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⁶ A review article on the theory of Coulomb excitation, containing also an analysis of experimental data, is being prepared in collaboration with A. Bohr, T. Huus, B. Mottelson, and C. Zupančič.

Rf-Induced Transitions of Nuclear Spins at the Electronic Resonance Frequency*

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A REASONABLE explanation is offered to account for the observed increased transition rate of nuclear spins at the electronic spin resonance frequency, as reported in the preceding Letter by Honig. It is known from the work of Bloembergen¹ that an important mechanism of nuclear spin-lattice relaxation in solids is through the coupling of the nuclear magnetic moments to the electron magnetic moments of paramagnetic impurities. The power spectrum associated with the relaxation of the electron moment determines the rate at which nuclear spins are flipped. If the electron spin-lattice relaxation time is very long, even a weak rf field at the electron resonance frequency will have a major effect on the power spectrum. In this way nuclear flips are enhanced when the electron resonance is observed."

The Hamiltonian for a single arsenic site, when the dipolar interactions are neglected, is

$$\mathcal{H} = a\mathbf{I} \cdot \mathbf{S} + H_0(g_e\mu_B S_z - g_N\mu_N I_z) + H_x e^{i\omega t} g_e\mu_B S_x - H_x e^{i\omega t} g_N\mu_N I_x, \quad (1)$$

where the nuclear moment is considered to be positive. For the experimental magnetic field of 3000 oersteds the use of I_z and S_z as good quantum numbers gives the correct energy levels within 2 to 3 percent. The off-diagonal interaction term $\frac{1}{2}aS^+I^-$ can be made to induce nuclear transitions to first order in the sense of time-dependent perturbation theory, by considering the rf-induced transitions of S^+ as a source of power. This is similar to the argument made by Bloembergen, Purcell, and Pound² in which they use the time dependence of the magnetic dipolar interactions caused by the rotational and translational motion of the spins to obtain the power necessary for a nuclear spin relaxation. The correlation function for $S^+(t)$ is assumed, following

Bloembergen,¹ to be proportional to

$$|S^+|^2 e^{i\omega_0 t} \exp(-t/T_{rf}^{el}),$$

where T_{rf}^{el} is the electronic transition time given as²

$$T_{rf}^{el} = (\frac{1}{2}\mu_B^2 H_1^2 T_2^{el}/\hbar^2)^{-1}, \quad (2)$$

and T_2^{el} is the half-maximum value of the electronic line shape function.

Corresponding to this correlation function is the power spectrum,

$$|S^+|^2 \frac{2T_{rf}^{el}/\pi}{1 + 4(T_{rf}^{el})^2(\omega - \omega_0)^2}, \quad (3)$$

where ω_0 is the electronic Larmor frequency. Substituting into the standard formula for transition probability,³ we have for the nuclear transition time,

$$T_{nuc} = \left[\frac{2\pi}{\hbar^2} \left| \frac{a}{2} S^+ I^- \right|^2 \frac{2T_{rf}^{el}/\pi}{1 + 4(T_{rf}^{el})^2(\omega_0' - \omega_0)^2} \right]^{-1}. \quad (4)$$

Note that the component of the power spectrum is taken at the nuclear Larmor frequency ω_0' .

Next we will show how the same result can be obtained by second-order perturbation theory. Here we consider the transition between the initial spin state, $m_S = \frac{1}{2}$, $m_I = \frac{3}{2}$, and the final state, $m_S = \frac{1}{2}$, $m_I = \frac{1}{2}$. If standard perturbation theory is used, the expansion coefficient for the final state satisfies the equation,

$$\dot{a}_f^{(2)} = (i\hbar)^{-1} \sum_n H_{fn} a_n^{(1)} e^{i\omega_{fn} t}, \quad (5)$$

where

$$H = H_1 g_e \mu_B S_x e^{i\omega t} + \frac{1}{2} a S^+ I^- + \frac{1}{2} a S^- I^+,$$

and

$$a_n^{(1)} = (i\hbar)^{-1} \left\{ \langle n | H_1 g_e \mu_B S_x | i \rangle \times \left[\frac{\exp[i(\omega + \omega_{ni}^{(1)} + i/2T_{rf}^{el})t] - 1}{i(\omega + \omega_{ni}^{(1)} + i/2T_{rf}^{el})} \right] + \langle n | \frac{1}{2} a S^+ I^- + \frac{1}{2} a S^- I^+ | i \rangle \times \left[\frac{\exp[i(\omega_{ni}^{(2)} + i/2T_{rf}^{el})t] - 1}{i(\omega_{ni}^{(2)} + i/2T_{rf}^{el})} \right] \right\}.$$

It is to be noted that we have represented the initial state by a decaying exponential.⁴ The only intermediate state which gives a nonzero contribution is $m_S = -\frac{1}{2}$, $m_I = \frac{3}{2}$, and thus only the matrix element containing H_1 contributes to $a_n^{(1)}$. Therefore we set $\omega_{ni}^{(1)} = -\omega_0$ and $\omega_{fn} = \omega_0 - \omega_0'$. The transition probability is wanted at the electronic resonance frequency, so setting $\omega = \omega_0$ the expansion coefficient then satisfies the equation

$$\dot{a}_f^{(2)} = - (i\hbar)^{-2} (2T_{rf}^{el}) \langle f | \frac{1}{2} a S^+ I^- | n \rangle \langle n | H_1 g_e \mu_B S_x | i \rangle \times [\exp(-t/2T_{rf}^{el} + i\omega_0 t - i\omega_0' t) - \exp(i\omega_0 t - i\omega_0' t)]. \quad (6)$$

The first two terms can be made to conserve energy between the initial and final states by choosing the Fourier component of $\exp(-t/2T_{rf}^{el}) \exp(i\omega_0 t)$ at the frequency ω_0' . Formally carrying this out gives just the transition probability in Eq. (4).

By taking the local field at the electrons as due to the electronic dipoles on the other arsenics, a $T_2^{e1} \approx 4 \times 10^{-6}$ sec is found, yielding $T_{rf}^{e1} \approx \frac{1}{2} \times 10^{-4}$ from Eq. (2) (if one takes $H_1 = 10^{-2}$ oersted). The hyperfine splitting of 73 oersteds gives a value of the hyperfine parameter a equal to $\frac{3}{2} \times 10^{-18}$ erg. Then, with an applied frequency of 9×10^9 cps and an rf field of 10^{-2} oersted, the value for the rf-induced nuclear transition time becomes, from Eq. (4),

$$T_{nuc} = [(6 \times 10^{54})(2 \times 10^{-36})(4 \times 10^{-18}/3)]^{-1} \approx 6 \times 10^{-2} \text{ sec.}$$

This is to be compared to the nuclear spin relaxation time of 16 sec.

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Results of a Phase Shift Calculation of High-Energy Electron Scattering*

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USING the phase shift analysis described previously,¹ an attempt has been made to fit the experimental data of Hofstadter *et al.*² on the elastic scattering of electrons on gold at energies of 84, 126, 154, and 183 Mev. Only scattering from a static spherically symmetric charge distribution is considered; all other interactions are neglected. The one-parameter charge distributions such as uniform, Gaussian, and exponential considered earlier¹ are clearly ruled out by the experimental data because of both the shape of the individual cross sections and their energy dependence. The present results are for "smoothed uniform" distributions, characterized by a central region of almost constant density and a surface region in which the density drops to zero. Of the two parameters required to identify a distribution of this type (a radius and a surface thickness), it turns out that the surface thickness is small enough compared with the electron's de Broglie wavelength λ that the detailed surface structure is unimportant; this is not true at higher energies, of course.

We define³ a radius c by

$$c = \int_0^\infty \rho(r) dr / \rho(0), \quad (1)$$

and a surface thickness s by

$$s^2 = -4 \int_0^\infty (r-c)^2 (d\rho/dr) dr / \rho(0). \quad (2)$$

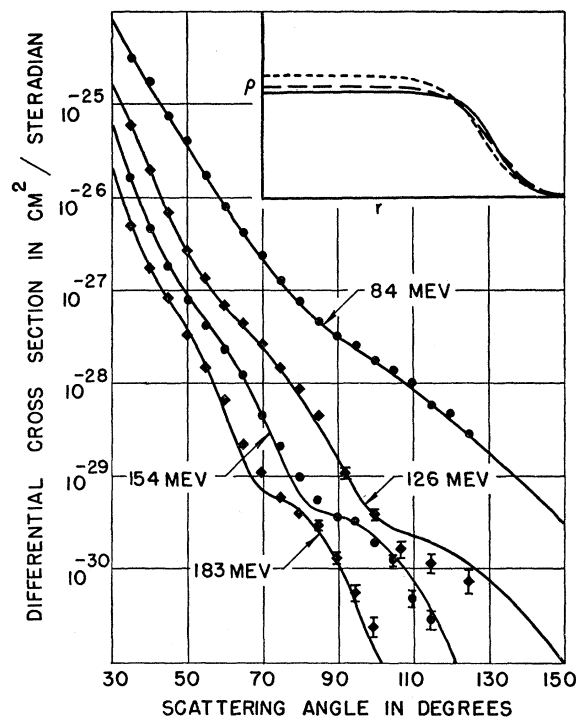


FIG. 1. Cross sections at 84, 126, 154, and 183 Mev for the elastic scattering of electrons by gold, using the charge distribution (4) with the parameters $K=2.20$, $c=6.63$ ($r_0=1.20$, $s=1.65$). (Distances are in units of 10^{-13} cm.) Inset is the charge distribution (full line), together with that for Fig. 2 (dashed line) and Fig. 3 (dotted line). The experimental values are of Hofstadter *et al.* (reference 2).

To relative order $(s/c)^4$, the root-mean-square radius is then given by

$$\begin{aligned} \langle r^2 \rangle^{\frac{1}{2}} &= \left(\frac{3}{5} \right)^{\frac{1}{2}} c \left(\frac{1 + (5/2)(s/c)^2}{1 + (3/4)(s/c)^2} \right)^{\frac{1}{2}} \\ &= (3/5)^{\frac{1}{2}} r_0 A^{\frac{1}{3}} \times 10^{-13} \text{ cm.} \end{aligned}$$

In the results presented here, we have used for the charge density the form

$$\rho = \rho_0 [1 + e^{K(r-c)}]^{-1}. \quad (4)$$

For $e^{Kc} \gg 1$ the c of Eq. (4) is the same as that defined by Eq. (1). The surface thickness s is $2\pi/\sqrt{3}K$ ($=3.63/K$), which is slightly smaller than the previously defined "smoothing distance"¹ ($=4.40/K$). To confirm theoretical arguments that the cross section depends only on c and s , and is independent of the surface shape,³ calculations at 183 Mev have been carried out for several shapes having the same value of s and c as those of Fig. 1. The most extreme of these has a surface which varies linearly with r . For angles less than 105° the cross section for all of these shapes agreed to within 10 percent; by a slight alteration of parameters to allow for the terms in $(s/c)^4$ this agreement could be improved still further.

In Figs. 1-3 we compare the results of the phase shift calculations with the experimental data, which have