

# Magnetostriction and Crystal Anisotropy of Single Crystals of Hexagonal Cobalt

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Magnetostriction at saturation has been measured in various crystallographic directions in a single crystal of cobalt. Results are described by Mason's formula [see Eq. (1)] with 4 constants, which are now evaluated. Magnetization causes contractions as large as  $100 \times 10^{-6}$ , and expansions up to  $150 \times 10^{-6}$ , depending on crystallographic direction. A fractional decrease in volume, associated with domain orientation, is observed to be as large as  $26 \times 10^{-6}$ . Superposed on this contraction is a small isotropic increase in volume of  $0.6 \times 10^{-9}$  per oersted. Magnetic crystal anisotropy constants are determined and used in a calculation of the magnetostriction as dependent on the magnitude of the field applied to a disk with given demagnetizing factor. Calculation and observation agree, and show that the direction and magnitude of magnetization and of true field can be calculated in any applied field as long as it is considerably above the coercive force.

## INTRODUCTION

THE magnetostriction of cubic crystals of iron, nickel, and various alloys of these metals has been investigated and the constants determined.<sup>1,2</sup> As a result, one can calculate the expansion or contraction that occurs in any crystallographic direction when a magnetic field is applied in sufficient strength to saturate the crystal in the same or any other crystallographic direction. The dimensional changes are usually described adequately by two constants.

Mason<sup>3</sup> has derived the expression for the saturation magnetostriction of a hexagonal crystal in terms of the direction cosines of the magnetization and the change in length. In the approximation used, which contains the direction cosines to the fourth power, there are four constants, which are determined by the measurements described below. To this approximation the magnetostriction has cylindrical symmetry.

The expression for the magnetic crystal anisotropy has also been derived by Mason,<sup>3</sup> and in accordance with the 2-constant expression generally used there is cylindrical symmetry when fourth-order terms are the highest used. Hexagonal symmetry appears only when sixth-power terms are used. In cobalt these terms are negligibly small.

The magnetostriction of a single crystal of cobalt was studied in 1929 by Nishiyama,<sup>4</sup> who used a maximum field of 7000 oersteds and observed a maximum contraction of about  $30 \times 10^{-6}$ . His results indicate cylindrical symmetry, with a maximum contraction at about  $70^\circ$  to the hexagonal axis; these observations are confirmed in the present study, and are extended to 25 000 oersteds.

<sup>1</sup> R. Becker and W. Döring, *Ferromagnetismus* (Verlag Julius Springer, Berlin, 1939).

<sup>2</sup> R. M. Bozorth, *Ferromagnetism* (D. Van Nostrand Company, Inc., New York, 1951).

<sup>3</sup> W. P. Mason, *Phys. Rev.* **96**, 302 (1954).

<sup>4</sup> Z. Nishiyama, *Science Repts. Tôhoku Imp. Univ.* **18**, 341 (1929).

## EXPRESSIONS FOR $K$ AND $\lambda$

The expression for magnetostriction at saturation in a hexagonal crystal is:

$$\begin{aligned} \lambda = & \lambda_A [(\alpha_1\beta_1 + \alpha_2\beta_2)^2 - (\alpha_1\beta_1 + \alpha_2\beta_2)\alpha_3\beta_3] \\ & + \lambda_B [(1 - \alpha_3^2)(1 - \beta_3^2) - (\alpha_1\beta_1 + \alpha_2\beta_2)^2] \\ & + \lambda_C [(1 - \alpha_3^2)\beta_3^2 - (\alpha_1\beta_1 + \alpha_2\beta_2)\alpha_3\beta_3] \\ & + 4\lambda_D (\alpha_1\beta_1 + \alpha_2\beta_2)\alpha_3\beta_3. \end{aligned} \quad (1)$$

Here  $\lambda$  is the fractional change in length in the direction defined by the direction cosines  $\beta_1, \beta_2$ , and  $\beta_3$ , when the crystal is magnetized to saturation in the direction defined by  $\alpha_1, \alpha_2, \alpha_3$ . The  $z$  or 3 axis is chosen parallel to the hexagonal  $c$  axis, the  $x$  axis parallel to the crystallographic  $a$  axis and the  $y$  axis at right angles to  $x$  and  $z$ . It is assumed in accordance with the data for cobalt at room temperature that the direction of easy magnetization lies along the  $c$  axis so that  $\lambda=0$  when the magnetization is in this direction ( $\alpha_3=1$ ). The change in length is then measured from this initial state.

When  $I_s$  is parallel to the direction of measurement of  $\lambda$  (i.e.,  $\alpha_i=\beta_i$ ), Eq. (1) reduces to

$$\lambda_{II} = \lambda_A [(1 - \alpha_3^2)^2 - (1 - \alpha_3^2)\alpha_3^2] + 4\lambda_D (1 - \alpha_3^2)\alpha_3^2. \quad (2)$$

If  $\theta$  is the angle between  $I_s$  and the  $z$  axis, Eq. (2) reduces to

$$\lambda_{II} = (\frac{1}{4}\lambda_A + \frac{1}{2}\lambda_D) - \frac{1}{2}\lambda_A \cos 2\theta + (\frac{1}{4}\lambda_A - \frac{1}{2}\lambda_D) \cos 4\theta, \quad (3)$$

containing two constants. The cylindrical symmetry is here apparent.

In any plane parallel to the hexagonal axis (e.g.,  $y, z$  plane), the magnetostriction measured at right angles to the magnetization is

$$\begin{aligned} \lambda_{\perp} = & (\frac{1}{4}\lambda_A + \frac{1}{2}\lambda_C - \frac{1}{2}\lambda_D) + \frac{1}{2}\lambda_C \cos 2\theta \\ & - (\frac{1}{4}\lambda_A - \frac{1}{2}\lambda_D) \cos 4\theta. \end{aligned} \quad (4)$$

In all directions in the hexagonal ( $x, y$ ) plane  $\lambda_{II} = \lambda_A$ , and  $\lambda$  measured perpendicular to  $I_s$  is  $\lambda_{\perp}' = \lambda_B$ . In the  $y, z$  plane  $\lambda_{\perp} = \lambda_C$  for  $\theta=0$  and  $\lambda_{II} = \lambda_D$  for  $\theta=45^\circ$ . The constants are evaluated from the measurements made in this way, as described below.

According to Mason,<sup>3</sup> the magnetic anisotropy energy density is

$$E = K_1 \sin^2\theta + K_2 \sin^4\theta + K_3 \sin^6\theta + K_4 \sin^6\theta \cos 6\varphi, \quad (5)$$

where  $\theta$  is the polar and  $\varphi$  the azimuthal angle. The first two terms comprise the well-known expression, and were the only ones used in the present study.

### SPECIMENS

A single crystal of cobalt was obtained from Horizons, Inc., where it was prepared by Mr. Straughan by slow cooling of the melt.<sup>5</sup> The material from which it was grown contained as principal impurities Ni (0.3 percent), Mn (0.08 percent), and Fe (<0.1 percent).

Specimens were cut for magnetostriction and anisotropy measurements as described in Table I. Surfaces were checked to an estimated  $10'$  by using an x-ray spectrometer, and directions to about  $0.5^\circ$  by using Laue photographs. All surfaces were carefully ground and electrolytically polished until no strain was evident from observation of Laue spots.

Specimens were cut from the same lot for measurement of the elastic constants by McSkimin.<sup>6</sup>

### MEASUREMENTS

Magnetostriction was measured using strain gauges as described by Goldman.<sup>7</sup> The crystal, to which was fixed a single gauge, was placed in a 2-in. gap between the 11-in. diameter poles of an electromagnet in which

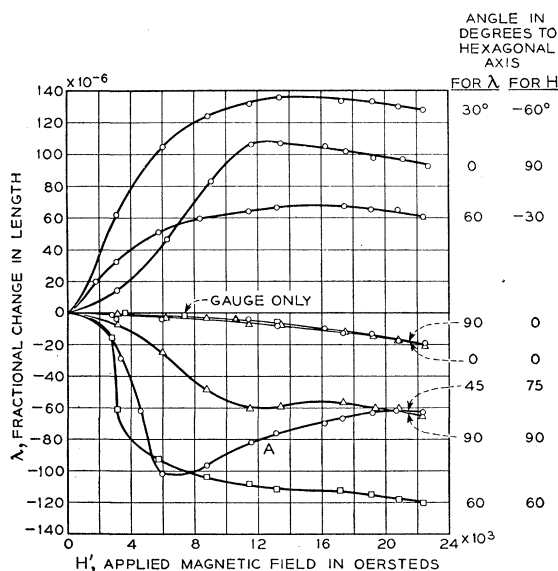


FIG. 1. Magnetostriction in (10·0) plane as dependent on applied field for various orientations of field and gauge with respect to  $z$  axis, as noted.

<sup>5</sup> A similar specimen was grown some years ago by Cioffi, Williams, and Bozorth, Phys. Rev. **51**, 1009 (1937).

<sup>6</sup> McSkimin, Williams, and Bozorth, Phys. Rev. **95**, 616 (1954).

<sup>7</sup> J. E. Goldman, Phys. Rev. **72**, 529 (1947).

a field strength of over 20 000 oersteds could be produced.

For control a second gauge from the same lot was fixed to brass, and a third was stuck to wood with Scotch tape. The apparent changes in length observed when the second and third gauges were separately subjected to a field were in agreement with each other and were used as corrections to the readings for the gauge on the cobalt crystal.

The difference between the magnetostriction of the disk used and that of an infinite sheet was calculated<sup>8</sup> from the dimensional ratio and the elastic constants and found to be about  $1 \times 10^{-6}$ , and was neglected.

### RESULTS

Some of the curves of  $\lambda$  vs  $H'$ , the applied field, are shown in Fig. 1 for various directions of  $H'$  and gauge. These measurements are uncorrected for the effect of the field on the gauge alone. After such correction the saturation  $\lambda$ 's for the  $\lambda \parallel H$  and  $\lambda \perp H$  curves were determined and are given in Fig. 2. The constants are

TABLE I. Description of specimens cut for magnetostriction and anisotropy measurements.

No.	Shape	Dimensions (in.)	Orientation
1	disk	0.592×0.080	(10·0)
2	plate	0.472×0.295×0.078	(00·1), [10·0], [12·0]
3	plate	0.408×0.250×0.021	(11·0), [11·0], [00·1]
4	disk	0.116×0.017	(11·0)
5	disk	0.134×0.032	(10·0)
6	disk	0.136×0.050	(00·1)

derived from these data for Specimen 1 and from other data for Specimen 2 plotted in the same figure.

Measurements were also made of  $\lambda$  at  $45^\circ$  to the hexagonal axis in the (10·0) plane (Specimen 1) when the field was applied in various directions in the plane. Saturation magnetostriction is shown by the crosses in Fig. 3.

The constants derived from the data are:

$$\begin{aligned} \lambda_A &= -45 \times 10^{-6}, & \lambda_B &= -95 \times 10^{-6}, \\ \lambda_C &= +110 \times 10^{-6}, & \lambda_D &= -100 \times 10^{-6}. \end{aligned}$$

With these values, the saturation magnetostriction in the (10·0) plane, measured parallel and perpendicular to the applied field, are:

$$\begin{aligned} \lambda_{\parallel} &= \frac{1}{2} \lambda_A (1 - \cos 2\theta) - \left( \frac{1}{4} \lambda_A - \frac{1}{2} \lambda_D \right) (1 - \cos 4\theta) \\ &= (-61 + 22 \cos 2\theta + 39 \cos 4\theta) \times 10^{-6}, \quad (6) \end{aligned}$$

$$\begin{aligned} \lambda_{\perp} &= \frac{1}{2} \lambda_C (1 + \cos 2\theta) + \left( \frac{1}{4} \lambda_A - \frac{1}{2} \lambda_D \right) (1 - \cos 4\theta) \\ &= (94 + 55 \cos 2\theta - 39 \cos 4\theta) \times 10^{-6}, \quad (7) \end{aligned}$$

as shown by the dotted lines of Fig. 2.

The saturation magnetostriction in the (10·0) plane at  $45^\circ$  to the hexagonal axis, when  $H$  is applied at

<sup>8</sup> W. J. Carr and R. Smoluchowski, Phys. Rev. **83**, 1236 (1951). Note omission of a minus sign in the middle of their Eq. (A-7).

angle  $\theta$ , is

$$\lambda_{45} = \left(\frac{1}{4}\lambda_A + \frac{1}{4}\lambda_C\right)(1 - \cos 2\theta) - \left(\frac{1}{4}\lambda_A + \frac{1}{4}\lambda_C - \lambda_D\right) \sin 2\theta \\ = (16 - 16 \cos 2\theta - 116 \sin 2\theta) \times 10^{-6}. \quad (8)$$

This is the equation for an ellipse, and when plotted on a polar diagram has the form shown by the solid line on the right-hand half of Fig. 3. The major axis of the ellipse is rotated a few degrees from  $\theta = 45^\circ$ .

Calculated curves for  $\lambda_0$  and  $\lambda_{90}$  ( $\theta = 0^\circ$  or  $90^\circ$ ) are also shown there in quadrants 2 and 3.

It is evident from Figs. 2 and 3 that the largest contraction occurs when  $H$  is applied at about  $50^\circ$  to the hexagonal axis, and that it then diminishes as the direction of  $H$  approaches  $90^\circ$ , as was also observed by Nishiyama.<sup>4</sup> It is at first surprising that the maximum magnetostriction is not at  $\theta = 90^\circ$ , as it would be in the simplest possible model in which the strain is represented by an ellipsoid of revolution with its axis

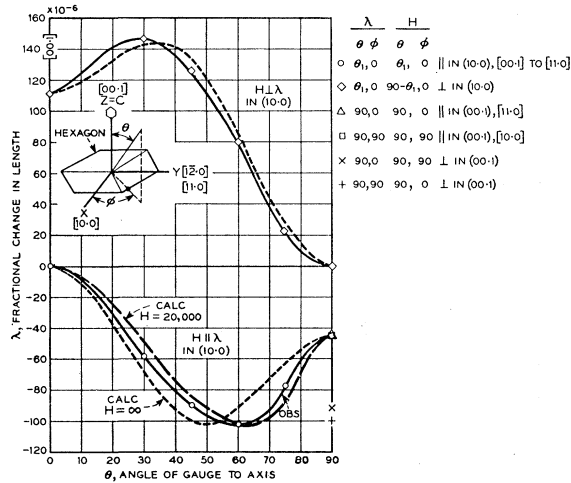


FIG. 2. Magnetostriction when applied field is  $H' = 20\,000$  oersteds, measured  $\parallel$  and  $\perp$   $H'$ , in (10·0) plane.

parallel to the magnetization. This is a true model, however, only when  $\lambda_D = \frac{1}{2}\lambda_A$ .

The maximum contraction now observed at about  $50^\circ$  to the hexagonal axis may be connected with the fact that in the close-packed hexagonal structure of cobalt this is the direction of *next*-nearest neighbors, the direction in which there are no nearest neighbors.

A noticeable feature of the curves of Fig. 1 is the initial large contraction and subsequent expansion of the specimen as shown in curve marked A. The explanation of this, and a calculation that shows good agreement with experiment, are given in the second section following.

#### VOLUME EFFECTS

The fractional change in volume associated with domain rotation is

$$\omega = \lambda_{11} + \lambda_{11}' + \lambda_{11}'', \quad (9)$$

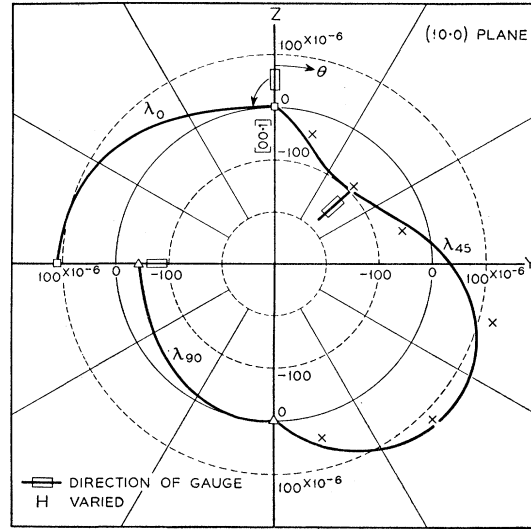


FIG. 3. Quadrants 1 and 4:  $\lambda$  measured at  $45^\circ$  to  $z$ -axis as dependent on strength of field applied at various angles to  $z$ -axis in (10·0) plane. Quadrant 2: Magnetostriction measured  $\parallel$   $z$  axis with  $H$  at various angles to  $z$  axis. Quadrant 3: Same with gauge  $\perp$   $z$  axis.

where  $\lambda_{11}$  and  $\lambda_{11}'$  are measured in the two directions at right angles to the direction of magnetization and to each other. Direct measurement of this change is obtained by combining the data for Specimens 1 and 2. Results are:  $\lambda_{11} = -45 \times 10^{-6}$ ,  $\lambda_{11}' = -93 \times 10^{-6}$  in the (00·1) plane,  $\lambda_{11}'' = +112 \times 10^{-6}$  in the (10·0) plane, so  $\omega = -26 \times 10^{-6}$ . This compares with  $\omega$  calculated<sup>3</sup> from the chosen constants:

$$\omega = (\lambda_A + \lambda_B + \lambda_C) \sin^2 \theta = -30 \times 10^{-6}. \quad (10)$$

This is the first unambiguous determination of a volume change associated with rotation, although a possible change of this kind has been reported<sup>9</sup> in an iron-nickel alloy containing 78 percent nickel.

The calculated volume magnetostriction of polycrystalline cobalt with crystals oriented at random is<sup>3</sup>

$$\bar{\omega} = \frac{2}{3}(\lambda_A + \lambda_B + \lambda_C) = -20 \times 10^{-6}. \quad (11)$$

The isotropic volume change associated with the change of spontaneous magnetization with field can readily be observed in principle by measuring the magnetostriction along the [00·1] axis of easy magnetization, parallel to which the domains lie, because the change in the sense of the magnetization results in zero magnetostriction [Eq. (1) with  $\theta = 0$ ]. Measured changes in this direction, with corrections applied for the effect of the field on the gauge alone, yield the data plotted in the upper part of Fig. 6 (see below). The slope of this line is approximately  $0.2 \times 10^{-9}$ /oersted, so that  $\omega/H = 0.6 \times 10^{-9}$ /oersted. This is consistent with the data of Kornetzki<sup>10</sup> who measured the total

<sup>9</sup> R. M. Bozorth and R. W. Hamming, Phys. Rev. **89**, 865 (1953).

<sup>10</sup> M. Kornetzki, Z. Physik **87**, 560 (1934).

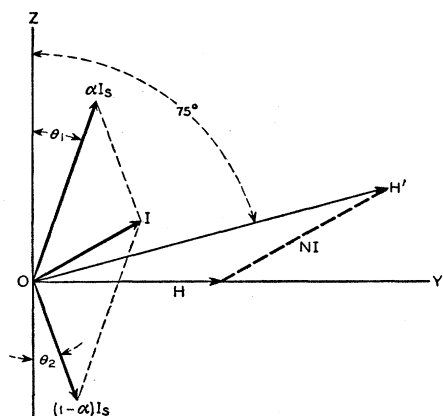


FIG. 4. Vectors used in calculation of domain distribution:  $H'$ , applied field at  $75^\circ$  to  $z$  axis;  $\alpha I_s$ , fraction of magnetization oriented  $\theta_1^\circ$  from  $z$  axis (near side);  $(1-\alpha)I_s$ , fraction at  $\theta_2^\circ$  from  $z$  axis (far side);  $I$ , magnetization of specimen as a whole;  $NI$ , demagnetizing field;  $H$ , true field, calculated to lie  $\parallel z$  axis when  $\alpha < 1$ .

volume change, including that attributable to domain rotation, in fields up to 10 500 oersteds. After an initial contraction, presumably caused by domain rotation, he observed a net volume expansion of  $0.5 \times 10^{-9}$ /oersted when  $H > 9500$ .

The magnitude of the isotropic volume change is not here determined with high accuracy. The sign of the change is such that any error in aligning the gauge parallel to the  $z$  axis would cause the observed change to be too small.

#### FIELD DEPENDENCE OF MAGNETOSTRICTION

The general form of Curve A of Fig. 1 can be understood in terms of the curve of  $\lambda_{45}$  of Fig. 3. A field is applied at  $75^\circ$  to the hexagonal axis, and the gauge measures change in length at  $45^\circ$  to this axis. Then as the field is increased in strength the angle of the

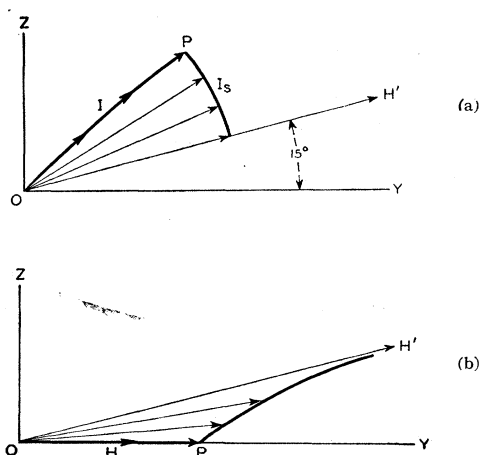


FIG. 5. (a) Increase of magnitude and direction of  $I$  with increase in  $H'$  applied  $75^\circ$  to  $z$  axis. (b) Change of (true) field  $H$  with applied field  $H'$ .

magnetization vector varies from  $\theta = 0^\circ$  to  $75^\circ$ . The maximum contraction takes place, in accordance with Curve  $\lambda_{45}$  of Fig. 3 and Eq. (8), when  $\theta \approx 45^\circ$ . After that the contraction diminishes and  $\lambda$  approaches a limit of  $-28 \times 10^{-6}$ .

A quantitative consideration of the magnitudes involved was then carried out. In order to do this it is necessary to evaluate the various energies involved and then to determine the variables corresponding to minimum total energy.

The energies are (1) the magnetic crystal anisotropy, (2) the mutual energy between field and magnetization, and (3) the energy of demagnetization, which is originally derived from (2). In accordance with Fig. 4 the applied field is  $H'$ , a fraction  $\alpha$  of the domains of magnetization  $I_s$  is oriented at the angle  $\theta_1$  from the "near" hexagonal axis,  $(1-\alpha)$  is at  $\theta_2$  from the "far" axis. It is assumed that  $\alpha = \frac{1}{2}$  at  $H' = 0$  and that as  $H'$  increases some of the domains change from one direction to the other by the movement of domain boundaries between domains inclined to each other originally at  $180^\circ$  and generally at  $180^\circ - \theta_1 - \theta_2$ .

The demagnetizing factor  $N$  is multiplied by the net magnetization  $I$  to obtain the demagnetizing field, and from this and  $H'$  the actual field  $H$  is obtained. The expressions for the energies per unit volume are:

$$\begin{aligned} E_1 &= \alpha(K_1 \sin^2 \theta_1 + K_2 \sin^4 \theta_1), \\ E_2 &= (1-\alpha)(K_1 \sin^2 \theta_2 + K_2 \sin^4 \theta_2), \\ E_3 &= \alpha H' I_s \cos(75^\circ - \theta_1) \\ &\quad + (1-\alpha) H' I_s \cos(75^\circ + \theta_2), \\ E_4 &= \frac{1}{2} N I^2. \end{aligned} \quad (12)$$

The anisotropy constants  $K_1$  and  $K_2$  were measured as described in a later section, using Specimens 4 and 5, with the results  $K_1 = 4.3 \times 10^6$ ,  $K_2 = 1.2 \times 10^6$  ergs  $\text{cm}^{-3}$ . These are close to the values derived<sup>2</sup> from the magnetization curves of Honda and Masumoto.<sup>11</sup> The demagnetizing factor is calculated to be  $N = 1.1$ , assuming it is the same as that for an ellipsoid of the same major and minor axes as the disk.

The values of the parameters  $\alpha$ ,  $\theta_1$ , and  $\theta_2$  for given values of  $H'$  up to  $H' = 20\,000$  oersteds were obtained by Miss M. C. Gray with the help of the analog computer and numerical calculations. By calculation and by analysis she also showed that  $\theta_1 = \theta_2$ .

Calculation of  $H$  from  $H'$  and  $NI$  showed that  $H$  is parallel to the  $y$  axis when  $\alpha < 1$ , and that when  $\alpha = 1$  it approaches the direction of  $H'$  as  $H'$  increases. This is also a consequence of the identity of  $\theta_1$  and  $\theta_2$  because these angles will be equal only when  $H$  makes the same angle with  $\alpha I_s$  and  $(1-\alpha)I_s$ . A similar situation exists in a disk cut from a cubic crystal: the actual field  $H$  tends to be nearer the direction of hard magnetization

<sup>11</sup> K. Honda and H. Masumoto, Science Repts. Tôhoku Imp. Univ. 20, 323 (1931).

than does the applied field  $H'$ , as was shown qualitatively previously.<sup>12</sup>

As a consequence of these results, the direction of magnetization in weak fields can be calculated for a disk with given  $N$ . If the  $z$  axis makes the angle  $\theta_0$  with  $H'$ , and  $\Phi$  with  $I$ , then analysis shows that in weak fields ( $\alpha \approx \frac{1}{2}$ ),

$$\tan \theta_0 = \frac{\frac{1}{2}NI_s^2 + K_1}{\frac{1}{2}NI_s^2} \cdot \tan \Phi. \quad (13)$$

When  $\alpha$  just becomes equal to 1 (point  $P$  in Fig. 5), analysis gives

$$\tan \theta_0 = \frac{(K_1 + K_2 + \frac{1}{2}NI_s^2) \sin 2\Phi - \frac{1}{2}K_2 \sin 4\Phi}{\frac{1}{2}NI_s^2(1 + \cos 2\Phi)}, \quad (14)$$

which reduces to Eq. (13) when  $K_2 = 0$ . The calculated course of the  $I$  vector in Fig. 5(a), is then a straight or slightly curved line over the length  $OP$ . In general the line may be concave upwards or downwards depending on the sign of  $K_2$ .

TABLE II. Values of  $\alpha$  and  $\theta = \theta_1 = \theta_2$ , for specimen 1 with various applied fields.

$H'$ (oersteds)	$\alpha$	$\theta$ (deg)	$H'$ (oersteds)	$\alpha$	$\theta$ (deg)
500	0.542	3.6	6000	1	38.4
1000	0.584	7.2	7000	1	42.3
2000	0.671	14.3	8000	1	45.9
3000	0.766	21.1	10 000	1	51.7
4000	0.874	27.6	15 000	1	60.8
5000	0.999	33.9	20 000	1	65.4

The values of  $\alpha$  and  $\theta = \theta_1 = \theta_2$  for various applied fields, for Specimen 1, are given in Table II. From these values the magnetostriction can easily be calculated from Eq. (8). The resulting  $\lambda_{45}$  vs  $H'$  curve is shown in Fig. 6. The data were taken in a special run made with more care than for Curve A of Fig. 1.

The agreement between calculation and experiment indicates strongly that the assumed mechanism of magnetization is correct. The distribution of the domains among the two directions, which themselves are calculated from the crystal anisotropy and applied field and demagnetizing factor, is thus generally calculable from known expressions for the various energies involved.

### CRYSTAL ANISOTROPY

The crystal anisotropy constants  $K_1$  and  $K_2$  of Eq. (5) were determined by measuring the torque on a disk subjected to a high field in its plane. The most careful measurements were made on the (10·0) disk, Specimen 5, using an applied field of 25 000 oersteds. Data are given as solid circles in Fig. 7.

<sup>12</sup> R. M. Bozorth and H. J. Williams, Phys. Rev. **59**, 827 (1941).

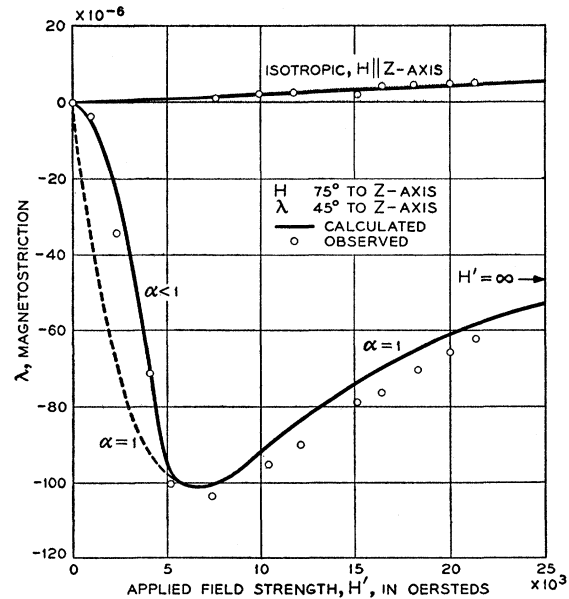


FIG. 6. Magnetostriction measured at  $45^\circ$  to  $z$  axis when field is applied at  $75^\circ$  to  $z$  axis. Points, observed, after correction for effect of gauge alone; solid line, calculated as described in text; broken line, calculated for zero demagnetizing factor. Upper line, isotropic magnetostriction measured with  $H$  and  $\lambda \parallel z$  axis;  $\omega/H = 3\lambda/H = 0.6 \times 10^{-9}$ /oersted.

The sum  $K_1 + K_2$  was put equal to the area under the torque curve from  $\theta = 0$  to  $\theta = 90$ . Correction was then applied to the torque curve for lack of saturation, in accordance with a suggestion of H. J. Williams. Since in reasonably high fields the torque is

$$L = -HI_s \sin \alpha,$$

the angle  $\alpha$  between  $H$  and  $I_s$  can be calculated and the observed  $L$  vs  $\theta$  curve can be corrected by displacing each point along the  $\theta$  axis by the appropriate amount  $\alpha$ . Data so corrected are shown in Fig. 7 by crosses. The maximum value of  $\alpha$  was found to be  $9^\circ$ .

Separate values of  $K_1$  and  $K_2$  were determined by a least-squares fit with the corrected curve. Results are

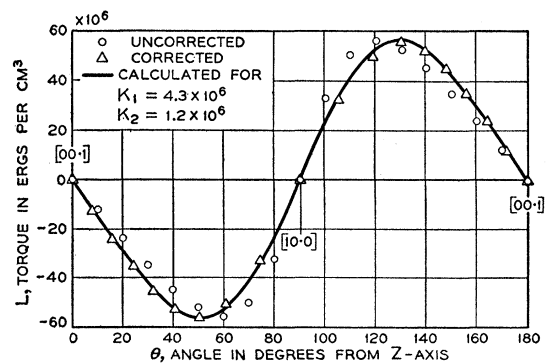


FIG. 7. Torque vs angle with  $z$  axis, in (10·0) plane. Circles, observed; triangles, corrected; line, theoretical with  $K_1 = 4.3 \times 10^6$ ,  $K_2 = 1.2 \times 10^6$  ergs  $\text{cm}^{-3}$ .

$K_1=4.3\times 10^6$ ,  $K_2=1.2\times 10^6$  ergs  $\text{cm}^{-3}$ . The torque curve calculated from these constants, using

$$L=-(K_1+K_2)\sin 2\theta+\frac{1}{2}K_2\sin 4\theta, \quad (15)$$

derived from Eq. (5) with  $K_3=K_4=0$ , is shown by the solid line in the figure. The values previously derived from the magnetization curves of Honda and Masumoto are  $K_1=4.0\times 10^6$ ,  $K_2=2.0\times 10^6$ . A negligible value of  $K_3$  is suggested by the agreement between the corrected torque data and the torques calculated using only  $K_1$  and  $K_2$ .

Measurements on Specimen 6, a disk cut parallel to (00·1), were made to estimate the magnitude of  $K_4$ . A slight  $6\theta$  component was detected, corresponding to

$K_4=+3000$  ergs  $\text{cm}^{-3}$ . This is about  $10^{-3}K_1$  and is considered negligible and uncertain.

The field necessary to saturate cobalt at right angles to the hexagonal axis is calculated to be

$$H_s=(2K_1+4K_2)/I_s=9500.$$

I am indebted to Mr. R. C. Sherwood for preparation of the specimens from the crystals, and to Mrs. E. F. Tilden and Mr. A. J. Williams for assistance with the measurements. My appreciation is also due to Miss M. C. Gray for evaluating the constants used in calculating magnetostriction as dependent on field strength, and to Mrs. G. J. Rowe for setting up and operating the analog computer used in this evaluation.

## Superconductivity of Vanadium at 24 000 Mc/sec

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The high-frequency resistance of vanadium was measured at frequencies in the vicinity of 24 000 Mc/sec in both the normal and superconducting states using resonant cavity techniques.

The normal conductivity was not affected by the "anomalous skin effect" as much as the "soft superconductors." The inner surfaces of the cavities were mechanically polished to a good surface finish. The normalized surface resistance extrapolated to absolute zero yields a value of 0.007 times its initial resistance.

### INTRODUCTION

**S**UPERCONDUCTIVITY of vanadium is of interest because this metal belongs to the class of "hard superconductors." Most of the metals in this class do not exhibit a sharp transition between the normal and superconducting state. The "soft superconductors" almost always exhibit an extremely sharp transition. The behavior of vanadium at high frequencies is of interest since very little work has been done on the "hard superconductors" at high frequencies. McLennan<sup>1</sup> and co-workers investigated tantalum at 10

Mc/sec many years ago and found an abnormally high resistance.

### APPARATUS AND TECHNIQUES

The present work was done at frequencies near 24 000 Mc/sec. The method used was to measure the electric  $Q$  factor of a resonant cavity and from these measurements calculate the surface resistance. The method of  $Q$  measurements and techniques<sup>2</sup> are described in an earlier paper and will not be discussed here. In the earlier paper the cavities operated in the  $TE_{111}$  mode, in this particular case the  $TE_{114}$  mode is used. The ratio of diameter to length is 1.0. The cavity was formed from two pieces that were machined from a vanadium ingot. The junction of the two parts was along a plane which has no transverse currents flowing. The interior surface of the cavity was polished mechanically to a good finish. The polishing compounds used were: first, No. 275 carborundum, then No. 400 carborundum and the final polishing was done with Linde "b" polishing compound using distilled water and aerosol as a wetting agent. Because of the difficulty of attaching vanadium to standard wave guides a special holder was constructed of brass. A gold O ring was used to seal the two parts of the holder. This arrangement was satisfactory in the normal helium region.

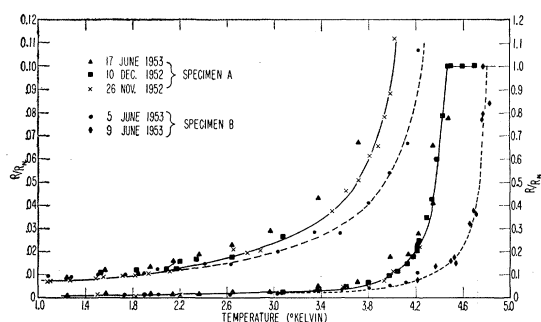


FIG. 1. Surface resistance of vanadium in superconducting state.

<sup>1</sup> McLennan, Burton, Pitt, and Wilhelm, Proc. Roy. Soc. (London) **136**, 52 (1931).

<sup>2</sup> C. J. Grebenkemper and J. P. Hagen, Phys. Rev. **86**, 673 (1952).