

TABLE VI. Comparison between correct and approximate formulas for $F(\theta)$ and $G(\theta)$ at $Z=1$.

		30°		90°		150°	
		Approx	Corr.	Approx	Corr.	Approx	Corr.
$\beta=0.6$	Re F	0.0943	0.0940	-0.00285	-0.00281	-6.61×10^{-3}	-6.66×10^{-3}
	Im F	0.500	0.500	0.500	0.500	0.500	0.500
	Re G	0.0854	0.0848	0.00617	0.00610	4.82×10^{-4}	$4.38 \times 10^{-4} \sim 10\%$ ^a
	Im G	-0.00614	-0.00163	2.50×10^{-5}	2.53×10^{-5}	4.86×10^{-6}	4.86×10^{-6}
$\beta=0.8$	Re F	<i>0.00715</i>	<i>0.00784</i>	-0.00203	-0.00215	-0.00487	-0.00501
	Im F	0.500	0.500	0.500	0.500	0.500	0.500
	Re G	0.0642	0.0637	0.00465	0.00458	3.73×10^{-4}	$3.29 \times 10^{-4} \sim 12\%$ ^a
	Im G	-9.45×10^{-4}	-9.34×10^{-4}	9.26×10^{-6}	1.04×10^{-5}	2.27×10^{-6}	2.36×10^{-6}

^a Values italicized show largest disagreement and percentage.

Mott series (*viz.*, an expansion in powers of α)¹ which should be valid for small Z . The comparison is shown in Table VI.

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Measurement of Heat Capacity of Microscopic Particles at Low Temperatures

W. F. GIAUQUE

Department of Chemistry and Chemical Engineering, University of California, Berkeley, California

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The problem of measuring the heat capacity of microscopic particles at temperatures up to a few degrees absolute is discussed. In order to avoid the heat effects of helium adsorption, it appears that such measurements must be made with the individual particles out of equilibrium with each other. In practice this restricts such investigations to paramagnetic substances which can be measured by the techniques of adiabatic demagnetization.

IT is evident that particle size should have an appreciable effect on heat capacity in the temperature regions available by means of liquid helium and adiabatic demagnetization. In considering this problem, it became evident that the experimental determination of the heat capacity of small or microscopic particles involves some unusual features. Since it seems likely that other experimental work in this laboratory will delay an attack on this problem for some time, it seems desirable to set forth some of our ideas with respect to it.

In conventional calorimetry, a measured amount of heat is introduced by means of a heater and some conducting gas is used to transfer heat between particles so that equilibrium may be attained. Even in our work with rather large crystals in the liquid helium range, we have found that great care must be used in adding helium gas to the sample tube in order to avoid thermal effects due to adsorbed helium. Stout and Giauque¹ made an experimental investigation of the adsorption of helium on $\text{NiSO}_4 \cdot 7\text{H}_2\text{O}$ and were able to evaluate the rather large heat effects. They found that the

degree of adsorption and thus the heat effect depends on time as well as temperature. This is a very obnoxious combination.

For their adiabatic demagnetization experiments with gadolinium sulfate octahydrate, Giauque and MacDougall² compacted crystals into the sample under very high pressure in order to attain a filling factor of about unity. They also hoped that the hydrated crystals would grow into a continuous mass, with an improvement in the thermal conductivity. The glazed appearance of the sample at first led them to believe that they had succeeded. However, the difficulties encountered at low temperatures made it clear that the pressure had fractured the crystals into very small sizes with a large surface and poor heat conductivity. Helium added to conduct heat appeared to be "cleaned up" at liquid helium temperatures, and under some conditions portions of the sample were at different temperatures. Various experiences of this kind, including some unpublished later ones, which were valueless because of the use of too much helium, make it clear that it would be very undesirable to use helium gas

¹ J. W. Stout and W. F. Giauque, J. Am. Chem. Soc. **60**, 393 (1938).

² W. F. Giauque and D. P. MacDougall, J. Am. Chem. Soc. **57**, 1175 (1935).

conduction in an investigation of the heat capacity of a substance in the form of small particles at liquid helium temperatures.

Consideration of the above problem makes it seem necessary to accept conditions in which the particles are not in equilibrium with each other during the determination of heat capacity. Energy must be added in some way so that each particle receives its proportional amount of energy simultaneously so that all will increase in temperature at the same rate from a common initial temperature. It will also be necessary to have some means of measuring the temperature of the particles without being able to have an auxiliary thermometer in thermal contact with them. These conditions suggest the general techniques of adiabatic demagnetization as possibly the only practicable solution in the special cases where they can be applied. For example, the accurate determination of the heat capacity of microscopic diamagnetic magnesium oxide at the temperatures of liquid helium would involve some unsolved experimental problems. Dielectric heating could be used, but the dielectric constant may safely be assumed to have no appreciable temperature coefficient so that temperature could not be denoted by this property. It is at least conceivable that at a sufficiently low temperature, magnesium oxide plus some liquid helium could be brought into contact with a calibrated resistance thermometer-heater. There may be a range well below 1°K in which the helium would have a sufficiently low heat capacity and yet retain enough heat conductivity to enable equilibrium to be attained. A check on the heat capacity of adsorbed helium with a paramagnetic substance would be necessary and it would also be necessary to assume that the adsorption on differing surfaces was similar. This possible method is not very attractive since the degree of adsorption is known to change with time. There is also the question as to whether particles in helium would act like particles alone.

In favorable paramagnetic cases, heat can be added by doing magnetic work on the sample either reversibly or irreversibly. The temperature of the particles can be measured by some of the magnetic techniques which have been applied in adiabatic demagnetization experiments. A critical discussion of methods of heat introduction has recently³ been given and it seems necessary to comment on only three of them here.

A sample could be cooled by adiabatic demagnetization with subsequent heating by γ rays as has been done on macroscopic crystals by Kurti, Laine, and Simon.⁴ This procedure is somewhat questionable even with large crystals because of the production of defects involving considerable amounts of energy and thus range and time of effect. In the case of microscopic crystals it seems inevitable that some particles would

be heated and some not. Thus the method must be ruled out.

Heat introduction by paramagnetic relaxation has been used by de Klerk, Steenland, and Gorter.⁵ This would be a satisfactory method of measuring heat capacity when the temperature can be determined as a function of magnetic susceptibility. After demagnetization it would be necessary to remove the cooled sample from the magnet and from all appreciable adjacent metal so that energy losses from the bridge circuit could be accurately evaluated as heat added to the sample. Correction could be made for heat developed by eddy currents and other irreversible processes in adjacent objects held at constant temperature, but any large effect of this kind will reduce accuracy.

It should be possible to determine the heat capacity by means of reversible magnetization in the manner described previously³ for determining absolute temperature near 0°K without introducing heat below 1°K. This would involve the determination of I , the intensity of magnetization, as a function of H , the field strength, along paths of known and constant entropy.

$$S_{\text{micro}} - S_{\text{macro}} = \Delta S.$$

S_{micro} and S_{macro} do not need to be absolute values; each may be referred to its own convenient reference state, thus ΔS plus some constant would be the known quantity. In such a method the absolute temperature can be calculated as a function of I and H along each constant entropy path.

$$\left(\frac{\partial(\Delta S + \text{const})}{\partial T} \right)_H = \frac{C_{\text{micro}} - C_{\text{macro}}}{T}.$$

This method would be considerably more involved than one utilizing only the initial magnetic susceptibility and some means of determining entropy values for a series of demagnetizations. It would, however, make it possible to determine the difference in the heat capacities of the two states within as well as out of the field.

There are certain general problems relating to either of the above two possible methods. The entropy of the magnetic system must be obtained in any method which is to be used below about 1°K since no accurate temperature scale is available without this knowledge. We are dubious concerning the assumption of simple ideal quantum magnetic functions for calculating the entropy, a procedure which we initiated from early necessity, and which seems to have become almost conventional. We suspect that actual entropy measurements may show the inadequacy of this procedure in all but a few exceptional cases.

In a rigorous experiment the entropy of magnetization for the several isentropics should be determined separately for both the macro- and micro-samples since, if the micro particles are very small, surface

³ W. F. Giaque, *Phys. Rev.* **92**, 1339 (1953).

⁴ Kurti, Laine, and Simon, *Compt. rend.* **204**, 754 (1937).

⁵ de Klerk, Steenland, and Gorter, *Commun. Phys. Lab. Univ. Leiden* 278a (1949), 282a (1950); *Physica* **15**, 649 (1949); **16**, 571 (1950).

conditions and related defects could affect the energy levels of the magnetic system of an appreciable fraction of the substance. If, as a first approximation, one wishes to assume that a magnetically dilute system has the same magnetic properties under the same *effective* field, it will be necessary to apply the appropriate filling and demagnetization factors.⁶ A check on the combined factor can be obtained by an investigation of the magnetic properties of both macro- and microscopic forms in the temperature range in which they can be kept in thermal contact with liquid helium. Magnetically dilute systems with minimal cooperative effects would seem most likely to avoid possible effects of particle size on the magnetic properties. The investigation of this subject is an interesting problem in itself.

There is another point which seems most difficult of all. One must know at some time before adiabatic demagnetization that the known temperature is uniform throughout the magnetized sample. We see no alternative to allowing it to stand for a very long time with just enough helium gas to bring equilibrium eventually. Also this should not be at a very low initial temperature or the amount of adsorption will spoil the experiment. It will be very difficult to decide when equilibrium has been reached, since the only kind of magnetic suscepti-

bility which can be measured in the field is the adiabatic differential susceptibility, $(\partial I/\partial H)_s$, and this is very insensitive to temperature under the necessary experimental conditions. A sensitive carbon thermometer used only to indicate equilibrium just before demagnetization is one possibility. Another possible method of determining the time required for equilibrium would be a study at zero field so that the full sensitivity of a magnetic thermometer would be available.

If the measurements are confined to the region above 1°K, the problem becomes somewhat simpler. The obvious method would seem to be the determination of initial susceptibility as a function of temperature while the sample is in thermal contact with helium and the later introduction of heat to the isolated sample by magnetic relaxation. There is, however, another problem which arises at temperatures much above 1°K, namely the heat capacity of the container which would have some contact with outer particles. A single crystal can be suspended without a container. It would appear that the best that could be done with a powder is to enclose it in an extremely thin blown glass bubble which could be suspended and left open to the space within its enclosing vessel. This would avoid the necessity of strength to support pressure changes. We believe that glass is superior to plastic for this purpose.

⁶H. B. G. Casimir, *Magnetism and Very Low Temperatures* (Cambridge University Press, Cambridge, 1940), pp. 9-12.

Theoretical Analysis of Buildup of Current in Transient Townsend Discharge

YASUNORI MIYOSHI

Nagoya Institute of Technology, Gokiso-Cho, Showa-Ku, Nagoya, Japan

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The buildup of current in the transient Townsend discharge has been calculated fully, taking the positive-ion and photon mechanisms at the cathode into account. The two cases of continuous and instantaneous electron supply have been treated. The calculated results are useful for the explanation of the buildup of current during the formative time lag of sparks under a sudden application of overvoltage, and for the transient state of a Townsend discharge in the case of undervoltage application.

I. INTRODUCTION

THE growth of current during the formative time of spark breakdown before the development of space charge effects has been studied by several authors.¹⁻⁶ Recently Bandel⁵ carried out experiments and a theoretical analysis of the current buildup, but his solution is approximately accurate only for times longer than the positive-ion transit time, and not for shorter times. Following Bandel, Auer⁶ made a further

analysis of this subject; his analysis is also limited to the special case of γ_p action alone, and it seems to be much too complicated for numerical computation.

In the present paper, a theoretical treatment of the same problem is carried out, based upon the fundamental equations of continuity for the electron and positive-ion streams in a parallel plane gap, from a mathematically more rigorous standpoint than those of the previous investigations.⁷ Our analysis shows that the current-time characteristics cannot be given by a single form, but require different forms appropriate to the following three time ranges: (a) times shorter than the electron transit time (t_-); (b) times longer than the

¹R. Schade, *Z. Physik* **104**, 487 (1937).

²W. Bartholomeyczzyk, *Z. Physik* **116**, 355 (1940).

³A. von Engel and M. Steenbeck, *Elektrische Gasentladungen* (Verlag Julius Springer, Berlin, 1934), Vol. 2, p. 178.

⁴Dutton, Haydon, Jones, and Davidson, *Brit. J. Appl. Phys.* **4**, 170 (1953).

⁵H. W. Bandel, *Phys. Rev.* **95**, 1117 (1954).

⁶P. L. Auer, *Phys. Rev.* **98**, 320 (1955).

⁷More recently P. M. Davidson [*Phys. Rev.* **99**, 1072 (1955)] has discussed this problem from another standpoint.