π^- scattering in terms of the phase shifts is

$$\frac{d\sigma}{d\omega}=\lambda^2\left[\left(\frac{2\alpha_1}{3}+\frac{\alpha_3}{3}+\frac{2\alpha_{33}}{3}\cos\chi\right.\right.$$
$$\left.\left.+\frac{1}{137\beta\pi(1-\cos\chi)}\right)^2+\frac{\alpha_{33}^2}{9}\sin^2\chi\right].$$

This is under the assumption $\alpha_{31}=\alpha_{13}=\alpha_{11}=0$. This equation can be solved for $(2\alpha_1+\alpha_3)$ if the value of α_{33} at 26 Mev is known. There is good evidence that $\alpha_{33}=0.235\eta^3$ in this energy region.[4] Using this value for α_{33} we have $2\alpha_1+\alpha_3=(4.7°\pm2.7°)$ at 26 Mev. The upper and lower limits are the values corresponding to a cross section of 1.7 and 0.6 mb, respectively, for scatterings greater than 50°.

This result when combined with other recent results lead to the conclusion that the simple energy dependence of momentum to the power $(2l\pm1)$ is quite reasonable for the phase shifts α_3, α_1 and α_{33} up to 80 Mev. The following paper develops this conclusion in detail.

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Low-Energy Behavior of the Phase Shifts in Pion-Proton Scattering*

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A best fit has been made to all pion-proton scattering phase-shift information under 80 Mev assuming the energy dependence of the phase shifts is momentum to the power $(2l+1)$ which would be the case for strong short-range interactions. The values obtained for the Fermi-type solutions are $\alpha_{33}=0.235\eta^3$, $\alpha_3=-0.11\eta$, and $\alpha_1=0.16\eta$. These phase shifts, along with Coulomb forces, fit all scattering experiment data quite closely including the 5-Mev cloud chamber results at Columbia. However, at zero energy they predict $(\alpha_1-\alpha_3)=0.27\eta$, while recent photoproduction results and the Panofsky effect as evaluated here predict $(\alpha_1-\alpha_3)=0.21\eta$ with rather large uncertainties.

INTRODUCTION

NECESSARY parameters for devising a theory of meson-nucleon forces are the scattering lengths, which in principle can be determined from low-energy pion-proton scattering experiments. Recently, considerable low-energy pion-proton scattering information has become available, and has been analyzed in terms of s- and p-wave phase shifts. At low energies the energy dependence of such phase shifts can be expanded in powers of the momentum, where the first term is the power $(2l+1)$. Furthermore, wave mechanics shows that if the two particle interaction is strong compared to the kinetic energy in a region $r<r_0$ and if there is essentially no interaction beyond this region, then for $kr_0\ll1$ the first term of the expansion in momentum is much larger than the higher order terms. In this energy region we would expect

$$\alpha_l\propto\eta^{2l+1}, \tag{1}$$

where η is the center-of-mass momentum divided by $m_\pi c$. Empirically the phase shifts up to 80 Mev seem to follow this energy dependence. We have taken such an empirical approach in selecting 80 Mev as the upper energy limit of the available data which is used in obtaining a best fit to the theoretical shape η^{2l+1}.

The scattering results have fairly successfully been fitted with s- and p-wave phase shifts assuming conservation of isotopic spin. In the various phase shift solutions at low energies there are two fairly large s-wave phase shifts α_3 and α_1 (corresponding to $T=\frac{3}{2}$ and $T=\frac{1}{2}$) and one large p-wave phase shift. We shall consider only that type solution (commonly called the Fermi-type solution) where α_{33} ($T=\frac{3}{2}$, $J=\frac{3}{2}$) is the largest p-wave phase shift.[1] We shall make the additional restriction that α_{33} be positive. This choice is based on preliminary 115-Mev π^+-p scattering results which show a destructive Coulomb interference.[2] Also the solutions with negative α_{33} are inconsistent with Eq. (1).

At the 1954 Rochester Conference, Bethe[3] and Noyes[3] discussed the possibility of a Jastrow-type potential for α_3 which had previously been suggested by Marshak.[4] Such a potential would cause α_3 to depart violently from Eq. (1) at about $\eta=0.3$ and in some of the proposals it would change sign at about $\eta=0.5$.[3,5] Bethe and de Hoffman discuss the possibility that $\alpha_3\sim0$ up to $\eta\sim0.3$ in their forthcoming book.[6] One of the motiva-

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[1] Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1953).
[2] J. Orear (to be published).
[3] *Proceedings of the Fourth Rochester Conference on High Energy Physics* (University of Rochester, Rochester, 1954).
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tions for such phase-shift behavior is an attempt to fit a zero-energy prediction from earlier photoproduction data of $(\alpha_1-\alpha_3)=0.16\eta$ with a value of $(\alpha_1-\alpha_3)=0.3\eta$ predicted by the scattering experiments. Also the preliminary 5-Mev cloud chamber results of Lederman[7] in some cases were possibly misinterpreted (the actual data have not yet been published). We find that the most recent scattering data is consistent with the simpler approach of Eq. (1) and that the zero-energy prediction of $(\alpha_1-\alpha_3)$ as reevaluated in the Appendix is not unreasonably off from this.

At present there is useful low-energy information at 0,[8] 5,[7] 20,[9] 26,[10] 30,[9] 40,[11] 45,[12] 61.5[13] and 78 Mev.[1] Fermi-type phase shift solutions have been obtained from the experiments at 40, 45, 61.5, and 78 Mev. The remaining information, although in the form of certain phase-shift combinations, is still quite useful and will be utilized fully in our analysis. The results of this analysis are $\alpha_{33}=0.235\eta^3$, $\alpha_3=-0.11\eta$, and $\alpha_1=0.16\eta$. As can be seen from Figs. 1 and 2, these values fit all the scattering data well within the errors which are usually about 20 percent. At energies above 80 Mev it appears that α_1 varies slower and that α_3 varies faster than the η^{2l+1} law.

DISCUSSION

p-Waves

In the Fermi-type solution, α_{33} is considerably larger than the other three *p*-wave phase shifts. So far α_{33} is the only *p*-wave phase shift which can be established by the experiments to any degree of accuracy. It is given by π^+-meson scattering experiments at 40, 45, 61.5, and 78 Mev. In the case of the 40 and 61.5 Mev solutions, π^- data is also used. In our analysis equal weights are given to these four independent results, even though the phase shifts are less well determined at the lower energies. This is compensated by the possibility that at higher energies departures from the η^{2l+1} law might begin to appear.

TABLE I. Phase-shift values.

Energy Mev	$a=\alpha_1/\eta$	$b=\alpha_3/\eta$	$(a-b)$	$(2a+b)$	α_{33}/η^3	Reference
0			0.21			8
20			0.25			9
26				0.141		10
30			0.26			9
40	0.164	−0.112			0.260	11
45		−0.138			0.206	12
61.5	0.191	−0.11			0.228	13
78		−0.109			0.248	1

[7] Rinehart, Sargent, Rogers, and Lederman (private communication) and reference 3.
[8] See Appendix for discussion and references.
[9] W. J. Spry, Phys. Rev. **95**, 1295 (1954).
[10] Orear, Slater, Lord, Eilenberg, and Weaver, preceding paper [Phys. Rev. **96**, 174 (1954)].
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[12] Orear, Lord, and Weaver, Phys. Rev. **93**, 575 (1954).
[13] Bodansky, Sachs, and Steinberger, Phys. Rev. **93**, 1367 (1954).

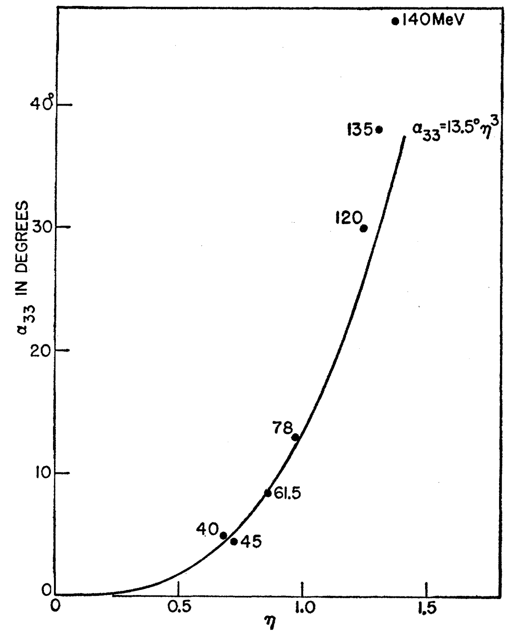

FIG. 1. α_{33} in degrees *vs* η of the pion in the center-of-mass system. Only the first 4 points were used in making the fit.

At the lowest energy the phase shifts are least well specified. For example, the paper of Roberts and Tinlot[11] quotes a preliminary Fermi-type analysis of H. P. Noyes: $\alpha_{33}=4.9°$, $\alpha_3=-3.7°$, and $\alpha_1=8.3°$. Bethe and de Hoffmann,[6] in section 33 of their forthcoming book, give $\alpha_{33}=4.5°$, $\alpha_3=-5°$, $\alpha_1=4.4°$ for the same Fermi-type solution to the 40-Mev data. Both these solutions give values less than 1.7° for the other *p*-wave phase shifts. The main explanation for the differences in these two solutions is the room available for personal judgments in handling the various 40-Mev data. In our analysis we have always used the average of these two solutions for the 40-Mev phase shifts.

These four values of α_{33}/η^3 are given in Table I. The average gives as our best fit:

$$\alpha_{33}=0.235\eta^3. \qquad (2)$$

The standard deviation due to the spread of the four values used is only 4.7 percent. This supports our choice of η^3 as the energy dependence since each of these values of α_{33} is probably only determined within limits of 20 percent. Equation (2) and the experimental values of α_{33} up to 140 Mev are plotted in Fig. 1.

s Waves

As in the above case, α_3 is determined at 40, 45, 61.5, and 78 Mev. α_1 is determined at 40 and 61.5 Mev with large errors. More information about α_1 can be obtained from low-energy measurements of the total charge-exchange cross section and the total elastic π^- cross section in the energy region below 40 Mev. In this energy region the *p*-wave phase shifts become

smaller than the s-wave phase shifts and the effects of α_{31}, α_{13}, and α_{11} have been neglected as is done in Eqs. (3) and (4).

Assuming that $\alpha_{31}=\alpha_{13}=\alpha_{11}=0$ and that $\sin\alpha\approx\alpha$, we have

$$\frac{d\sigma}{d\omega}(\pi^-,\pi^0)=\frac{2\bar{\lambda}^2}{9}\frac{v_0}{v_-}\times[(\alpha_3-\alpha_1+2\alpha_{33}\cos\chi)^2+\alpha_{33}^2\sin^2\chi], \quad (3)$$

$$\frac{d\sigma}{d\omega}(\pi^-,\pi^-)=\bar{\lambda}^2\left[\left(\frac{2\alpha_1+\alpha_3}{3}+\frac{2\alpha_{33}}{3}\cos\chi+\frac{1}{137\beta_\pi(1-\cos\chi)}\right)^2+\frac{\alpha_{33}^2}{9}\sin^2\chi\right], \quad (4)$$

where $\beta_\pi c=$ initial velocity of pion in lab system.[14] The total charge-exchange cross section at zero energy can be inferred indirectly by using the Panofsky effect[15] and photoproduction results. This is done in the Appendix and gives $(\alpha_1-\alpha_3)=0.21\eta$. Because of various uncertainties, this evaluation should be considered less reliable than the direct experiments. Also the total charge-exchange cross section has been measured at 20 Mev and 30 Mev by Spry.[9] If the values for α_{33} given by Eq. (2) are used in Eq. (3), and an additional correction made for the process $(\pi^-+p\rightarrow\gamma+n)$, Spry's results give $(\alpha_1-\alpha_3)=0.25\eta$ at 20 Mev and 0.26η at 30 Mev.

Likewise Eqs. (2) and (4) give a determination of $(2\alpha_1+\alpha_3)$ at 26 Mev based on the results of the preceding paper.[10] Preliminary results of total π^- elastic cross section at an average energy of about 5 Mev using the diffusion cloud-chamber technique were reported by Lederman[7] at the 1954 Rochester Conference. Since these data have not been presented in a final form, they will not be used in making the best-fit determination. However, the predictions from our best-fit phase shifts compare favorably with Lederman's data. This also holds for the more extensive 12-Mev experiments[16] now in progress.

A least-squares analysis is made to all these data (shown in Table I) which gives the best-fit values of α_1 and α_3 assuming the theoretical forms $\alpha_1=a\eta$ and $\alpha_3=b\eta$. In the least-squares method assumptions must be made about the errors of each of the pieces of data. We have assumed the 40 Mev and above data to have the same relative errors, and the lower energy data to have relative errors twice this amount.

The results of this calculation are

$$\alpha_3=-0.11\eta, \quad \alpha_1=0.16\eta. \quad (5)$$

These curves are plotted in Fig. 2 along with the data

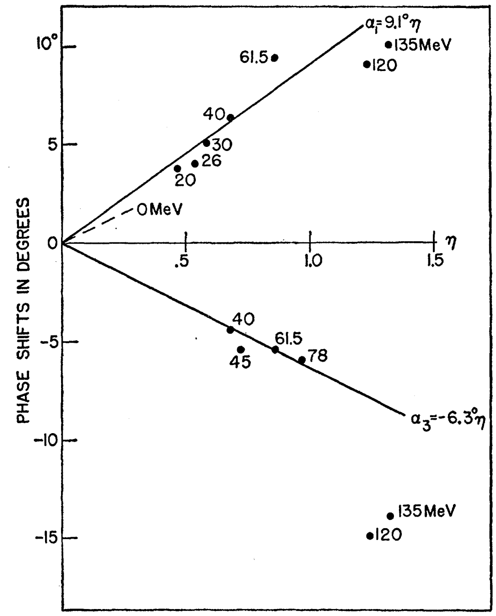

FIG. 2. α_1 and α_3 *vs* η. The 0 Mev slope and the 20, 26, and 30 Mev values of α_1 are obtained from experiments when α_3 is assumed $=-0.11\eta$.

of Table I. The values of $(a-b)$ and $(2a+b)$ are shown by plotting the α_1 obtained when using $\alpha_3=-0.11\eta$. The zero-energy value of $(\alpha_1-\alpha_3)=0.21\eta$ is expressed by a dotted line showing the predicted slope of α_1 when using $\alpha_3=-0.11\eta$. It is encouraging that all phase shifts and phase-shift combinations below 80 Mev fit these curves within their experimental errors. If the phase shifts of Eq. (5) are inserted in Eq. (4) and integrated from 60° to 180° at 5 Mev one gets a π^- elastic cross section of 6.5 mb. The preliminary Columbia cloud chamber results[3] are 4 events in 610 g/cm^2 of hydrogen which is a cross section of 10.9 mb. This value does not agree very well with the phase-shift energy dependence proposed by others.[4,5] For example, sets I and III of Noyes and Woodruff[5] predict 25 mb for 5 Mev. As mentioned earlier, the Columbia cloud chamber 12-Mev experiments, which involve about $4\frac{1}{2}$ times more track, so far show no sign of disagreement with our choice of phase shifts.

Although the Rochester 40-Mev phase shifts fit in fairly well with this scheme, the π^- transmission experiment[17] might seem out of line. Subtracting the total charge-exchange cross section from the transmission cross section gives (5 ± 2.5) mb (using their errors). This still includes the contribution from $(\pi^-+p\rightarrow\gamma+n)$. Assuming an energy-independent matrix element over the region of photon energy from 130 to 160 Mev, this contribution can be evaluated to be 1.8 mb using our phase shifts and the Panofsky effect.[15] This leaves (3.2 ± 2.5) mb for elastic scattering between 55° and 180°. Our values of α_{33}, α_3, and α_1 give 1.7 mb when the

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[17] C. E. Angel and J. P. Perry, Phys. Rev. **92**, 835 (1953).

other p-wave phase shifts are neglected. This may no longer be justified at 40 Mev.

CONCLUSIONS

All possible data relevant to the low-energy dependence of the phase shifts α_{33}, α_3, and α_1 can be fitted within experimental errors by $\alpha_{33}=0.235\eta^3$, $\alpha_3=-0.11\eta$, and $\alpha_1=0.16\eta$. Furthermore, potential models which make α_3 approach zero in the region $\eta=0.5$ do not give any better fit and actually tend to contradict the 5-Mev and 26-Mev total elastic π^- cross sections. If α_3 is to be zero at 26 Mev, the largest permissible value for α_1 within the errors of the experiment is 3.7°. This is considerably lower than the value such fits give for α_1. As already pointed out, the phase shifts given by the model of Noyes and Woodruff predict a 5-Mev cross section considerably larger than observed. Also their phase shifts at 26 Mev predict 8 mb as opposed to the observed (1.15 ± 0.6) mb.[10] The behavior of α_3 at $\eta=0.5$ can be settled once and for all by a measurement of the total (π^+-p) cross section at 20 Mev. This experiment is now in progress using the techniques described in the preceding paper.[10] This cross section for angles greater than 60° should be 5.5 mb if our phase shifts are used. If α_3 were zero or positive, then this cross section would be less than 1.1 mb.

The biggest discrepancy at present to our best fit is the zero-energy prediction of $(\alpha_1-\alpha_3)=0.21\eta$ using Panofsky effect and photoproduction data. Our phase shifts give $(\alpha_1-\alpha_3)=0.27\eta$. This evaluation could be made independent of the photoproduction links by a direct measurement of $\pi^-+p\rightarrow\gamma+n$. Conversely, such a measurement would help explain what happens in photoproduction on deuterium since the photoproduction on free neutrons would then be established by detailed balancing.

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APPENDIX

Zero-Energy Charge Exchange

The value of $(\alpha_1-\alpha_3)/\eta$ at zero energy can be obtained indirectly by making use of several other experiments and making assumptions about their interpretation. One of these experiments, photoproduction on deuterium, has rather large errors, at least in the sense that independent measurements have given different numbers. Of the several assumptions involved in the reasoning, perhaps the weakest is that one can obtain the threshold photoproduction on free neutrons $(\gamma+n\rightarrow\pi^-+p)$ by multiplying the threshold cross section $(\gamma+p\rightarrow\pi^++n)$ by the π^- to π^+ ratio from threshold photons on deuterium. We feel at least one should make a Coulomb correction due to the presence of the extra proton in $(\gamma+d\rightarrow\pi^-+p+p)$. A reasonable model is to assume that in π^- production the photon has interacted only with the neutron in the deuteron. This neutron then becomes a proton with a rather small recoil velocity when the pion is produced. Thus it will also have a fairly small relative velocity (compared to the total phase space available) with respect to the other proton, and its wave function in the final state will be accordingly suppressed as compared to the π^+ production case. We find that our knowledge of the structure of the proton and deuteron is insufficient to make a precise calculation of this effect. We estimate a 10 to 30 percent effect. A suppression greater than 30 percent is obtained when $2\pi\alpha/(e^{2\pi\alpha}-1)$ is used for the Coulomb penetration factor, where $\alpha=e^2/\hbar v$.[18] We will use a 20 percent suppression as an educated guess. The latest results obtained by nuclear emulsions exposed at the University of Illinois[19] give an average π^- to π^+ ratio of 1.53 ± 0.1 in the photon energy region of 180 Mev. The Coulomb correction makes this ratio 1.9. This emulsion result of 1.53 is somewhat higher than the recent (May 1954) California Institute of Technology result of 1.3 at 200 Mev.[20]

Bernardini and Goldwasser[21] have made an analysis of the s-wave component of the reaction $(\gamma+p\rightarrow\pi^++n)$. Their value is $1.55\times10^{-28}\eta$ cm^2. Multiplying by 1.9 gives $\sigma(\gamma+n\rightarrow\pi^-+p)=2.94\times10^{-28}\eta$ cm^2 at threshold. Detailed balancing gives for very low-energy pions

$$\sigma(\pi^-+p\rightarrow\gamma+n)=2\left(\frac{p_\gamma}{p_\pi}\right)^2\sigma(\gamma+n\rightarrow\pi^-+p)$$
$$=\frac{1}{\beta_{\pi^-}}(5.1\times10^{-28}).$$

From the Panofsky effect[15] branching ratio of 0.94, we have

$$\sigma(\pi^-+p\rightarrow\pi^0+n)=\frac{0.94}{\beta_{\pi^-}}\times5.1\times10^{-28}.$$

Now making the final assumption that all violations of isotopic spin conservation for pion kinetic energies much lower than the π^--π^0 mass difference are explicitly taken into account in Eq. (3), we obtain $(\alpha_1-\alpha_3)=0.21\eta$. The more recent Columbia result for the pion mass difference is used.[22]

We feel that a 25 percent error should be assigned to this determination of $(\alpha_1-\alpha_3)$ at zero energy due to the experimental uncertainties alone. In addition there are unknown uncertainties in the assumptions made.

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