logical theory of Brueckner and Watson. This is discussed in reference 1.

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Production of Pions in Nucleon-Nucleon Collisions at Cyclotron Energies

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All available data on the production of pions in nucleon-nucleon collisions at cyclotron energies are compiled and are compared with the phenomenological theory of Watson and Brueckner. The principle of conservation of isotopic spin allows all these cross sections to be written in terms of only three independent cross sections, whose excitation functions are predicted by the phenomenological model. The theory represents satisfactorily the excitation functions and the angular distributions that are known experimentally; however the experimental ratio $\sigma(P+P\rightarrow\pi^++N+P)/\sigma(P+P\rightarrow\pi^++D)$ appears to be about a factor 2 larger than predicted.

Using a notation for total cross section in which the first subscript indicates the isotopic spin of the initial state, and the second that of the final state of the two nucleons, and expressing all cross sections in millibarns, we find:

$$\begin{aligned}
\sigma_{10}(D) &= \sigma(P+P\rightarrow\pi^++D) &&= 0.14\eta+1.0\eta^3,\\
\sigma_{10}(N+P) &= \sigma(P+P\rightarrow\pi^++N+P)-\sigma_{11} &&\approx 1.5\eta^4,\\
\sigma_{01} &= 2\sigma(N+P\rightarrow\pi^\pm)-\sigma_{11} &&\lesssim 0.3\eta^4,\\
\sigma_{11} &= \sigma(P+P\rightarrow\pi^0) &&\approx 0.2\eta^8.
\end{aligned}$$

η is the maximum c.m. momentum available to the pion, measured in units of μc.

I. INTRODUCTION

IN 1951 several authors[1–3] succeeded in interpreting satisfactorily the data then available on the production of pions near threshold. The only reaction that had been studied extensively was $P+P\rightarrow\pi^++D$, mainly using 340-Mev protons, corresponding to a pion energy of 22 Mev in the c.m. system. By assuming that the pions are created mainly in p states and by treating the nucleon-nucleon interaction phenomenologically it was possible to explain the energy spectrum and angular distribution of the pions, and to predict the excitation function for the reaction.

Since 1951 the information available on pion production has been greatly extended.[4] There are now data on N-P as well as P-P collisions; the energy range studied has been extended downwards to 10 Mev above threshold (where s-state production of pions should become relatively important) and cross sections have been published up to 50 Mev above threshold. Currently the Pittsburgh and Chicago synchrocyclotrons are being used to study the energy region 46 to 92 Mev. At Chicago eight experiments either are in progress or else have been recently completed.

There is by now some experimental confirmation of the principle of conservation of isotopic spin in pion production.[5–7] According to this principle it is possible to express all pion production data for each energy in terms of only three independent differential cross sections;[3] moreover the phenomenological treatment makes certain predictions about the excitation functions for these cross sections. We mean "phenomenological" to include the assumption of charge independence and also all the selection rules that will be discussed below.

We shall show that the new data are consistent with the phenomenological model, even when the pions are relativistic. The only appreciable discrepancy is that the experimental ratio $\sigma(P+P\rightarrow\pi^++N+P)/\sigma(P+P\rightarrow\pi^++D)$ seems to be about twice as large as predicted.

We have compiled the available cross sections for pion production in nucleon-nucleon collisions at cyclotron energies, and we have adjusted unknown parameters in the phenomenological treatment so as to give a best fit to the data.

Several authors[8,9] have proposed that various reactions could be sorted out by polarization experiments.

[1] K. Brueckner, Phys. Rev. **82**, 598 (1951).
[2] Chew, Goldberger, Steinberger, and Yang, Phys. Rev. **84**, 581 (1951).
[3] K. M. Watson and K. A. Brueckner, Phys. Rev. **83**, 1 (1951).
[4] References to all experiments are given in Table III.
[5] R. H. Hildebrand, Phys. Rev. **89**, 1090 (1953).
[6] R. A. Schluter, Phys. Rev. **95**, 639(A) (1954).
[7] R. H. Hildebrand and A. H. Rosenfeld (to be published).
[8] K. M. Watson and C. Richman, Phys. Rev. **83**, 1256 (1951).
[9] R. E. Marshak and A. M. L. Messiah, Nuovo cimento **11**, 337 (1954).

Inserting our parameters into their expressions we find how large a polarization effect can be expected and at what energy it can most easily be detected. We discuss the results of such experiments.

II. SUMMARY OF THE THEORY

The conservation of isotopic spin places severe restrictions on the production of pions in nucleon-nucleon collisions.

All the possible reactions can be expressed in terms of only three independent reactions, whose cross sections we shall denote by σ_{10}, σ_{01}, and σ_{11}. The notation is the following: the first index represents the isotopic spin of the initial state, and the second that of the final state of the two nucleons. There is no σ_{00}, since this reaction cannot conserve isotopic spin.

The simplest results are obtained for the total cross sections and are given in Table I. The word "nucleons" indicates that the final nucleons may be either bound or unbound; the symbol D is reserved for a deuteron and we shall use $N+P$ to indicate unbound nucleons; for example $\sigma(P+P\rightarrow\pi^++\text{nucleons})=\sigma(P+P\rightarrow\pi^++D)+\sigma(P+P\rightarrow\pi^++N+P)=\sigma_{10}(D)+\sigma_{10}(N+P)+\sigma_{11}$.

Exactly the same table applies to cross sections (integrated over angle) per unit pion energy. Thus the unbound part of reaction (A) can be written

$$\frac{d\sigma}{dT}(P+P\rightarrow\pi^++N+P)=\frac{d\sigma_{10}}{dT}+\frac{d\sigma_{11}}{dT}, \qquad (\mathrm{A}')$$

where T is the c.m. energy of the pion.

By chance, none of the reactions listed in Table I involves more than two of the fundamental cross sections. This fact simplifies the discussion. It may seem surprising that σ_{11} does not contribute to reaction (E), but the appropriate Clebsch-Gordan coefficient is zero.

Note that reactions (B) and (H) are experimentally accessible in two different ways; i.e., $\sigma_{10}(D)$ can be determined directly in a pion production experiment, but it can also be inferred from $\sigma(\pi^++D\rightarrow P+P)$ using the principle of detailed balancing.

A table equivalent to Table I can be constructed for cross sections *per steradian*, but is slightly more complicated.[10]

TABLE I. Pion production cross sections in terms of the three independent cross sections.

$\sigma(P+P\rightarrow\pi^++\text{nucleons})$	$=\sigma_{10}+\sigma_{11}$	(A)
$\sigma(P+P\rightarrow\pi^++D)$	$=\sigma_{10}(D)$	(B)
$\sigma(P+P\rightarrow\pi^0)$	$=\sigma_{11}$	(C)
$\sigma(N+P\rightarrow\pi^\pm)$	$=\frac{1}{2}[\sigma_{11}+\sigma_{01}]$	(D)
$\sigma(N+P\rightarrow\pi^0+\text{nucleons})$	$=\frac{1}{2}[\sigma_{10}+\sigma_{01}]$	(E)
$\sigma(N+P\rightarrow\pi^0+D)$	$=\frac{1}{2}[\sigma_{10}(D)]$	(F)
$\sigma(N+N\rightarrow\pi^-+\text{nucleons})$	$=\sigma_{10}+\sigma_{11}$	(G)
$\sigma(N+N\rightarrow\pi^-+D)$	$=\sigma_{10}(D)$	(H)

[10] Van Hove, Marshak, and Pais, Phys. Rev. 88, 1211 (1952).

It should be noted that even if charge independence should be essentially valid, all the relations above are subject to correction because of Coulomb forces and the π^+-π^0 and N-P mass differences.

Selection Rules

All of the experimental information available is at energies such that the pion has 65 Mev or less in the c.m. system, with the exception of Stadler's work which extends to 94 Mev. A 65-Mev pion has a momentum η, measured in units of μc, of 1.1. Figure 6 of an accompanying paper[11] gives the experimental pion spectrum when 63.7 Mev is available in the c.m. system; it shows that a typical pion carries away 50 Mev, leaving an internal energy of the residual two-nucleon system of only 10 to 15 Mev. Now we may imagine the phenomenon of pion production to take place at a characteristic distance R from the center of collision, where R must be of the order of $\hbar/\mu c$. (Arguments based on the data in this paper in fact indicate that R is slightly less than $\frac{1}{2}\hbar/\mu c$.) Therefore, in the energy region under consideration, the product $k_\pi R$ or $k_{\text{nuc}}R$ is $\lesssim 1$. We may suppose, then, that the pion will be emitted in an s or p state with respect to the two-nucleon system, and that the two nucleons will be in an S or P state relative to one another.[12] In the case of the nucleons the strong attractive force in the S state (as contrasted with the relatively weak interaction between P-state nucleons at low energies) will strongly favor S states as well as enhancing the preference for low nucleon energies. In the case of the pions, experiment indicates that the p state is preferred (except close to threshold). This is consistent with the large p-state interaction found for pion-proton scattering, and with the fact that the pion is a pseudoscalar particle.

Let us adopt a shorthand notation for the final states in which the first letter stands for the orbital angular momentum of the two-nucleon system and the second for the orbital angular momentum of the pion. The discussion above can then be summarized as follows. If pion production near threshold is analyzed in terms of the angular momentum of the final states, angular momentum 2 or more may be neglected, and the remaining partial waves can be classified in order of decreasing intensity Sp, Ss, Pp, and Ps.

In Table II, all the allowed reactions of class Sp and Ss (and some others) are listed, grouped according to isotopic spin of the initial and final nucleons (see Table I) and then further grouped according to intensity class. The notation is defined in the caption. The construction of the table will be described by discussing the first few rows. We shall show below that the most important elementary cross section is σ_{10}, so the reactions contributing to this cross section are taken first.

[11] A. H. Rosenfeld, preceding paper [Phys. Rev. 95, 130 (1954)].

[12] We use lower-case letters to indicate the angular momentum of the pion, capitals for the angular momentum of the two-nucleon system.

Among these we list first the most important intensity class Sp. The pion is a pseudoscalar particle, so this class will have even parity. If the final nucleons are to have isotopic spin $T=0$, the S state must have spin 1 and be a triplet with over-all angular momentum $J=0$, 1, or 2. There is only one possible initial state ($^1S_0$) that has $T=1$, $J=0$, and even parity. This feeds the $^3S_1\,p_0$ final state and is accordingly listed in front of it. There is no appropriate initial state with $J=1$, so this reaction is absent. Only $^1D_2$ can feed $^3S_1\,p_2$. In the absence of interference between the various final states, any reaction with $J=0$ must be isotropic, and any reaction $^1D_2 \rightarrow {}^3S_1\,p_2$ must have an angular distribution proportional to $\frac{1}{3}+\cos^2\theta$. We may point out that this latter anisotropic reaction should dominate over the isotropic one because it is favored by a statistical factor $2J+1=5$. Experimentally this is found to be the case. So far the only assumptions incorporated into the table are that the pion is pseudoscalar and that isotopic spin is conserved. In calculating the total cross sections, however, excitation functions are derived taking into account nuclear forces, and undetermined parameters are then adjusted to fit the data of Sec. III.

Table II shows that for σ_{11} the intense reaction class Sp is absent. It turns out to be impossible to explain the experimental excitation function by invoking class Ss alone. Therefore, for σ_{11} only, we must consider also reactions of class Pp and Ps.

TABLE II. Analysis of pion production near threshold in terms of the angular momentum (S or P) of the final two-nucleon system and the angular momentum (s or p) of the pion. The spin and angular momentum of the two-nucleon system are indicated in the conventional way. The last subscript to the final-state symbols indicates the over-all angular momentum. The "Initial state" column gives all the two-nucleon states that have the same isotopic spin, angular momentum, and parity as a given final state. The angular distributions are those which would apply in the absence of interferences. The total cross sections are calculated in Secs. II and III. Classes which are intensified by the $(\frac{3}{2},\frac{3}{2})$ pion-nucleon interaction are in boldface.

Isotopic reaction	Class	Initial state	Final state	Angular distribution	Total cross section in mb
σ_{10}	**Sp**	$^1S_0$	$^3S_1\,p_0$	isotropic	$1.0\eta^3$ for deuteron formation; $1.5\eta^4$ for $N+P$
		none	$^3S_1\,p_1$	...	
		$^1D_2$	$^3S_1\,p_2$	$\frac{1}{3}+\cos^2\theta$	
	Ss	$^3P_1$	$^3S_1\,s_1$	isotropic	0.14η
σ_{01}	Sp	$^3S_1$ or $^3D_1$	$^1S_0\,p_1$	isotropic or $\frac{1}{3}+\cos^2\theta$	$0.3\eta^4$
	Ss	none	$^1S_0\,s_0$	...	0
σ_{11}	Sp	none	$^1S_0\,p_1$	...	0
	Ss	$^3P_0$	$^1S_0\,s_0$	isotropic	$\sim 0.01\eta^2$
	Pp	$^3P_{0,1}$	$^3P_0\,p_1$	$c+\cos^2\theta$	$0.2\eta^8$
		$^3P_{0,1,2}$ or $^3F_2$	$^3P_1\,p_{0,1,2}$		
		$^3P_{1,2}$ or $^3F_{2,3}$	$^3P_2\,p_{1,2,3}$		
	Ps	$^1S_0$	$^3P_0\,s_0$	isotropic	$\propto \eta^6$
		none	$^3P_1\,s_1$	...	
		$^1D_2$	$^3P_2\,s_2$	isotropic	

Although there are six possible reactions of class Sp or Ss the $^1D_2 \rightarrow {}^3S_1\,p_2$ reaction (associated with σ_{10}) is the only one in which this $(\frac{3}{2},\frac{3}{2})$ state is represented. The $(\frac{3}{2},\frac{3}{2})$ interaction can also enhance the reactions of class Pp and Ps which contribute to σ_{10} and σ_{11}. It does not affect σ_{01} because a nucleon-pion state with isotopic spin $\frac{3}{2}$ cannot combine with a nucleon to give overall isotopic spin 0.

Using meson theory, Aitken *et al.*[13] have calculated the increase in cross section resulting from the $(\frac{3}{2},\frac{3}{2})$ interaction. They assume that the effect is energy-independent, and find an enhancement of two or three. Experimentally we find a factor of about 5 (see the discussion following Eq. (27).

In discussing Table II we pointed out that the $^1D_2 \rightarrow {}^3S_1\,p_2$ reaction was already favored by a statistical factor 5. With the extra $(\frac{3}{2},\frac{3}{2})$ enhancement it should be even more dominant.

Angular Distributions

Two or more of the reactions listed in Table II can interfere in the pion angular distribution only if the final nucleon states are identical. This immediately rules out the possibility of interference between σ_{10} and the other two cross sections. Even if the final nucleons are in the same state, there can be no interference unless the initial nucleons are in the same spin state (for an unpolarized beam). These rules, applied to Table II, show that there can be interference between the reactions within a given intensity class, but the only possibility of interference between classes is between the Sp reaction contributing to σ_{01} and the Ss reaction of σ_{11}. Only this interference can produce a forward-backward asymmetry, as is discussed below.

Relative Intensities

At the energies of interest here, σ_{10} is found experimentally to be considerably larger than the other two cross sections. There are two reasons why this should be the case. First there is the possibility of the production of deuterons; this added possibility roughly doubles the cross section. Second there is the effect of the strong interaction between a pion and a nucleon when they are in a two-particle state characterized by isotopic spin $\frac{3}{2}$ and angular momentum $\frac{3}{2}$, the state which is known to have a large cross section for pion-proton scattering.

The Pion Spectrum and Excitation Function for σ_{10}

We shall now discuss in greater detail the most important reaction, namely the production of p-state pions associated with ^3S nucleons.

Watson[14] takes into account phenomenlogically the attraction between the final nucleons; neglecting terms of order $\mu/2M$ he finds a pion production cross section

[13] A. Aitken *et al.*, Phys. Rev. **93**, 1349 (1954).
[14] K. M. Watson, Phys. Rev. **88**, 1163 (1952).

per unit pion energy, integrated over angle,

$$\frac{d\sigma_{10}'}{dT}=b_{10}(2\mu)^{-\frac{3}{2}}\frac{P^3(T_0-T)^{\frac{1}{2}}}{B+T_0-T}, \tag{1}$$

where b_{10} is a constant, T and P are the c.m. kinetic energy and momentum of the pion, T_0 is the kinematic maximum for T for the continuous part of the spectrum, and B is the binding energy of the deuteron. We have adopted the convention of writing a prime after cross sections derived from the phenomenological model. The numerator of (1) is the product of P^2 from the square of the matrix element for the production of a p-state pion and $P(T_0-T)^{\frac{1}{2}}$ which is the phase space available to the three final particles. The nuclear force introduces the denominator $|B|+T_0-T$. The spectrum is plotted at the bottom of Fig. 6 of the accompanying paper.

Equation (1) can be integrated over pion energy to give a total cross section for the production of pions associated with unbound ^3S nucleons. Using the nonrelativistic relation $P^2/2\mu=T$ we get the simple result

$$\sigma_{10}'(N+P,\ p\text{-state})=b_{10}\pi T_0^2F(y), \tag{2*}$$

where

$$F(y)=\tfrac{3}{8}-y^{\frac{1}{2}}(1+y)^{\frac{3}{2}}+\tfrac{3}{2}y+y^2, \tag{3}$$

and $y=|B|/T_0$. (An asterisk after the number of an equation means that it applies in the nonrelativistic limit.) Note that the nuclear force serves not only to raise the cross section, but also to change the form of the excitation function, which would otherwise be proportional to T_0^3.

Exactly the same set of assumptions give a cross section for the formation of deuterons:

$$\sigma_{10}'(D,p)=b_{10}(2\mu)^{-\frac{3}{2}}2\pi B^{\frac{1}{2}}P_m^3\equiv\beta_{10}\eta^3, \tag{4}$$

where P_m is the c.m. momentum of the pions and η is $P_m/\mu c$. It should be emphasized that the ratio σ_{N+P}/σ_D is fixed by the theory without the introduction of any arbitrary parameters. Nonrelativistically, (4) can be better compared with (2*) by writing $P_m^2=2\mu T_m=2\mu T_0(1+y)$, so

$$\sigma_{10}'(D,p)=b_{10}2\pi T_0^2y^{\frac{1}{2}}(1+y)^{\frac{3}{2}},$$

and

$$\frac{\sigma_{10}'(N+P)}{\sigma_{10}'(D)}=\frac{F(y)}{2y^{\frac{1}{2}}(1+y)^{\frac{3}{2}}}. \tag{5*}$$

In writing (1) and (4) we have used Watson's results, but have tried to write them in a form that applies to relativistic pions, i.e., to distinguish between $P^2/2\mu$ and T. Phase space is treated relativistically, and we assume that the matrix element for the production of a p-state pion is $p/\sqrt{w}$, where w is the pion energy (it enters because of the normalization of the wave function), p comes from a gradient operator, and we have neglected terms in higher powers of p because we do not know what they are. Even the expression $p/\sqrt{w}$, while reasonable, is only a guess. It has the virtue, however, that it fits the experimental results and leads to some particularly simple algebraic expressions.

The approximations which are used to treat the S-wave nucleon-nucleon interaction also become worse if there is an appreciable probability that the nucleons may acquire more than about 20 Mev. Watson's expression (1) is derived on the assumption that the presence of the pion does not affect the nuclear force. It is then reasonable that the enhancement of the cross section by this force should be proportional to $|\psi(R)|^2$, where $\psi(R)$ is the amplitude of the nucleon wave function at the small distance R characteristic of pion production. The simple denominator $|B|+T_0-T$ of (1) comes from writing $\psi(R)$ in terms of effective-range parameters and dropping terms involving the square of the nucleon energy. It can be shown that the approximations used to obtain (1) result in an underestimate of the enhancement when the nucleon energies are large particularly if R is really much smaller than $\hbar/\mu c$. To estimate the error, we computed $|\psi(0)|^2$ exactly (assuming a square potential well of effective range 1.7×10^{-13} cm) and averaged it over the pion energy spectrum for a single representative high-energy case, $T_0=57.8$ Mev ($\eta=1$). This calculation gives a branching ratio $\sigma'(N+P)/\sigma'(D)$ which is 20 percent larger than given by (5*). (The relatively good agreement arises from the cancellation of errors from several approximations.) We feel that an approximation that is good to 20 percent is adequate, particularly since there is probably a much larger error introduced by the assumption that the pion has no effect upon the nuclear force. This difficulty is discussed in Sec. III.

Equation (1) can be integrated over pion energy to give the relativistic version of (2*),

$$\sigma_{10}'(N+P,\ p)=b_{10}\pi T_0^2\left\{F(y)-\frac{3z}{32(2+z)}\right\}(1+z/2)^2, \tag{6}$$

where $z=T_0/\mu c^2$, and $F(y)$ is given by (3). This differs from the nonrelativistic expression (2*) by somewhat less than 50 percent at $\eta=1$.

Using expressions (6) and (4), we calculate the fraction of unbound nucleons which should be formed at various energies. We find for 341-Mev protons (Berkeley) $\sigma'(N+P,\ p)/[\sigma'(N+P,\ p)+\sigma'(D,p)]=17$ percent; for 400-Mev neutrons (Chicago) 32 percent; for 440-Mev protons (Chicago and Pittsburgh) 35 percent.

We have so far considered only the production of pions in p states. The equivalent expressions for s-state production are obtained by replacing η^3 by η in the expressions for the cross section since the matrix element involved is constant rather than proportional to P. We then have

$$\sigma_{10}'(D,s)=\alpha_{10}\eta, \tag{7}$$

and

$$\frac{\sigma_{10}'(N+P,\ s)}{\sigma_{10}'(D)}=\frac{G(y)}{2y^{\frac{1}{2}}(1+y)^{\frac{1}{2}}}, \tag{8}$$

where
$$G(y)=\tfrac{1}{2}+y-y^{\frac{1}{2}}(1+y)^{\frac{1}{2}},\quad y=|B|/T_0. \tag{9}$$

Because s-state production is relatively important only at low energy there is no point in treating (8) relativistically. The fraction (8) of unbound nucleons is higher for s- than for p-state production. From (8) we find for 341-Mev protons (Berkeley) the fraction is 27 percent, for 440-Mev protons (Chicago), 47 percent.

In Sec. III we shall determine the p-state parameter β_{10} by fitting (4) to the experimental cross sections, but the s-state parameter α_{10} has already been determined indirectly by Brueckner, Serber, and Watson.[15] They make use of the known branching ratio between the two processes $\pi^-+D\to 2N$ and $\pi^-+D\to 2N+\gamma$, following the capture (from an s state) of π^- by deuterium.[16] Using detailed balancing and taking the exclusion principle into account, they relate α_{10} to experimental information on photopion production near threshold. Taking the latest information available, $\sigma(\gamma+D\to\pi^-)=(1.4\pm0.2)\sigma(\gamma+D\to\pi^+)$;[17] and $\sigma(\gamma+P\to\pi^++N)=0.14$ mb $\times\eta$,[18] we find
$$\alpha_{10}=(0.14\pm0.05)\text{ mb}. \tag{10}$$

The Angular Distribution for $\sigma_{10}(D)$

We now want to consider the addition and interference of the three reactions which contribute to $\sigma_{10}(D)$, i.e.,

$J=0$,	${}^1S_0\to{}^3S_1\,p_0$,	(isotropic),	amplitude $\mu_0\propto\eta$
$J=2$,	${}^1D_2\to{}^3S_1\,p_2$,	$\frac{1}{3}+\cos^2\theta$,	amplitude $\mu_2\propto\eta$
$J=1$,	${}^3P_1\to{}^3S_1\,s_1$,	isotropic,	amplitude $\mu_1\propto$ const.

To each reaction has been assigned a complex transition amplitude μ_J. These amplitudes are understood to be proportional to S-matrix elements, so that the partial cross sections $\sigma_J\propto(2J+1)|\mu_J|^2$. Thus
$$\frac{\sigma_{10}(D,s)}{\sigma_{10}(D,p)}=\frac{\sigma_1}{\sigma_0+\sigma_2}=\frac{3|\mu_1|^2}{|\mu_0|^2+5|\mu_2|^2}=\frac{\alpha_{10}}{\beta_{10}\eta^2}.$$

Angular distributions and polarization relations depend only upon ratios of transition amplitudes; to simplify the equations we define
$$r_0=\frac{\mu_0}{(5)^{\frac{1}{2}}i^2\mu_2};\quad r_1=\frac{(3)^{\frac{1}{2}}i\mu_1}{(5)^{\frac{1}{2}}i^2\mu_2}.$$

Note that r_0 is independent of pion energy, but $r_1\propto\eta^{-1}$.

The $J=0$ and $J=2$ reactions, which both arise from an initial singlet state, interfere to give an angular distribution of the form $X+\cos^2\theta$, where
$$X=\left\{\left|\frac{2-\sqrt{2}r_0}{1+\sqrt{2}r_0}\right|^2-1\right\}^{-1}. \tag{11}$$

Equation (4), which describes the Sp contribution to $\sigma_{10}(D)$ can now be written in the form of an angular distribution,
$$4\pi\frac{d\sigma_{10}'(D,p)}{d\Omega}=\beta_{10}\eta^3\frac{X+\cos^2\theta}{X+\frac{1}{3}}. \tag{12}$$

Adding the contribution (7) of the noninterfering s wave, we find
$$4\pi\frac{d\sigma_{10}'(D)}{d\Omega}=(\alpha_{10}\eta+\beta_{10}\eta^3)\frac{A+\cos^2\theta}{A+\frac{1}{3}}, \tag{13}$$
where
$$A=X+(X+\tfrac{1}{3})\frac{\alpha_{10}}{\beta_{10}}\frac{1}{\eta^2}. \tag{14}$$

For the energy range in which this treatment is valid A should be a linear function of η^{-2}. In Sec. III (Fig. 2) we plot the experimental values of A *vs* η^{-2} and obtain a best fit value of X.

The discussion has been restricted to the case of deuteron formation because of the simple form of the excitation functions and because there are no good experimental angular distributions for the "unbound" reaction. Equation (11) still applies to the unbound case, however, and the isotropic contribution A to the angular distribution should resemble that drawn in Fig. 2.

Pion Spectrum and Excitation Function for σ_{01}

Of the reactions considered in Table II, only those two leading to the final state ${}^1S_0\,p_1$ can contribute to σ_{01}. The pion spectrum is of the same form as (1) except that $B=2.227$ Mev must be replaced by $|B_{\text{singlet}}|\approx 70$ kev. Because of the low value of $|B|$ the pion spectrum is very peaked near T_0 and the quantity $y=|B|/T_0$ in (2*) and (3) becomes almost zero, so that the expression for the excitation function (2*) reduces to
$$\sigma_{01}'\approx b_{01}\pi T_0{}^2\cdot\tfrac{3}{8}\equiv\beta_{01}\eta^4. \tag{15*}$$

A similar simplification arises in the relativistic expression (6).

Excitation Function for σ_{11}

For σ_{11} reactions of class Sp are absent. Reactions of class Ss give a cross section
$$\sigma_{11}'(Ss)=\alpha_{11}\eta^2. \tag{16*}$$

We shall see in Sec. III that (16*) fails utterly to describe the experimental excitation function, which is very steep, proportional to $\approx\eta^8$. We must therefore discuss class Pp and Ps. The relative importance of

[15] Brueckner, Serber, and Watson, Phys. Rev. **81**, 575 (1951).
[16] Panofsky, Aamodt, and Hadley, Phys. Rev. **81**, 565 (1951).
[17] Beneventano, Lee, Stoppini, Hanson, Yamagata, and Bernardini, quoted by G. Bernardini at 1954 Rochester Conference on High-Energy Nuclear Physics (University of Rochester, Rochester).
[18] G. Bernardini, 1954 Rochester Conference Report on High-Energy Nuclear Physics (University of Rochester, Rochester).

these classes is not surprising since they are reinforced by the strong pion-nucleon interaction (see Table II).

The effect of nuclear forces is relatively unimportant for reactions of class Pp or Ps, so that the excitation functions are simple. For class Ps the cross section is proportional to η^6, and for class Pp:

$$\sigma_{11}'(Pp)=\beta_{11}\eta^8. \tag{17*}$$

Mather and Martinelli[19] discuss the possibility of explaining the steep experimental excitation function in terms of a cross section $\propto\eta^6$. They note that a pion, created in a pure s state at a distance R from the center of mass has an s-state component kR about the center of mass. This cross section must certainly be present, but it can be shown to be somewhat too small to explain the data.

Angular Distribution for σ_{01} and σ_{11}

Inspection of Table I shows that only in the case of reaction D is a forward-backward asymmetry not ruled out by the principle of charge symmetry. According to the more restrictive principle of charge independence none of the various reactions contributing to a given elementary cross section (like σ_{01}) can interfere asymmetrically *among themselves*. (According to charge independence a neutron and a proton in a pure isotopic spin state behave like indistinguishable particles, so there is no distinction between forwards and backwards.)

It is experimentally observed that σ_{01} and σ_{11} do interfere to give a large forward-backward asymmetry

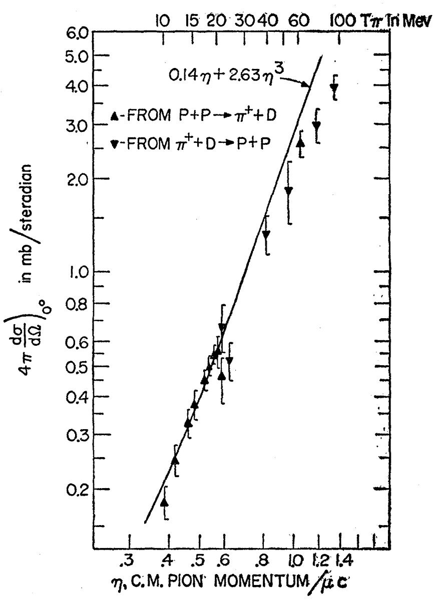

FIG. 1. Cross section per steradian in the c.m. at 0° for $P+P\rightarrow\pi^++D$. The lowest six points are the relative excitation results of Schulz and have been normalized to the absolute point of Crawford and Stevenson at $\eta=0.54$ (see Table III).

[19] J. W. Mather and E. A. Martinelli, Phys. Rev. 92, 780 (1953).

in reaction D. We can account for this asymmetry by invoking only the interference between reactions associated with S-state nucleons. Although this is what we do, it may not be correct. We have just mentioned that σ_{11} (Ss) is smaller than σ_{11} (Pp or Ps), which can interfere with σ_{01} (Pp or Ps). Until experimental data on σ_{01} become available at more than one energy we have no information on σ_{01} (Pp or Ps); if it should turn out that these Pp and Ps reactions contribute more than about 10 percent to σ_{01}, then the relations given below will have to be replaced with more complicated ones.

The only interfering reactions that we consider are

$$\sigma_{01},\ {}^3S_1\rightarrow{}^1S_0\ p_1,\ \text{amplitude } \mu_3;$$
$$\sigma_{01},\ {}^3D_1\rightarrow{}^1S_0\ p_1,\ \text{amplitude } \mu_4;$$
$$\sigma_{11},\ {}^3P_0\rightarrow{}^1S_0\ s_0,\ \text{amplitude } \mu_5.$$

In the present case the subscripts to the transition amplitudes do not correspond to any physical quantity, they simply distinguish the three reactions. We have again normalized the μ_i so that $(2J+1)|\mu_i|^2\propto\sigma_i(\eta)$.

Let us write the angular distribution for reaction (D) in the form

$$4\pi\frac{d\sigma}{d\Omega}\propto a+b\cos\theta+c\cos^2\theta. \tag{18}$$

Then we find

$$a=-3\sqrt{2}\mathrm{Re}\mu_3^*\mu_4+3|\mu_3|^2+\tfrac{3}{2}|\mu_4|^2+|\mu_5|^2 \tag{19}$$

$$b=2\sqrt{3}\mathrm{Im}\mu_5^*(\mu_3+\sqrt{2}\mu_4) \tag{20}$$

$$c=g/2|\mu_4|^2+g\sqrt{2}\ \mathrm{Re}\mu_3^*\mu_4 \tag{21}$$

$$\sigma\propto a+c/3=3|\mu_3|^2+3|\mu_4|^2+|\mu_5|^2 \tag{22}$$

$$\sigma=\sigma_{01}(Sp)+\sigma_{11}(Ss). \tag{23}$$

We must again caution that pion production associated with the formation of P-state nucleons has been ignored. While there is some hope that the equation for the asymmetry b may still be meaningful, the expressions for a and c are certainly incomplete; for example we shall see in Sec. III that at $\eta=0.9$ σ_{11} (Pp or Ps) represents about $\frac{1}{3}$ of the total cross section for reaction (D).

III. COMPARISON OF THEORY AND EXPERIMENT

In this section we shall show that there is reasonably good agreement between the experimental data and the excitation functions set forth in the last section.

Determination of Parameters for $\sigma_{10}(D)$

Equation (13) gives the energy and angular dependence of $d\sigma(D)/d\Omega$ in terms of the three parameters α_{10}, β_{10}, and X. To determine these parameters, we shall first make use of the 0° data, which are comparatively abundant at low energy. Figure 1 gives all the available 0° differential cross sections for reaction (B) and also the results from the inverse reaction con-

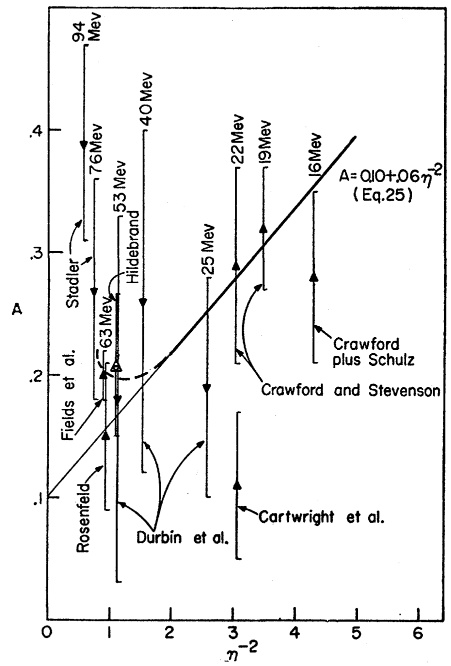

FIG. 2. Experimental values of A vs η^{-2}, where A is defined by $d\sigma_{10}'/d\Omega \propto A+\cos^2\theta$. The straight line comes from inserting into Eq. (14) the values $X=0.1$, $\alpha_{10}/\beta_{10}=0.14$. The dashed curve indicates that experimentally A deviates from (14). References are in Table II. Durbin *et al.* report values of A but not their errors; we have computed the errors using their published data.

verted by detailed balancing. The solid line was obtained by assuming $\alpha_{10}=0.14$ mb [see (10)], and adjusting the coefficient of η^3 to give a good fit at low energy. In this manner we obtain the relation $\beta_{10}(X+1)/(X+\frac{1}{3})=2.63$ mb [see (12)]. The data on the angular distribution as a function of energy, as shown in Fig. 2, can now be used to determine X. The data are evidently not adequate to confirm the linear dependence of A on η^{-2} given by (14); in fact they show that (14) does not apply for $\eta \gtrsim 1$. We have, however, drawn a straight line of A vs η^{-2}, with its slope determined by (14), which is a fair fit to the low-energy data. We find $X=0.10\pm0.03$. Then for the total cross section we find

$$\sigma_{10}'(D)=0.14\eta+1.0\eta^3, \tag{24}$$

and Eq. (14) becomes

$$A=1.0+0.06\eta^{-2}. \tag{25}$$

The only experimental result in serious disagreement with (25) is that of Cartwright *et al.*[20] at $\eta^{-2}=3$, but this is presumably to be superseded by the more recent work of Crawford and Stevenson[20] at the same energy. Nevertheless we must repeat that (25) should not be taken very seriously.

It should be noted that our fit to the experimental data is not too sensitive to the value of α_{10} assumed because at the energies and angles which we have considered so far the Sp reactions dominate over the Ss. To obtain more information on α_{10} we need more data at low energy and at 90° c.m. At 0° class Ss contributes more than Sp only from threshold to ≈ 4 Mev; however at 90° c.m. the Ss contribution dominates for energies up to ≈ 37 Mev.

We can now test our semiempirical excitation function (24) by comparing it with experimental total cross sections. The values of $d\sigma_{10}(D)/d\Omega$ displayed in Fig. 1 are given in Fig. 3 and Table III in the form of total cross sections, along with all other available total cross sections for the single production of charged and neutral pions in nucleon-nucleon collisions below 510 Mev. When only a differential cross section is published and an angular distribution is not reported, then the cross section is computed using the angular distribution (25). It can be seen that (24) fits the experimental data very well up to $\eta \approx 1$.

We believe that the parameters we have chosen give a good over-all fit to the data. Different parameters could, for example, improve the apparent fit in Fig. 3 for the total cross sections, but only at the expense of

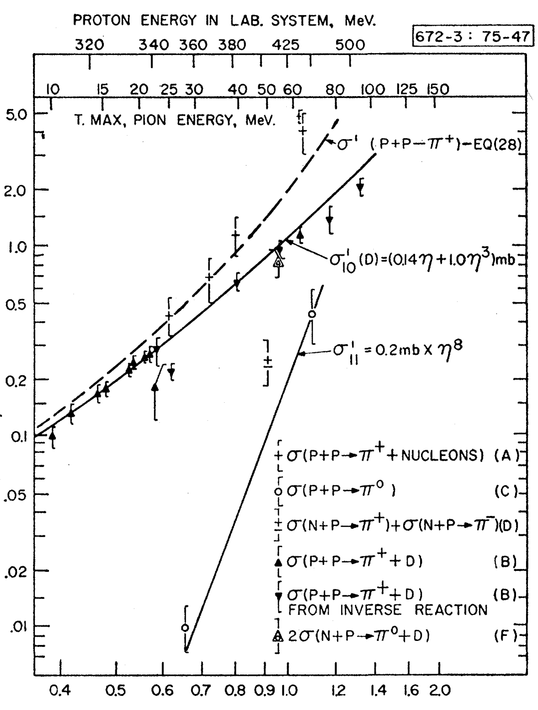

FIG. 3. Excitation function for the production of charged and neutral pions in nucleon-proton collisions in both direct and inverse reactions.

The abscissa η is the maximum momentum available to a pion in the particular reaction plotted. Note that the scale labelled "Proton Beam Energy" applies only to reaction (A); it cannot apply to all reactions because of the mass differences between neutral and charges pions, etc.

Note also that in the case of pion production by neutrons, twice the cross section has been plotted so as to display more directly the fundamental cross sections as defined in Table I.

References are in Table III.

[20] See Table III.

TABLE III. Production of charged and neutral pions in nucleon-proton collisions.

Reaction	Elementary cross section	T_{proton} Mev	$T_{m\text{ pion}}$[a] Mev	η[b]	A[c]	$\sigma_{\text{pion production}}$ mb	Reference
$P+P\rightarrow\pi^++D$	$\sigma_{10}(D)$	311	10.1	0.386		0.100 ± 0.013[d]	Schulz[e]
$P+P\rightarrow\pi^++D$	$\sigma_{10}(D)$	315	11.7	0.418		0.133 ± 0.016[d]	Schulz[e]
$P+P\rightarrow\pi^++D$	$\sigma_{10}(D)$	321	14.3	0.462		0.168 ± 0.018[d]	Schulz[e]
$P+P\rightarrow\pi^++D$	$\sigma_{10}(D)$	324	15.5	0.482	0.28 ± 0.07	0.178 ± 0.016	Crawford[f]
$P+P\rightarrow\pi^++D$	$\sigma_{10}(D)$	330	18.1	0.523		0.228 ± 0.017[d]	Schulz[e]
$P+P\rightarrow\pi^++D$	$\sigma_{10}(D)$	336	20.6	0.560		0.264 ± 0.019[d]	Schulz[e]
$P+P\rightarrow\pi^++D$	$\sigma_{10}(D)$	332	19.0	0.535	0.32 ± 0.05	0.245 ± 0.013	Crawford,[f] Stevenson[g]
$P+P\rightarrow\pi^++D$	$\sigma_{10}(D)$	338	21.5	0.575	0.29 ± 0.08	0.269 ± 0.026	Crawford,[f] Stevenson[g]
$P+P\rightarrow\pi^++D$	$\sigma_{10}(D)$	340	22.3	0.585	0.11 ± 0.06	0.18 ± 0.06	Cartwright *et al.*[h]
$P+P\rightarrow\pi^++$nucl.	$\sigma_{10}+\sigma_{11}$	340	22.3	0.585		0.28 ± 0.10	Cartwright[i]
$\pi^++D\rightarrow P+P$	$\sigma_{10}(D)$	341	22.8	0.590		0.284 ± 0.050	Clark *et al.*[j]
$P+P\rightarrow\pi^0+P+P$	σ_{11}	341	27.8	0.660	isotropic	0.010 ± 0.003	Mather and Martinelli[k]
$P+P\rightarrow\pi^++$nucl.	$\sigma_{10}+\sigma_{11}$	345	24.5	0.618		0.43 ± 0.10	Passman *et al.*[l]
$P+P\rightarrow\pi^++$nucl.	$\sigma_{10}+\sigma_{11}$	365	33.0	0.728		0.68 ± 0.18	Passman *et al.*[l]
$P+P\rightarrow\pi^++$nucl.	$\sigma_{10}+\sigma_{11}$	381	39.5	0.806		1.15 ± 0.27	Passman *et al.*[l]
$\pi^++D\rightarrow P+P$	$\sigma_{10}(D)$	346.5	25	0.625	$0.19+0.09$	0.22 ± 0.02	Durbin *et al.*[m]
$\pi^++D\rightarrow P+P$	$\sigma_{10}(D)$	382	40	0.815	0.26 ± 0.14	0.66 ± 0.07	Durbin *et al.*[m]
$\pi^++D\rightarrow P+P$	$\sigma_{10}(D)$	413.5	53	0.960	0.18 ± 0.15	0.97 ± 0.10	Durbin *et al.*[m]
$N+P\rightarrow\pi^\pm+$nucl.	$\frac{1}{2}(\sigma_{01}+\sigma_{11})$	405	48	0.915		0.16 ± 0.04[n]	Yodh[o]
$N+P\rightarrow\pi^0+D$	$\frac{1}{2}\sigma_{10}(D)$	400	53	0.970	0.21 ± 0.06		Hildebrand[p]
$N+P\rightarrow\pi^0+D$	$\frac{1}{2}\sigma_{10}(D)$	400	53	0.960	$0.28^{+0.24}_{-0.14}$	0.41 ± 0.07	Schluter[q]
$P+P\rightarrow\pi^++$nucl.	$\sigma_{10}+\sigma_{11}$	437	63.0	1.05		4.75 ± 0.24	de Carvalho *et al.*[r]
$P+P\rightarrow\pi^++D$	$\sigma_{10}(D)$	437	63.0	1.05	0.20 ± 0.02	1.15 ± 0.13	Fields *et al.*[s]
$P+P\rightarrow\pi^++$nucl.	$\sigma_{10}+\sigma_{11}$	440	63.7	1.07	0.15 ± 0.06	4.0 ± 1.0	Rosenfeld[t]
$P+P\rightarrow\pi^0+P+P$	σ_{11}	430	64.4	1.11	isotropic	0.45 ± 0.15	Marshall *et al.*[u]
$\pi^++D\rightarrow P+P$	$\sigma_{10}(D)$	470	76	1.17	0.27 ± 0.09	1.39 ± 0.23	Stadler[v]
$\pi^++D\rightarrow P+P$	$\sigma_{10}(D)$	515	94	1.33	0.39 ± 0.08	2.04 ± 0.22	Stadler[v]

[a] T_m pion is the maximum energy available to the pion in the center-of-mass system.
[b] η is the momentum corresponding to T_m in units of μc.
[c] A represents the fraction of isotropic contribution on the assumption that $d\sigma/d\Omega\propto(A+\cos^2\theta)$. Where the angular distribution was not measured, we assume Eq. (25) for $d\sigma_{10}/d\Omega$ and isotropy for $d\sigma_{11}/d\Omega$ in calculating the total cross section.
[d] Schulz measured only an excitation function, which is here normalized to Crawford and Stevenson's absolute cross sections.
[e] A. H. Schulz, University of California Radiation Laboratory Report UCRL-1756, 1952 (unpublished).
[f] F. S. Crawford, University of California Radiation Laboratory Report UCRL-2187, 1953 (unpublished).
[g] M. L. Stevenson, University of California Radiation Laboratory Report UCRL-2188, 1953 (unpublished).
[h] Cartwright, Richman, Whitehead, and Wilcox, Phys. Rev. **91**, 677 (1953).
[i] W. F. Cartwright (private communication).
[j] Clark, Roberts, and Wilson, Phys. Rev. **83**, 649 (1951).
[k] J. W. Mather and E. A. Martinelli (reference 19).
[l] Passman, Block, and Havens, Phys. Rev. **88**, 1247 (1952).
[m] Durbin, Loar, and Steinberger, Phys. Rev. **84**, 581 (1951).
[n] Yodh's cross section represents the average value for π^+ and π^- production. This is a preliminary result.
[o] G. B. Yodh (to be published).
[p] R. H. Hildebrand (see reference 5).
[q] R. A. Schluter (see reference 6).
[r] de Carvalho *et al.* (to be published).
[s] Fields, Fox, Kane, Stallwood, and Sutton, Phys. Rev. **95**, 638(A) (1954).
[t] A. H. Rosenfeld, Phys. Rev. **95**, 638(A) (1954).
[u] Marshall, Marshall, Nedzel, and Warshaw, Phys. Rev. **88**, 632 (1952).
[v] H. L. Stadler, Phys. Rev. (to be published).

the agreement with the angular distributions shown in Fig. 2. We should remark that the ratio $\alpha_{10}/\beta_{10}=0.14$ from (24) is in close agreement with the estimate made by Brueckner, Serber, and Watson[21] in 1951.

It is interesting to note that if the differential cross section in the c.m. system is of the form $A+\cos^2\theta$, a measurement at a single angle can give the total cross section directly: The quantity,

$$\frac{d\sigma}{d\Omega}\bigg/\sigma=\frac{1}{4\pi}\frac{A+\cos^2\theta}{A+\frac{1}{3}},$$

becomes independent of A for $\cos^2\theta=\frac{1}{3}$ (55° or 125°). For the reaction $P+P\rightarrow\pi^++D$, the "representative" c.m. angle of 125° transforms to an angle close to 90° in the laboratory system (87° for a bombarding energy of 440 Mev, and 83° for 380 Mev). The experiments of Passman *et al.*[20] which were done at 90° in the lab., thus give a total cross section comparatively insensitive to assumptions about the angular distribution.

[21] Brueckner, Serber, and Watson, Phys. Rev. **84**, 258 (1951).

(3/2, 3/2) Interaction

We can insert our value of X into (11) and obtain an expression for the ratio r_0 of the transition amplitudes relating the two reactions of class Sp. It should be remembered that the important pion-nucleon state of isotopic spin $\frac{3}{2}$ and angular momentum $\frac{3}{2}$ is not represented at all in one of these reactions (${}^1S_0\rightarrow{}^3S_1\,p_0$), but is important for the other (${}^1D_2\rightarrow{}^3S_1\,p_2$).

Equation (11) restricts r_0 to a circle in the complex plane, i.e.,

$$r_0=-\surd\tfrac{1}{2}(1+3X)+(3/\surd 2)(X^2+X)^{\frac{1}{2}}e^{i\omega_0}. \quad (26)$$

For $X=0.10$,

$$r_0=-0.92+0.70e^{i\omega_0}, \quad (27)$$

and the circle lies entirely to the left of the origin. Near the threshold for pion production it can be shown that the phase of r_0 is related to proton-proton scattering phase shifts. We consider it more probable that the phase is closer to 0° or 180° than to 90°. Making the assumption that r_0 is real and less than unity, we find $r_0\approx0.2$, so that the relative intensity of the $J=2$

(enhanced) and $J=0$ (unenhanced) reactions is $r_0^{-2}\approx 25$. Part of this factor ≈ 25 must arise from the purely statistical factor $2J+1=5$, but the remaining factor ≈ 5 probably arises from the strong $(\frac{3}{2},\frac{3}{2})$ pion-nucleon interaction.

Conservation of Isotopic Spin

In Fig. 2 we should call attention to the result of Hildebrand[5,20] which is an established piece of evidence for the conservation of isotopic spin. According to this principle, only one of the three independent cross sections, namely σ_{10}, can lead to deuteron formation (thus Table I shows that reactions (B) and (F) should be identical except for a factor two). Hildebrand has measured the isotropic contribution A for reaction (F) at 53 Mev c.m. His result is seen to be in good agreement with all the rest of the data, which come from reaction (B).

A new piece of evidence is shown in Fig. 3; this is Schluter's total cross section for reaction (F). It is plotted as an open triangle at $\eta=0.96$. If isotopic spin were not conserved, there would be no reason why the cross sections for reactions (F) and (B) should agree. They do agree within the experimental error which however is still large for the neutron reaction. Schluter also reports a value for A, but his statistical error is large, so that we have not plotted his result in Fig. 2.

σ_{10}—The Branching Ratio of Unbound to Bound Nucleons

Reaction (A) of Table I can be written

$$\sigma(P+P\rightarrow\pi^{+}+\text{nucleons})$$
$$=\sigma_{10}(D)+\sigma_{10}(N+P)+\sigma_{11}. \quad (28)$$

We now have phenomenological expressions for the three cross sections on the right-hand side. $\sigma_{10}(D)$ is given by (24), $\sigma_{10}(N+P)$ is computed by inserting the known parameters α_{10} and β_{10} from (24) into (6), (7), and (8); σ_{11} will be given by (30′).

The sum of these three phenomenological cross sections is plotted as a dashed line in Fig. 3. It is not a simple power law, but may be approximated by

$$\sigma(P+P\rightarrow\pi^{+}+\text{nucleons})\approx 1.8\eta^{3}, \quad (29)$$

in the energy region of interest.

It can be seen that the dashed line falls somewhat below the experimental points, although no discrepancy is indicated at low energy. There is however a discrepancy which does not show up in Fig. 3. Certain experiments, which do not necessarily establish a total cross section, give the ratio $\sigma(D)/\sigma(N+P)$ directly. Such results are not displayed in Fig. 3, but are summarized in Table IV.

Cartwright *et al.* and Peterson *et al.* measured the pion energy spectrum, and were thus able to determine, or at least put a lower limit on, the branching ratio. Hildebrand counts the protons or deuterons in coincidence with the π^0; knowing the range and angle of the proton or deuteron, he can distinguish between them and measure the branching ratio directly. All three experiments give measured branching ratios about twice as large as those computed.

Thus both Table IV and the high-energy data of Fig. 3 give considerable evidence that the branching ratio $\sigma(N+P)/\sigma(D)$ is about twice as big as predicted by the phenomenological treatment. Some of this discrepancy can be explained in terms of P-state nucleon production, which we have neglected; we also pointed out in the discussion preceding Eq. (5) that we have made approximations which indeed do underestimate the probability of formation of unbound nucleons by about 20 percent. These two effects, combined with some experimental error, could conceivably explain the discrepancy, but more likely the error lies with the phenomenological treatment. The presence of the pion might, for instance, slightly decrease the interaction of the final-state nucleons. Equation (5*) shows that the branching ratio depends sensitively on $|B|/T_0$; in fact it is almost inversely proportional to $|B|$. If the presence of the pion reduced the effective deuteron binding energy from 2.227 Mev to, say, 1 Mev, the phenomenological equations would again fit all the available data.

Determination of σ_{11}

Reference to Table I shows that σ_{11} should in principle be easy to measure, since $\sigma_{11}=\sigma(P+P\rightarrow\pi^0)$. Unfortunately the cross section is small and the reaction has not been studied extensively. Cross sections are available at only two energies, and the accuracy is low.

In our phenomenological discussion of the excitation function [Eqs. (16*) and (17*)], we predicted a sum of $\alpha_{11}\eta^2$ and $\beta_{11}\eta^8$, with perhaps some additional terms in η^6 from the "displaced" s-state cross section discussed by Mather and Martinelli[19] and from reactions of class (Ps). For the high-energy experimental point $(\eta=1.1)$ the term $\beta_{11}\eta^8$ should be dominant. We have accordingly drawn a line

$$\sigma_{11}'(Pp)=0.2\text{ mb}\times\eta^{8}, \quad (30^*)$$

passing through this point. This line fits the meager data sufficiently well that we conclude the production of pions in s states is small and cannot be determined quantitatively at present.

TABLE IV. Measured and predicted branching fractions $\dfrac{\sigma(N+P)}{\sigma(N+P)+\sigma(D)}$.

Reaction	Measured fraction %	Predicted fraction %	η	Reference
$P+P\rightarrow\pi^+$	35 ± 10	20	0.59	Cartwright[a]
$P+P\rightarrow\pi^+$	45 ± 10	20	0.49	Peterson *et al.*[b]
$N+P\rightarrow\pi^0$	60 ± 15	32	0.95	Hildebrand[c]

[a] See reference 20.
[b] Peterson, Iloff, and Sherman, Phys. Rev. 84, 372 (1951).
[c] R. H. Hildebrand (to be published).

Both Wright and Schluter[22] and Yodh[23] find a large experimental forward-backward asymmetry for reaction (*D*); if the angular distribution is written in the form $4\pi d\sigma/d\Omega = a+b\cos\theta+c\cos^2\theta$, then the ratio of b to the total cross section $a+c/3$ is probably about one-half. We must now show that this asymmetry is consistent with our conclusion that *s*-state production contributes little to σ_{11}.

The transition amplitude μ_5 for *s*-state production is related to the coefficient of $\cos\theta$ in the angular distribution by (20) and (22). Using these two equations and the two semiempirical formulas (30*) and (32*) it follows that

$$\sigma'_{11}(Ss) > 0.01 \text{ mb}\times\eta^2. \tag{31*}$$

Thus $\sigma'_{11}(Ss)$ can be as small as 0.01 mb$\times\eta^2$ and still be large enough to account for the observed asymmetry. Such a small coefficient of η^2 would not modify the observed steep excitation for σ_{11}, which is proportional to η^6 or η^8.

Determination of σ_{01}

The only elementary cross section that cannot be measured directly is σ_{01}. It can best be determined by subtracting σ_{11} from $2\sigma(N+P\rightarrow\pi^\pm)$. Two experimental determinations of this latter cross section are now in progress using the Chicago neutron beam, and Yodh's tentative result[20] is plotted in Fig. 3. Adjusting the parameter β_{01} of (15*) to fit Yodh's cross section, we get

$$\sigma_{01}' = (0.3\pm0.1) \text{ mb}\times\eta^4. \tag{32*}$$

There is as yet no experimental way to test (32*) since data are available at only one energy.

IV. POLARIZATION

In the discussion entitled "Angular Distribution for $\sigma_{10}(D)$" in Part II of this paper, we established relations (11) and (14) between the two transition amplitude ratios r_0 and r_1 and the experimentally measurable quantities X and A in the angular distribution of σ_{10}, but we could not determine uniquely the magnitudes and phases of r_0 and r_1.

This can in principle be done by polarization experiments. Most of the relations which we give below have been derived previously,[8,9] but it may be worth while to restate them in our notation and to insert our values of X and A so as to see at what energies and angles interesting effects may be observed, and how large these effects may be. Two experiments are discussed below, and the second of these is now underway at several laboratories. The results will be summarized in a note-in-proof.

Polarized Deuterons

We first discuss that polarization experiment which is in practice the most difficult. The deuterons associated with the $\sigma_{10}(Ss)$ reaction are not polarized perpendicular to the plane of the reaction, but the two *Sp* reactions produce deuterons which are polarized perpendicular to this plane, with a degree of polarization given by

$$P_P = \frac{\sin\omega_0}{1+2X+2(X^2+X)^{\frac{1}{2}}\cos\omega_0} \cdot \frac{2(X^2+X)^{\frac{1}{2}}\sin\theta\cos\theta}{X+\cos^2\theta}, \tag{33}$$

where $-1<P_P<1$, and ω_0 [defined by (26)] is a parameter of the ratio r_0 relating the two *Sp* reactions. The first factor can vary from -1 to $+1$, and thus establishes the phase ω_0. The second factor can also vary from -1 to $+1$, depending upon the angle θ at which the deuterons are detected. The second factor is largest when $\cos2\theta = -1/(2X+1)$; for $X=0.1$ this corresponds to $\theta=73°$. If the phase ω_0 happens to be propitious (for $X=0.1$ this is 56°, which is a rather improbable angle), deuterons leaving the collision at about 73° could be completely polarized. On the other hand it should be pointed out that at 73° the cross section per steradian is not very large ($X+\cos^2\theta=0.18$), and despite the large possible polarization the experiment would probably be extremely difficult.

Polarized Beam of Protons

We have not yet discussed the phase of the transition amplitude μ_1 governing the *Ss* contribution to σ_{10}. According to (4) and (10), the magnitude of μ_1 is determined by the relation:

$$\frac{\alpha_{10}}{\beta_{10}\eta^2} = \frac{3|\mu_1|^2}{|\mu_0|^2+5|\mu_2|^2} = \frac{|r_1|^2}{1+|r_0|^2}, \tag{34}$$

but we have no information on its phase. Marshak and Messiah[9] have pointed out that it may be found by measuring the angular distribution of the reaction $P+P\rightarrow\pi^++D$ using polarized protons. Since the reactions of class *Sp* arise from initial singlet states, any asymmetry must be due to the *Ss* reaction. If the proton beam has a degree of polarization P_1 in the $+x$ direction then the angular distribution of the pions is given by

$$\frac{d\sigma_{10}}{d\Omega} \propto A+\cos^2\theta+P_1QA\sin\theta\sin\phi, \tag{35}$$

where

$$Q = \sqrt{\tfrac{1}{2}}\frac{2b}{1+b^2}\sin(\psi-\tau_1), \tag{36}$$

$$r_1 = |r_1|e^{i\tau_1}, \tag{37}$$

$$b = \left|\frac{r_0+\sqrt{\frac{1}{2}}}{r_1}\right|, \tag{38}$$

$$\psi = \arg(r_0+\sqrt{\tfrac{1}{2}}). \tag{39}$$

[22] S. C. Wright and R. A. Schluter (to be published).
[23] G. B. Yodh (to be published).

Unfortunately, (35) shows that the fractional asymmetry is the greatest at 90° where the cross section is a minimum.

For simplicity we shall adopt the point of view that quantities do not vary appreciably with energy in the limited range under consideration unless a variation is indicated by the phenomenological model. Thus, while we assume that $r_1 \propto \mu_1/\mu_2 \propto \eta^{-1}$, we shall consider the phases of r_1 and r_0 to be constant. The parameter of maximum asymmetry Q will reach its maximum value of $\sqrt{\frac{1}{2}}\sin(\psi-\tau_1)$ when $b=1$, i.e., when

$$|r_1|=|r_0+\sqrt{\tfrac{1}{2}}|; \tag{40}$$

or, using (34), when

$$\eta=\eta_c=\left(\frac{\alpha_{10}}{\beta_{10}}\right)^{\frac{1}{2}}\frac{(1+|r_0|^2)^{\frac{1}{2}}}{|(r_0+\sqrt{\frac{1}{2}})|}=\left(\frac{\alpha_{10}}{\beta_{10}}\right)^{\frac{1}{2}}\left(1+\frac{1}{3X}\right)^{\frac{1}{2}}. \tag{41}$$

This last equality can be checked using (11). Inserting the value $X=0.1$, we find

$$\eta_c=0.78\pm0.2, \tag{42}$$

which corresponds to a pion energy of 35 Mev. Equation (38) may now be rewritten as $b=\eta/\eta_c$. Putting this into (36), we obtain an energy-dependence of the asymmetry:

$$Q/Q_{\max}=2\eta\eta_c/(\eta^2+\eta_c^2). \tag{43}$$

The experiment under consideration has now been completed by three groups. Crawford and Stevenson[24] find a large asymmetry; they report $|Q|=0.39\pm0.05$ at $\eta=0.41$, with the pions produced preferentially to the right of a proton beam which had been polarized by a left scatter. Sutton *et al.*[25] find a very similar result at $\eta=0.96$, namely $|Q|=0.41\pm0.07$. Here too the majority of pions were found on the side opposite from the original scatter.

de Carvalho *et al.*[26] have a result obtained as a by-product of another experiment. Their arrangement was such as to detect not only the pions formed in association with deuterons, but also about 78 percent of the rest. We can hope that both reactions show the same asymmetry and try to interpret the experimental results merely by inserting into (43) an average value of η which we take to be 0.8. Then at 85° c.m. they find $|Q|=-0.22\pm0.12$, and at 98° $|Q|=-0.08+0.11$.

[24] F. S. Crawford and M. L. Stevenson, Phys. Rev. **95**, 1112 (1954).
[25] Roger Sutton (private communication).
[26] de Carvalho, Heiberg, Marshall, and Marshall, Phys. Rev. **94**, 1796 (1954).

If we express these four results in terms of $\sin(\psi-\tau_1)$ they should be independent of the energy at which the experiment was performed. We then find for $\sin(\psi-\tau_1)$:

$+0.67\pm0.03$ (Crawford and Stevenson)
$+0.6\ \pm0.1$ (Sutton *et al.*)
$+0.22\pm0.12$ (de Carvalho *et al.*, 85°)
-0.08 ± 0.11 (de Carvalho *et al.*, 98°).

If all of these results are correct it would indicate that the asymmetry of pions associated with unbound nucleons is almost equal and opposite to that of pions associated with deuterons.

V. CONCLUSIONS

We have compiled the available data on pion production in nucleon-nucleon collisions and have analyzed them using the approach of Watson and Brueckner, which uses the fact that the pion is a pseudoscalar particle, assumes that isotopic spin is conserved, and treats nuclear forces phenomenologically.

By adjusting undetermined parameters which give the relative amplitude and phase of several partial reactions, we establish semiempirical formulas which fit the available information within experimental error (see Fig. 3). This statement should not be taken as too striking, since none of the reactions except $P+P\rightarrow\pi^++D$ has been extensively studied. For the latter reaction, however, the agreement is good, even considerably above threshold.

There is probably a discrepancy in that the fraction of unbound nucleons formed per deuteron is higher than predicted.

Evidence is given for the conservation of isotopic spin in cases where the data are good enough to permit meaningful comparison of pion production in *P-P* and *N-P* collisions.

Using our best values for the relevant parameters we discuss the information attainable by experimenting with polarized beams.

Many of the ideas in the paper came from cooperation with Murray Gell-Mann and there is considerable overlap in content between this paper and a chapter on pion production by M. Gell-Mann and K. M. Watson to appear in Vol. 4 of *Annual Reviews of Nuclear Science* (1954); we have used similar notation.

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