.383	0.425	0.448	0.462	0.471	0.477
	Re G	1.55	5.27	4.69	4.13	3.65	3.27	2.95
	Im G	−13.6	−4.44	−2.92	−2.06	−1.53	−1.18	−0.936
	$d\sigma/d\Omega$[a]	6.98×10^{6}	3.84×10^{5}	1.41×10^{5}	5.80×10^{4}	2.50×10^{4}	1.04×10^{4}	3.41×10^{3}
	S	2.97×10^{-5}	-1.92×10^{-4}	-2.68×10^{-4}	-3.24×10^{-4}	-3.56×10^{-4}	-3.55×10^{-4}	-2.94×10^{-4}
30°	Re F	0.360	0.181	0.144	0.119	0.102	0.0887	0.0784
	Im F	0.346	0.466	0.479	0.486	0.490	0.492	0.494
	Re G	2.30	1.56	1.29	1.0	0.945	0.834	0.746
	Im G	−2.40	−0.613	−0.394	−0.275	−0.203	−0.157	−0.125
	$d\sigma/d\Omega$	4.54×10^{5}	2.50×10^{4}	9.13×10^{3}	3.75×10^{3}	1.61×10^{3}	6.63×10^{2}	2.17×10^{2}
	S	-7.90×10^{-4}	-1.80×10^{-3}	-2.13×10^{-3}	-2.36×10^{-3}	-2.45×10^{-3}	-2.35×10^{-3}	-1.92×10^{-3}
45°	Re F	0.209	0.0907	0.0703	0.0572	0.0481	0.0413	0.0361
	Im F	0.455	0.492	0.496	0.498	0.499	0.499	0.500
	Re G	1.27	0.693	0.560	0.471	0.406	0.357	0.319
	Im G	−0.589	−0.133	−0.0849	−0.0594	−0.0442	−0.0345	−0.0280
	$d\sigma/d\Omega$	9.52×10^{4}	5.20×10^{3}	1.89×10^{3}	7.72×10^{2}	3.28×10^{2}	1.34×10^{2}	43.3
	S	-3.00×10^{3}	-5.14×10^{-3}	-5.88×10^{-3}	-6.40×10^{-3}	-6.60×10^{-3}	-6.33×10^{-3}	-5.19×10^{-3}
60°	Re F	0.0866	0.0264	0.0184	0.0137	0.0106	0.00833	0.00664
	Im F	0.494	0.501	0.501	0.501	0.501	0.501	0.501
	Re G	0.711	0.364	0.293	0.246	0.212	0.186	0.166
	Im G	−0.130	−0.0236	−0.0149	−0.0107	−0.00831	−0.00685	−0.00589
	$d\sigma/d\Omega$	3.27×10^{4}	1.76×10^{3}	6.35×10^{2}	2.56×10^{2}	1.07×10^{2}	43.0	13.6
	S	-6.28×10^{-3}	-9.73×10^{-3}	−0.0110	−0.0120	−0.0125	−0.0121	−0.0101
75°	Re F	−0.00780	−0.0216	−0.0201	−0.0186	−0.0173	−0.0162	−0.0152
	Im F	0.502	0.502	0.502	0.502	0.502	0.502	0.502
	Re G	0.410	0.207	0.167	0.140	0.120	0.106	0.0948
	Im G	0.00213	0.00565	0.00362	0.00225	0.00131	6.55×10^{-4}	1.76×10^{-4}
	$d\sigma/d\Omega$	1.48×10^{4}	7.87×10^{2}	2.80×10^{2}	1.11×10^{2}	45.7	17.9	5.50
	S	-9.90×10^{-3}	−0.0147	−0.0167	−0.0184	−0.0194	−0.0192	−0.0164
90°	Re F	−0.0800	−0.0582	−0.0496	−0.0433	−0.0386	−0.0349	−0.0320
	Im F	0.496	0.49	0.500	0.501	0.501	0.501	0.502
	Re G	0.240	0.122	0.0984	0.0826	0.0712	0.0627	0.0561
	Im G	0.0354	0.0118	0.00754	0.00504	0.00346	0.00240	0.00166
	$d\sigma/d\Omega$	8.11×10^{3}	4.22×10^{2}	1.48×10^{2}	57.7	23.1	8.76	2.58
	S	−0.0130	−0.0191	−0.0220	−0.0244	−0.0263	−0.0269	−0.0240
105°	Re F	−0.135	−0.0862	−0.0722	−0.0623	−0.0550	−0.0494	−0.0450
	Im F	0.485	0.496	0.498	0.499	0.500	0.501	0.501
	Re G	0.138	0.0716	0.0579	0.0486	0.0420	0.0370	0.0332
	Im G	0.0361	0.0108	0.00688	0.00464	0.00325	0.00232	0.00168
	$d\sigma/d\Omega$	5.09×10^{3}	2.60×10^{2}	89.6	34.2	13.3	4.83	1.34
	S	−0.0150	−0.0220	−0.0256	−0.0290	−0.0321	−0.0341	−0.0324
120°	Re F	−0.175	−0.107	−0.0894	−0.0768	−0.0675	−0.0604	−0.0548
	Im F	0.473	0.492	0.496	0.498	0.499	0.500	0.500
	Re G	0.0764	0.0403	0.0327	0.0275	0.0238	0.0210	0.0188
	Im G	0.0268	0.00776	0.00494	0.00335	0.00236	0.00170	0.00125
	$d\sigma/d\Omega$	3.56	1.78	6.04	2.25	8.47	2.93	0.748
	S	−0.0152	−0.0226	−0.0266	−0.0309	−0.0353	−0.0394	−0.0406
135°	Re F	−0.205	−0.123	−0.102	−0.0875	−0.0768	−0.0686	−0.0622
	Im F	0.462	0.489	0.494	0.496	0.498	0.499	0.500
	Re G	0.0385	0.0207	0.0168	0.0142	0.0123	0.0108	0.00971
	Im G	0.0162	0.00462	0.00294	0.00199	0.00141	0.00102	7.54×10^{-4}
	$d\sigma/d\Omega$	2.74×10^{3}	1.34×10^{2}	44.8	16.3	5.94	1.94	0.446
	S	−0.0136	−0.0205	−0.0244	−0.0290	−0.0342	−0.0404	−0.0461
150°	Re F	−0.224	−0.134	−0.111	−0.0948	−0.0831	−0.0742	−0.0672
	Im F	0.453	0.486	0.492	0.495	0.497	0.499	0.500
	Re G	0.0158	0.00861	0.00702	0.00593	0.00514	0.00454	0.00407
	Im G	0.00745	0.00211	0.00134	9.13×10^{-4}	6.47×10^{-4}	4.71×10^{-4}	3.48×10^{-4}
	$d\sigma/d\Omega$	2.28×10^{3}	1.10×10^{2}	36.4	13.0	4.58	1.42	0.288
	S	−0.0102	−0.0155	−0.0187	−0.0226	−0.0276	−0.0343	−0.0444
165°	Re F	−0.236	−0.140	−0.116	−0.0991	−0.0869	−0.0775	−0.0702
	Im F	0.447	0.485	0.491	0.495	0.497	0.498	0.499
	Re G	0.00377	0.00207	0.00169	0.00143	0.00124	0.00110	9.82×10^{-4}
	Im G	0.00189	5.34×10^{-4}	3.41×10^{-4}	2.31×10^{-4}	1.64×10^{-4}	1.20×10^{-4}	8.87×10^{-5}
	$d\sigma/d\Omega$	2.05×10^{3}	98.0	32.1	11.3	3.90	1.16	0.211
	S	-5.48×10^{-3}	-8.38×10^{-3}	−0.0102	−0.0125	−0.0156	−0.0202	−0.0290

[a] $d\sigma/d\Omega$ is given in barns per steradian.

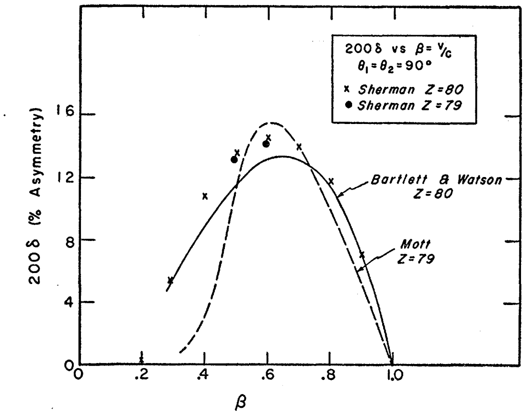

FIG. 1. Polarization asymmetry at 90° calculated by Mott,[1] Bartlett and Watson,[2] and the UNIVAC.

which is appropriate for these series.[6] This transformation is given by

$$\sum_{n=0}^{\infty}(-1)^n v_n=\frac{v_0}{2}+\frac{\Delta v_0}{4}+\cdots+\frac{\Delta^p v_0}{2^{p+1}}+\sum_{m=0}^{\infty}\frac{(-1)^m\Delta^{p+1}v_m}{2^{p+1}},$$

where

$$\Delta v_m=v_m-v_{m+1},$$

$$\Delta^p v_m=\Delta^{p-1}v_m-\Delta^{p-1}v_{m+1}.$$

RESULTS

Table I lists the values of $F(\theta)$, $G(\theta)$, $d\sigma(\theta)/d\Omega$, and $S(\theta)$ for mercury. Tables II and III present the corresponding quantities for cadmium and aluminum.

Figure 1 compares the results of Mott[1] and Bartlett and Watson[2] with those given here, for the polarization asymmetry at 95°. Although our values of δ and those of Bartlett and Watson (which may be more reliable than Mott's) disagree at $\beta=0.4$ by about 15%, the values of F and G are in agreement to within 2%. This demonstrated in Table IV. The small disagreement is magnified by taking differences of products ($S\sim F^*G+FG^*$) and then squaring ($\delta=S^2$).

Since the single-scattering cross section is sometimes expressed in the form of its ratio to the Rutherford scattering cross section,[2,3]

$$\sigma/\sigma_R=[1/\sigma_R(\theta)][d\sigma(\theta)/d\Omega],$$

where

$$\sigma_R(\theta)=\left(\frac{Ze^2}{m_0c^2}\right)^2\frac{(1-\beta^2)}{\beta^4(1-\cos\theta)^2},$$

this ratio is tabulated in Table V. (For further ease in comparing results with those of other investigators, this table also shows the electron's kinetic energy in Mev, corresponding to β, for each value of the latter quantity.) Table V can be used to verify the approximate agreement between these calculations and those of Doggett and Spencer[3] in the regions where the two calculations overlap. The closest overlap occurs for $Z=13$ at an energy of 0.2, and our results agree to about 1%.

As a last indication of the accuracy of the numerical calculations, F and G were calculated at $\theta=30°$, 90°, and 155°, $\beta=0.6$ and 0.8, for $Z=1$. These results were compared with the approximate expressions for the

TABLE IV. Comparison with the results of Bartlett and Watson[a] at $Z=80$ and $\theta=90°$.

β		Re F	Im F	Re G	Im G
0.292	Sherman	0.416	0.210	0.358	−1.193
	B and W	0.423	0.213	0.359	−1.192
0.390	Sherman	0.0372	0.510	0.976	−0.325
	B and W	0.0369	0.514	0.975	−0.329
0.585	Sherman	0.332	0.463	0.757	0.238
	B and W	0.329	0.474	0.759	0.241
0.974	Sherman	−0.470	0.401	0.510	0.270
	B and W	−0.475	0.406	0.509	0.272

[a] See reference 2.

TABLE V. Normalized cross sections σ/σ_R.

θ	β=0.2 E(Mev)=0.010	0.4 0.046	0.5 0.079	0.6 0.128	0.7 0.204	0.8 0.340	0.9 0.661
				$Z=80$			
15°	1.00	1.02	1.02	1.02	1.04	1.06	1.10
30°	1.01	1.00	1.02	1.08	1.14	1.22	1.29
45°	1.02	1.02	1.12	1.23	1.34	1.45	1.55
60°	0.976	1.12	1.27	1.41	1.55	1.67	1.78
75°	0.939	1.26	1.43	1.59	1.72	1.82	1.91
90°	0.964	1.41	1.58	1.71	1.80	1.86	1.89
105°	1.09	1.58	1.70	1.77	1.79	1.78	1.72
120°	1.33	1.74	1.80	1.79	1.72	1.61	1.44
135°	1.63	1.92	1.89	1.79	1.61	1.39	1.12
150°	1.93	2.06	1.95	1.76	1.50	1.18	0.809
165°	2.14	2.16	2.00	1.74	1.42	1.04	0.594
				$Z=48$			
15°	1.00	1.02	1.03	1.05	1.07	1.09	1.10
30°	1.00	1.06	1.10	1.13	1.15	1.18	1.20
45°	1.00	1.11	1.16	1.20	1.22	1.24	1.26
60°	1.02	1.16	1.20	1.23	1.24	1.25	1.25
75°	1.06	1.19	1.22	1.22	1.22	1.20	1.17
90°	1.02	1.20	1.21	1.20	1.16	1.11	1.05
105°	1.15	1.21	1.18	1.13	1.06	0.982	0.882
120°	1.21	1.21	1.16	1.07	0.961	0.841	0.701
135°	1.27	1.20	1.12	1.00	0.862	0.707	0.528
150°	1.32	1.20	1.08	0.944	0.782	0.593	0.386
165°	1.35	1.20	1.07	0.907	0.727	0.522	0.292
				$Z=13$			
15°	1.00	1.01	1.02	1.02	1.02	1.02	1.02
30°	1.01	1.02	1.02	1.02	1.01	1.01	1.00
45°	1.01	1.01	1.01	1.00	0.987	0.975	0.956
60°	1.02	0.999	0.986	0.966	0.939	0.912	0.875
75°	1.01	0.982	0.955	0.920	0.881	0.834	0.778
90°	1.01	0.958	0.919	0.871	0.810	0.743	0.664
105°	1.00	0.936	0.882	0.818	0.740	0.649	0.546
120°	0.995	0.910	0.844	0.764	0.669	0.559	0.433
135°	0.992	0.887	0.811	0.717	0.607	0.479	0.334
150°	0.986	0.870	0.787	0.683	0.560	0.419	0.258
165°	0.984	0.860	0.770	0.659	0.529	0.380	0.210

TABLE VI. Comparison between correct and approximate formulas for $F(\theta)$ and $G(\theta)$ at $Z=1$.

		30° Approx	30° Corr.	90° Approx	90° Corr.	150° Approx	150° Corr.
$\beta=0.6$	Re F	0.0943	0.0940	-0.00285	-0.00281	-6.61×10^{-3}	-6.66×10^{-3}
	Im F	0.500	0.500	0.500	0.500	0.500	0.500
	Re G	0.0854	0.0848	0.00617	0.00610	*4.82×10^{-4}*	*4.38×10^{-4}* $\sim$10%[a]
	Im G	-0.00614	-0.00163	2.50×10^{-5}	2.53×10^{-5}	4.86×10^{-6}	4.86×10^{-6}
$\beta=0.8$	Re F	*0.00715*	*0.00784* $\sim$10%[a]	-0.00203	-0.00215	-0.00487	-0.00501
	Im F	0.500	0.500	0.500	0.500	0.500	0.500
	Re G	0.0642	0.0637	0.00465	0.00458	*3.73×10^{-4}*	*3.29×10^{-4}* $\sim$12%[a]
	Im G	-9.45×10^{-4}	-9.34×10^{-4}	9.26×10^{-6}	1.04×10^{-5}	2.27×10^{-6}	2.36×10^{-6}

[a] Values italicized show largest disagreement and percentage.

Mott series (*viz.*, an expansion in powers of α)[1] which should be valid for small Z. The comparison is shown in Table VI.

These calculations were suggested by Professor K. M. Case who also provided detailed criticism and encouragement on several occasions. The author is grateful to Professor R. W. Pidd and to D. F. Nelson for illuminating discussions concerning their double-scattering experiments at the University of Michigan and to H. Hanerfeld for patient assistance in the early stages of programing the calculation for the UNIVAC computer.

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Measurement of Heat Capacity of Microscopic Particles at Low Temperatures

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The problem of measuring the heat capacity of microscopic particles at temperatures up to a few degrees absolute is discussed. In order to avoid the heat effects of helium adsorption, it appears that such measurements must be made with the individual particles out of equilibrium with each other. In practice this restricts such investigations to paramagnetic substances which can be measured by the techniques of adiabatic demagnetization.

IT is evident that particle size should have an appreciable effect on heat capacity in the temperature regions available by means of liquid helium and adiabatic demagnetization. In considering this problem, it became evident that the experimental determination of the heat capacity of small or microscopic particles involves some unusual features. Since it seems likely that other experimental work in this laboratory will delay an attack on this problem for some time, it seems desirable to set forth some of our ideas with respect to it.

In conventional calorimetry, a measured amount of heat is introduced by means of a heater and some conducting gas is used to transfer heat between particles so that equilibrium may be attained. Even in our work with rather large crystals in the liquid helium range, we have found that great care must be used in adding helium gas to the sample tube in order to avoid thermal effects due to adsorbed helium. Stout and Giauque[1] made an experimental investigation of the adsorption of helium on $NiSO_4\cdot7H_2O$ and were able to evaluate the rather large heat effects. They found that the degree of adsorption and thus the heat effect depends on time as well as temperature. This is a very obnoxious combination.

For their adiabatic demagnetization experiments with gadolinium sulfate octahydrate, Giauque and MacDougall[2] compacted crystals into the sample under very high pressure in order to attain a filling factor of about unity. They also hoped that the hydrated crystals would grow into a continuous mass, with an improvement in the thermal conductivity. The glazed appearance of the sample at first led them to believe that they had succeeded. However, the difficulties encountered at low temperatures made it clear that the pressure had fractured the crystals into very small sizes with a large surface and poor heat conductivity. Helium added to conduct heat appeared to be "cleaned up" at liquid helium temperatures, and under some conditions portions of the sample were at different temperatures. Various experiences of this kind, including some unpublished later ones, which were valueless because of the use of two much helium, make it clear that it would be very undesirable to use helium gas

[1] J. W. Stout and W. F. Giauque, J. Am. Chem. Soc. **60**, 393 (1938).

[2] W. F. Giauque and D. P. MacDougall, J. Am. Chem. Soc. **57**, 1175 (1935).