Hexagonal Close-Packed Lattice

We shall again follow exactly the notation of Herring.[2] The first Brillouin zone and the various kinds of symmetry point are shown in Fig. 4.

The double groups of U, Σ, R, and T are trivial; there being only one extra representation which has zero character for everything except the identity ($\chi(\epsilon)=-\chi(\bar{\epsilon})=2$) and translations. This representation of R, however, always occurs twice because of time reversal so the states are all fourfold degenerate on R. The only other symmetry position S has character Table XVI.

Depending on the limiting position of the point S we should also consider primary translations in the group of S. This would give the same results as in Herring's single tables and they are omitted here.

The compatibility relations for the lines Δ, P and S are nontrivial: $\Gamma_i^{\pm}\rightarrow\Delta_i(\Gamma)$ ($i=7, 8, 9$). $A_4\rightarrow\Delta_9(A)$, $A_5\rightarrow\Delta_9(A)$, $A_6\rightarrow\Delta_7(A)+\Delta_8(A)$. H_4+H_6, $H_5+H_7\rightarrow P_4(H)+P_5(H)$; H_8, $H_9\rightarrow P_6(H)$. K_7, $K_8\rightarrow P_6(K)$; $K_9\rightarrow P_4(K)+P_5(K)$. A_4+A_5, $A_6\rightarrow S_2(A)+S_3(A)+S_4(A)+S_5(A)$. $H_4\rightarrow S_2(H)$, $H_5\rightarrow S_3(H)$, $H_6\rightarrow S_5(H)$. $H_7\rightarrow S_4(H)$, $H_8\rightarrow S_3(H)+S_4(H)$, $H_9\rightarrow S_2(H)+S_5(H)$.

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Magnetization of Tin at the Superconducting Transition

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Measurements have been made of the steady-state magnetization of a tin cylinder in the presence of an external magnetic field and with an externally supplied current at the transition to superconduction. In accord with earlier work in Germany, a longitudinal flux in excess of that caused by the external field is observed for certain combinations of external field and current. The dependence of the effect on the external parameters is given and it is shown that the metal is not in the pure superconducting state. No satisfactory theory for the effect has been found, but some numerical relationships have been computed.

INTRODUCTION

SUPERCONDUCTION in metals is an electronic state characterized by zero electrical field and by zero magnetic induction. The independent variables which may determine whether a metal is in the superconducting state, if it becomes one at all, are the temperature T, the magnetic field B, and finally because a current in the specimen may produce its own field, we must consider the current I, as a variable. At sufficiently low values of each of these variables, the metal may be in the superconducting state; just how low is a characteristic of each superconductor.[1] In this research we dealt with very pure monocrystalline tin made in the form of a cylinder and we were concerned with some unusual properties just at the transition into and out of the superconducting state. At a temperature just below the zero field transition temperature for ordinary tin, 3.735°K, and in a small externally created magnetic field of 1 to 4×10^{-4} weber/sq meter applied along the long axis of the cylinder, a steady current of from 1 to 13 amperes was allowed to flow along the cylinder and we measured the magnetic flux content of the tin specimen. The work of Steiner[2] and of Meissner, Schmeissner, and Meissner[3] has shown that under these conditions the longitudinal flux through a conductor, instead of decreasing monotonically to zero at the transition to superconduction, first shows an *increase* provided one uses certain conditions of a large current, a small external magnetic field, and a temperature such that with the currents and fields used the super to normal transition may be effected. The results which we now report are in accord with the above authors and represent a continuation of the preliminary studies by Mendelssohn, Squire, and Teasdale[4] and by Teasdale and Rorschach.[5] Dr. Mendelssohn suggested these studies and indicated the method of measurement which we have continued to use since his work with us here in this laboratory. The present apparatus has

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[2] K. Steiner and H. Schoeneck, Physik. Z. **44**, 346 (1943); also K. Steiner, Z. Naturforsch. **4a**, 271 (1949).

[3] Meissner, Schmeissner, and Meissner, Z. Physik **130**, 521, 529 (1951); **132**, 529 (1952).

[4] Mendelssohn, Squire, and Teasdale, Phys. Rev. **87**, 589 (1952).

[5] T. Teasdale and H. E. Rorschach, Jr., Phys. Rev. **90**, 709 (1953).

allowed us to obtain quantitative results and to overcome many, if not all, difficulties which might interfere with the interpretation of the experimental results.

EXPERIMENTAL

The apparatus used in the present studies is shown schematically in Fig. 1. The method used to study the flux content of the tin cylinder specimen was to move a coil C from its position around the tin upward to a position around a copper cylinder. The difference in magnetic flux induced a current in a ballistic-type galvanometer. The copper, being a normal metal, was used as a reference flux standard and the superconducting Sn gave the characteristic zero flux behavior. Silk threads W were used to link the test coil with the iron slug I which could be moved with a strong solenoid (not shown) placed outside the gas-tight glass envelope housing the iron slug. The detecting coil C, made of 1900 turns of No. 40 copper wire wound upon a Lucite form, was fitted about the tin-copper specimen with just enough clearance to move easily. The exact position of the coil was controlled by the stops L which were made of Lucite.

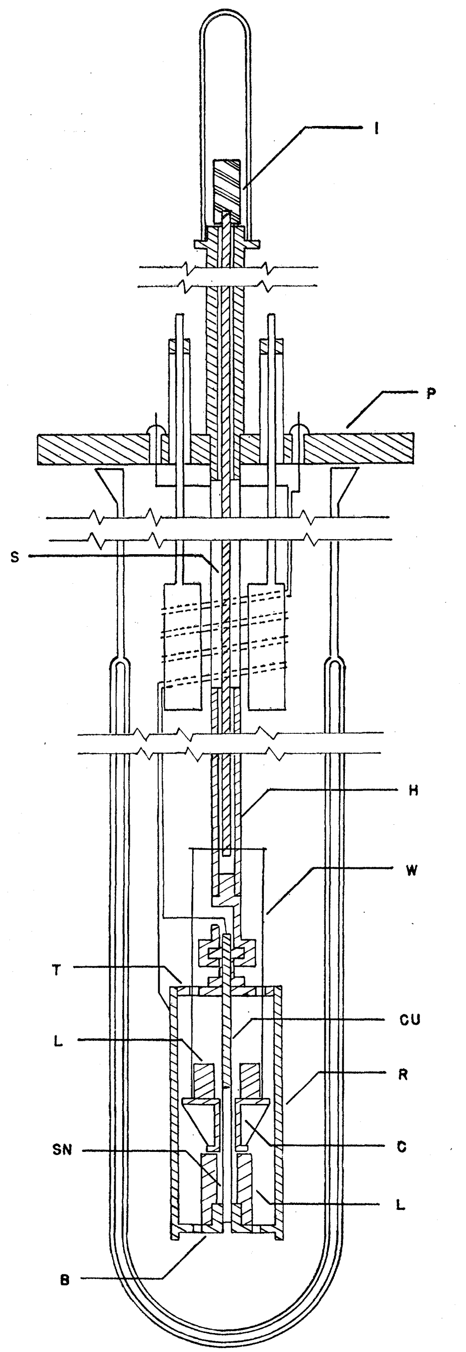

FIG. 1. Schematic diagram of experimental apparatus.

The direct current leads to the specimen are shown in Fig. 1 entering the top of the cryostat through gas-tight seals and then spiraling around a vessel which contained liquid nitrogen. Below that point and extending on down into the liquid helium vessel the current-carrying wires were of smaller diameter, such that neither heat leaks nor Joule heating effects were excessive. The current could be sent through the specimen in either direction by a reversing switch in the power supply (large Edison cells). It will be noted in Fig. 1 that the return lead is by the concentric copper shell R, and we must add that this shell was slotted so that liquid helium was always in direct contact with the specimen cylinder. Not shown in Fig. 1 are the external liquid nitrogen Dewar and the external wire-wound solenoid for producing small homogeneous fields (1 to 5×10^{-4} weber/m^2—i.e., 1 to 5 gauss) whose direction was along the specimen cylinder. The external magnetic field could also be reversed in direction. The arrangement in Fig. 1 allows one to measure the flux in the tin compared to that in the copper at any desired combination of values of current, external field, and temperature.

The tin specimen was from Baker Chemical Company 99.99-percent pure laboratory stock. The molten metal was drawn up into a glass tube by vacuum and then an electric furnace was slowly (6 cm/hr) removed in the well-known manner for growing single crystals. The glass tube was gently removed and the tin specimen was electrolytically polished with perchloric acid so that one could observe directly the crystal faces. The final cut shape was a cylinder 7.2 cm long and 0.615 cm in diameter, composed of a large single crystal in the central portion with a few small crystals at each cut end. The tin cylinder was butted to a copper cylinder of the same cross section and the two welded together.

RESULTS

A given cycle of measurements was taken as follows: the external parameters, T, B, and I were adjusted to the desired starting values (say 3.694°K, 1.0×10^{-4} weber/m^2, and 2.5 amp) and the detecting coil was

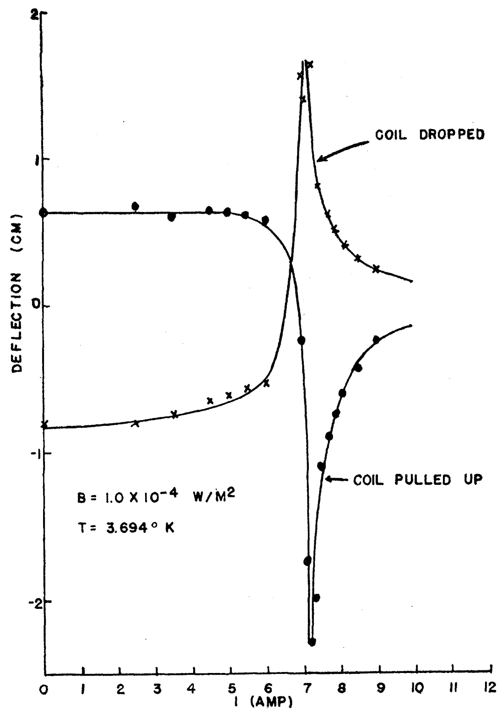

FIG. 2. Transition at constant temperature and field showing effect of direction of coil motion.

then raised quickly from its position about the tin to its position about the copper. The galvanometer deflection was noted and plotted as on Fig. 2. The detecting coil was then dropped and an opposite deflection noted. One external parameter, such as the current I, was slightly changed to a new steady-state value (say 3.0 amp) and the next point taken. In this way the current in the tin-copper cylinder was increased in small steps until the tin entered the normal state. Figure 2 clearly shows that the deflections for current values in the region of 7 amperes are such as to denote a greater flux in the tin than in the copper. The current in the cylinder was lowered, stepwise, and points taken as the tin was put back into the completely superconducting state. The plot of Fig. 2 is completely reversible. The existence of the increased magnetic flux content in the tin does not depend on whether the directions of I and B are parallel or antiparallel. Alternatively, the current could be set at a constant value (less then critical) and the transition made by changing the external longitudinal field. Likewise the temperature can be used[5] as the parameter moved across the phase transition. The effect is always observed with proper values of current and field.

The magnetic field at the surface of the cylinder caused by the current I is only 0.651×10^{-4} weber/m² per ampere. Thus, if there were enough heating to cause the temperature in the tin to be 0.01°K higher than the vapor pressure over the helium indicated, we would expect the critical current actually to be reached for a value 2.3 amperes lower than anticipated. This is illustrated in Fig. 3 showing another set of values of galvanometer deflection *vs* current with the expected critical current I_c marked at about 11 amperes for $T=3.680$°K and $B=3.0\times10^{-4}$ weber/m². The destruction of the Meissner effect, $B=0$, is observed at 9 amperes, and by 10 amperes we have the additional magnetic field produced by the circular motion of the current so that surely the tin is in the normal state at that point. The two curves in Fig. 3 are for field and current parallel and then antiparallel and their slight difference is considered insignificant. The curves are reversible going in and out of the superconducting state.

DISCUSSION

The detecting coil certainly informs us that the magnetic flux within the tin specimen is greater than that of the copper for certain values of I, B, and T. In the pure superconducting state the tin contains no flux. At temperatures of 4.2°K, well above the superconducting transition, the tin and copper are identical in flux content so far as the detecting coil can observe. The deflection of the galvanometer tells us directly and quantitatively the flux change. The increased flux contant in the tin over the copper is a maximum for a certain current I, and for fixed values of external field B, and of temperature T. Suppose at this maximum flux content we were to assume the induction

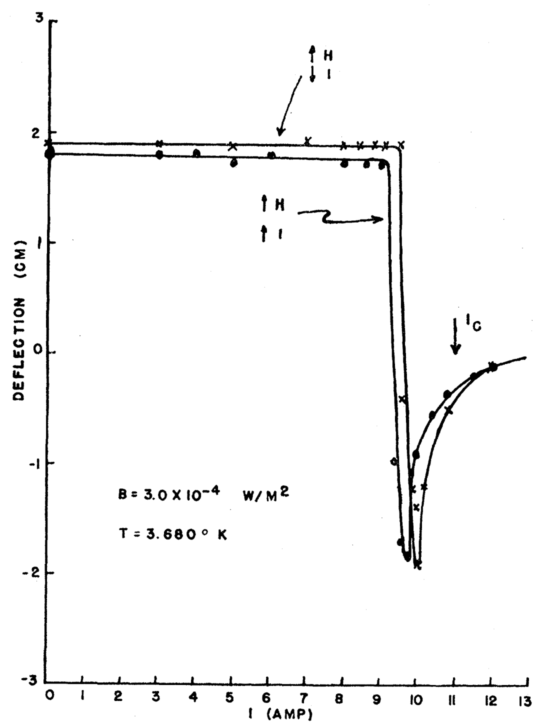

FIG. 3. Transition at constant temperature and field showing effect of current-field parallelism.

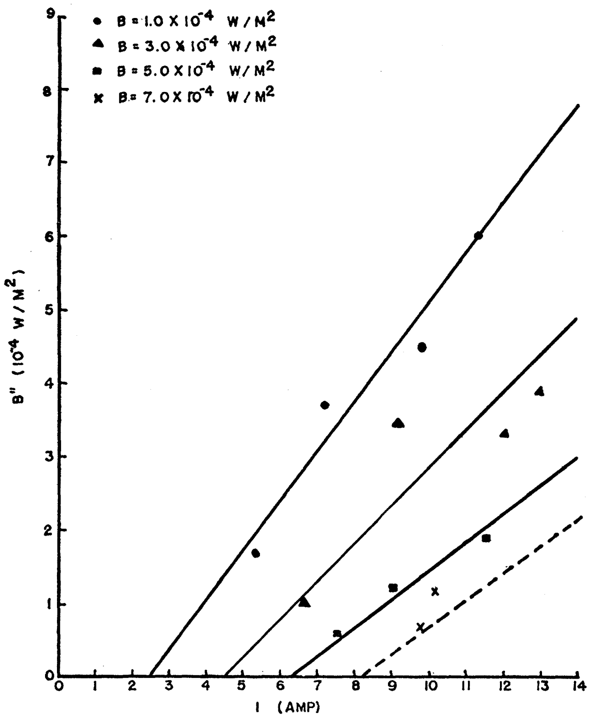

FIG. 4. Field produced in tin at maximum of "paramagnetic effect" as a function of current in specimen and for several fields.

vector B' to be constant over the area of cross section of the tin and assert that B' was the resultant of the external field B and a field B'' because of a circular component of the current in the specimen

$$B' = B + B''. \tag{1}$$

We know the value of B'' because the flux produced by it is just the extra flux observed by the test coil. The field B is also in the copper and we assume the copper does not produce a longitudinal field with its current. The field B'' *vs* the current I is plotted in Fig. 4 for a series of externally applied fields B. These are all taken from the maximum deflection on a curve such as Figs. 2 and 3. Thus, the more the current the more the field it can produce. The effect of the magnitude of the external field B is such as to effectively diminish the size of B'' and Fig. 5 shows this for a series of currents I. But this is on the basis that the galvanometer deflection being smaller indicated a smaller flux increase (at maximum). The flux is given by

$$\phi = B''A + BA. \tag{2}$$

The value of B'' could appear too small because we have taken the flux area to be constant, i.e., that of the cylinder cross section A.

Finally, in Fig. 6 we show the threshold curve in I and B to indicate that for any effect of flux increase to be observed at all, we must have currents and external fields in the so-called "paramagnetic" region. The experimental points were taken at successively lower temperatures such that higher currents and fields could be used. The equation of the line is

$$I = I_g + \gamma dB, \tag{3}$$

where $I_g = 1.2$ amperes is the intercept for the field $B = 0$, where $\gamma = 1.7 \times 10^6$ meters/henry, and d is the diameter of the cylinder in meters. This result is in accord with Meissner *et al.*[3]

The flux increase observed seems to arise from the circular component of the current and we would say that Meissner, Schmeissner, and Meissner[3] had proved this with an experiment in which they cut a slot in their cylinder thereby making such circular currents impossible and thereby destroying the effect. In Fig. 7 we show the cylinder with the field produced by the external solenoid B and the field produced by a straight through current, B_φ. The force on an electron moving with velocity v_f along the Z axis will tend to push it inward along the radius r. The velocity so acquired will then cause ($\mathbf{F} = e\mathbf{v}_r \times \mathbf{B}$) the electron to be pushed with a force orthogonal to these and hence might produce a helical flight path for the electron. But this is not too convincing and we must look for a more complete theory.

How many turns must a certain current make per unit length in order to produce the observed maximum flux? In our work (Fig. 2) a current of 7.2 amperes

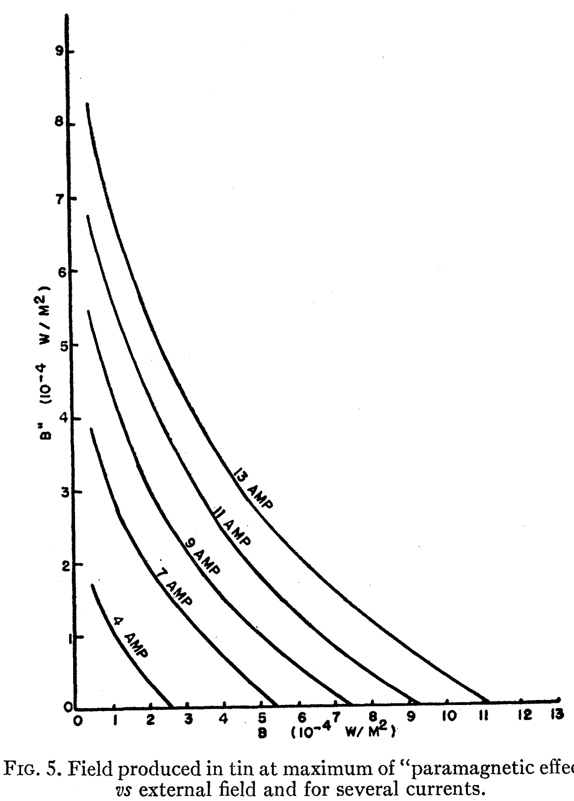

FIG. 5. Field produced in tin at maximum of "paramagnetic effect" *vs* external field and for several currents.

produced a $B''=3.7\times10^{-4}$ weber/m^2 by making 0.363 turns/cm, i.e., a total of 2.43 turns in passing through the tin specimen. The value of B'' is assumed constant over the specimen cross section to make up the observed flux. Likewise, we say the current is all on the surface of the tin. Now for a free electron the cyclotron angular velocity is $\omega=(e/m)B'$. So in the above example, with $B'=4.7\times10^{-4}$ weber/m^2, we have $\omega=8.3\times10^7$ rad/sec$=1.32\times10^7$ rev/sec. It will be recalled that in this example the external field B was 1×10^{-4} weber/m^2. The number of revolutions made by an electron traveling from one end of the tin specimen to the other (6.7 cm in the apparatus of Fig. 1) then depends upon the time of flight. If the electrons suffered no collisions, corresponding to a mean free path of the length of the tin specimen, then this time could be $t=6.7/v_f$, where v_f is the velocity of the electrons at the top of the Fermi distribution. This velocity is of the order of 10^8 cm/sec so that the electrons would make about 0.9 turn. This would agree exactly with our experimental value (2.4) if we choose a slightly slower velocity for v_f. Such a mean free path is acceptable for a superconductor, but it is too large by a factor of 10^5 for tin in the normal state at 4.2°K. The orbit of the electron has a radius given by: $R=v_t/\omega$, where v_t is the tangential velocity to the circular motion. If this is 2×10^7 cm/sec, a reasonable value, then the electron flight path easily fits inside the specimen used in our work. The above crude numerical relationships are in sore need of theoretical justification.

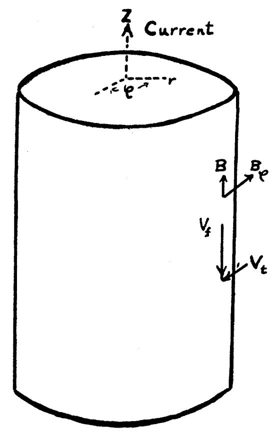

FIG. 7. The cylinder of tin showing possible velocity components of the electron in steady flight.

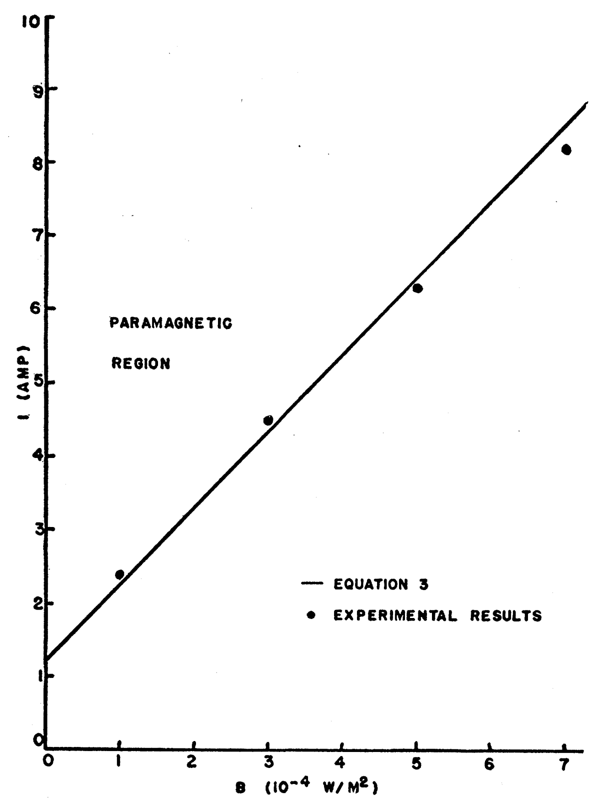

FIG. 6. Threshold for "paramagnetic effect."

CONCLUSION

The experiments reported in this paper have been carried out in such a way as to remove difficulties and objections to earlier experiments. There is certainly an interesting flux increase just as the tin specimen passes out of the pure superconducting state. There is need for a clear theoretical treatment of the effect.

The authors are indebted to Professor W. Meissner, Dr. H. Meissner, and to Dr. K. Mendelssohn for stimulating discussions. We wish also to thank several colleagues at The Rice Institute, in particular Professor W. V. Houston, for assistance. The research was supported in part by a grant from the National Science Foundation.